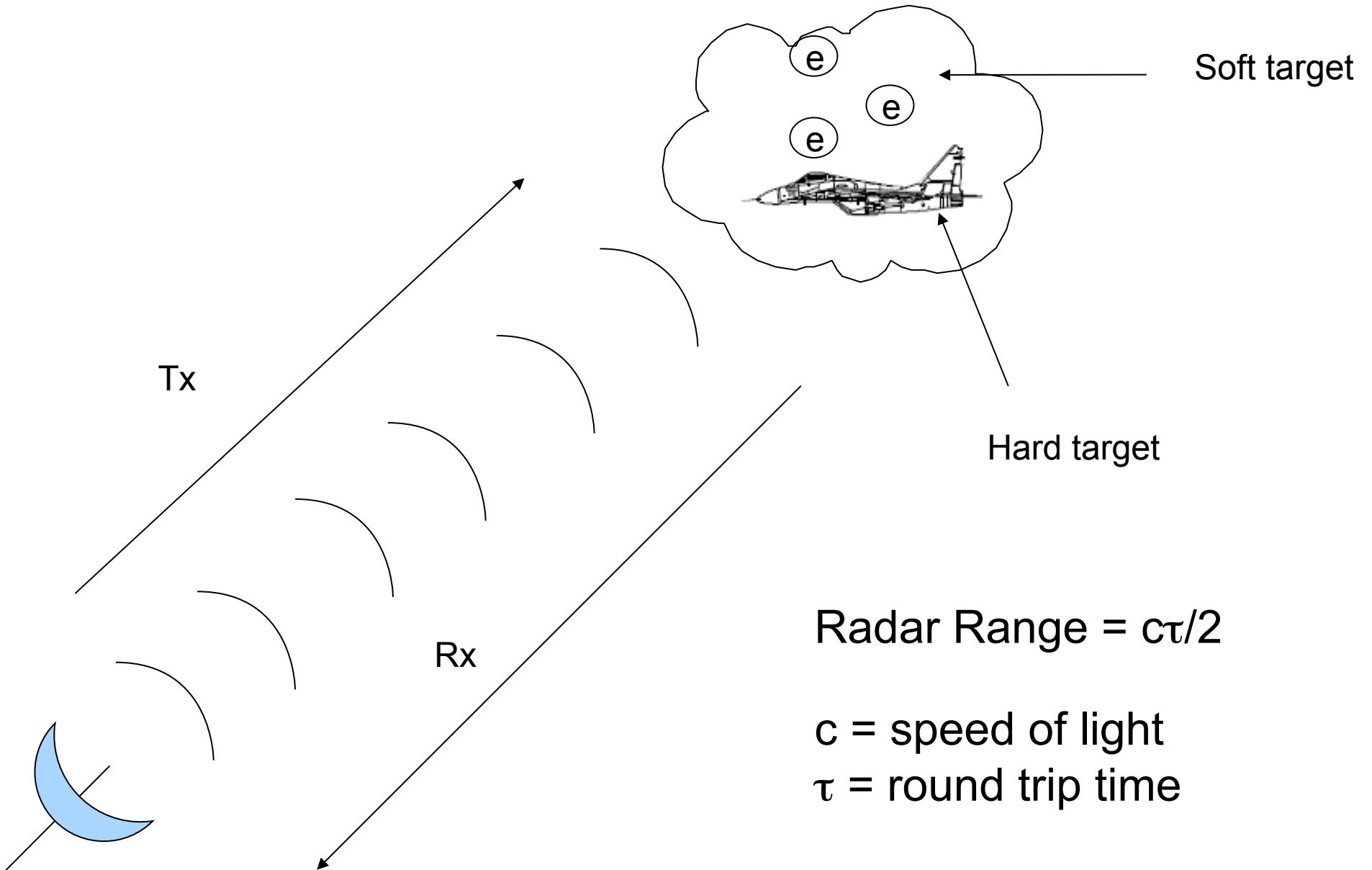


# RAdio Detection And Ranging



$$\text{Radar Range} = c\tau/2$$

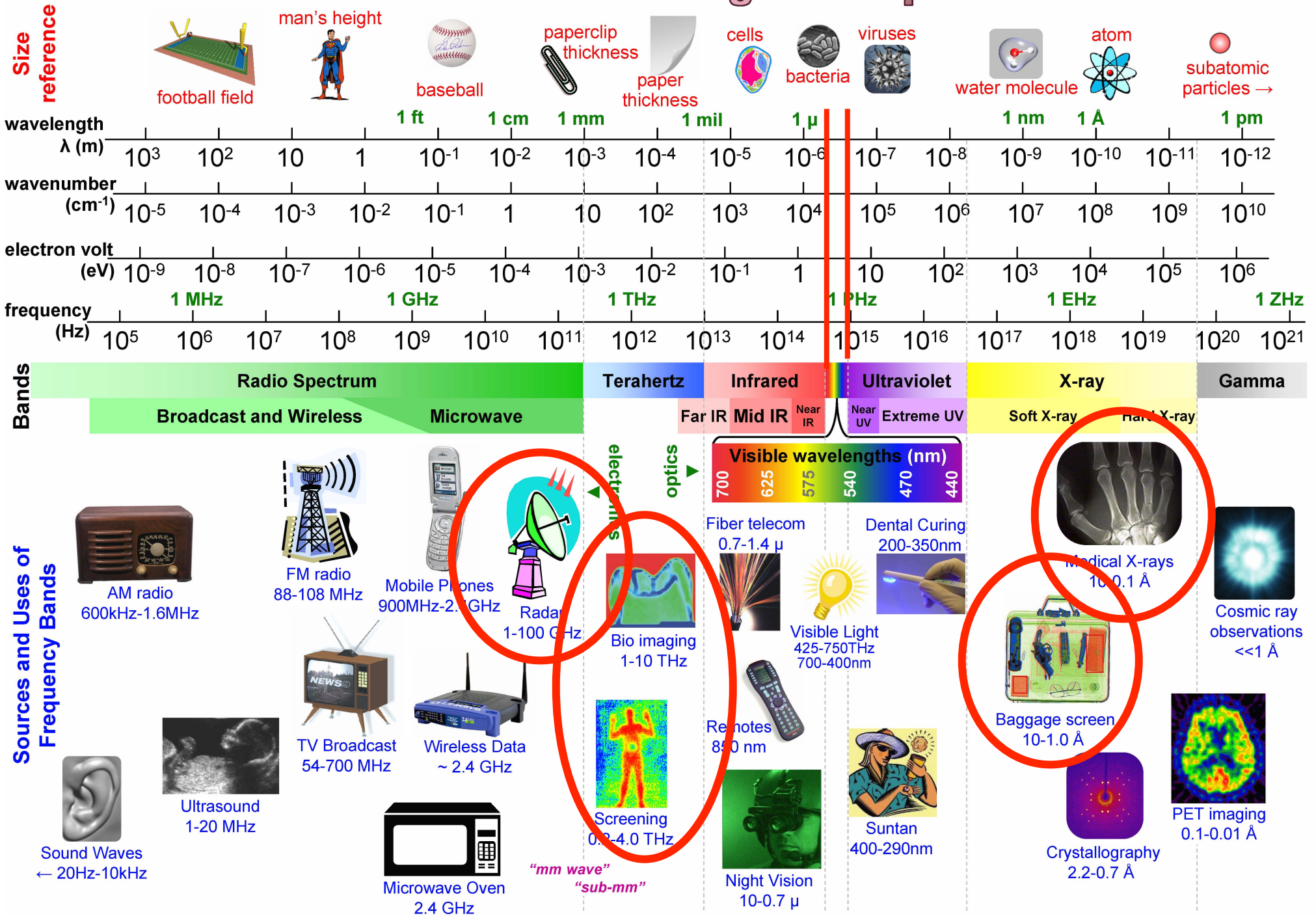
$c$  = speed of light

$\tau$  = round trip time

# Outline - Radar Basics

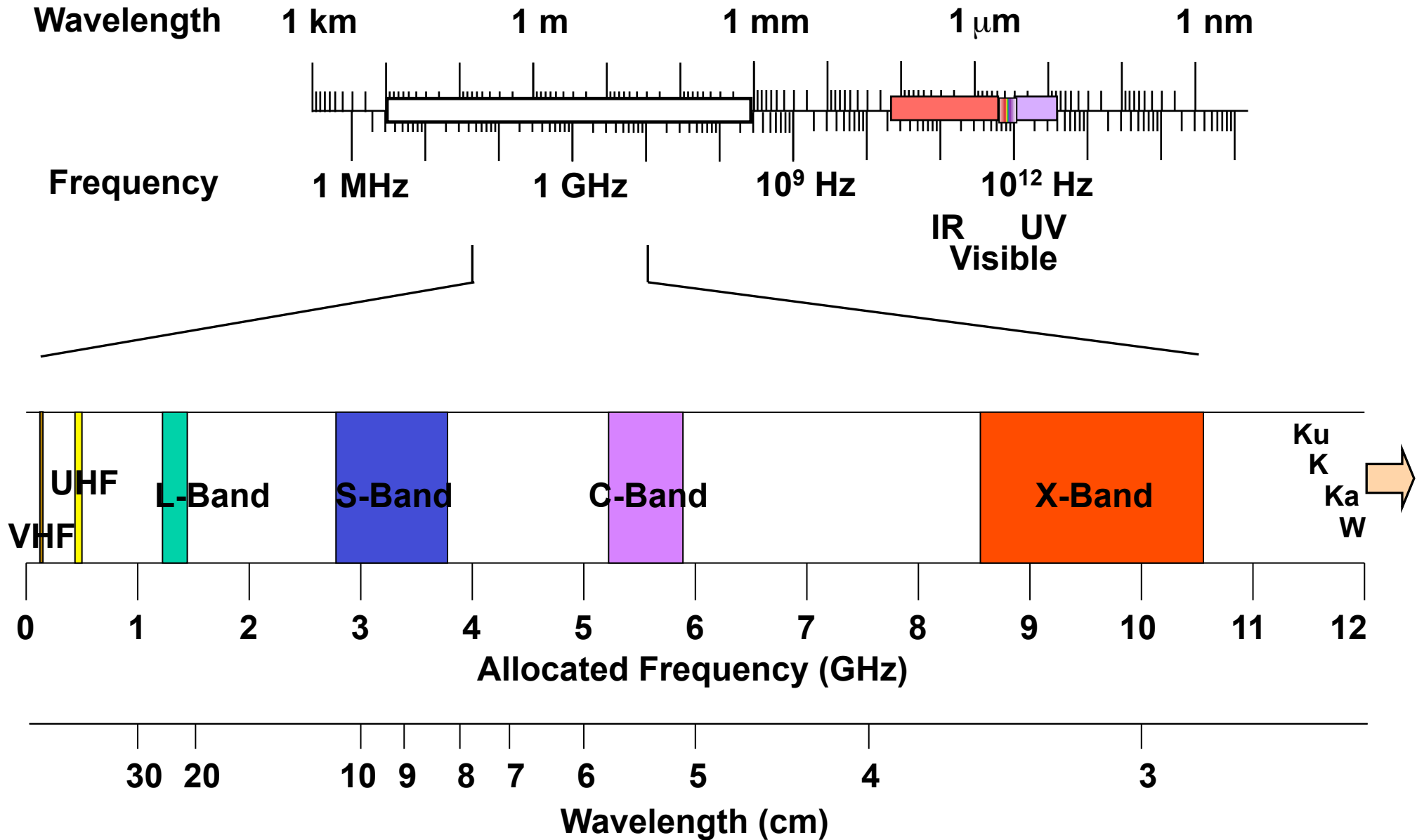
- Electromagnetic spectrum
- Radio Waves and Propagation
- Radar fundamentals
  - Radar equation
  - Range Resolution and pulsed radars
- Doppler

# Chart of the Electromagnetic Spectrum



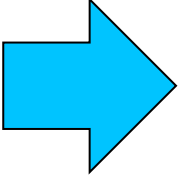
$$\lambda = 3 \times 10^8 / \text{freq} = 1 / (\text{wn} * 100) = 1.24 \times 10^{-6} / \text{eV}$$

# Radar Frequency Bands



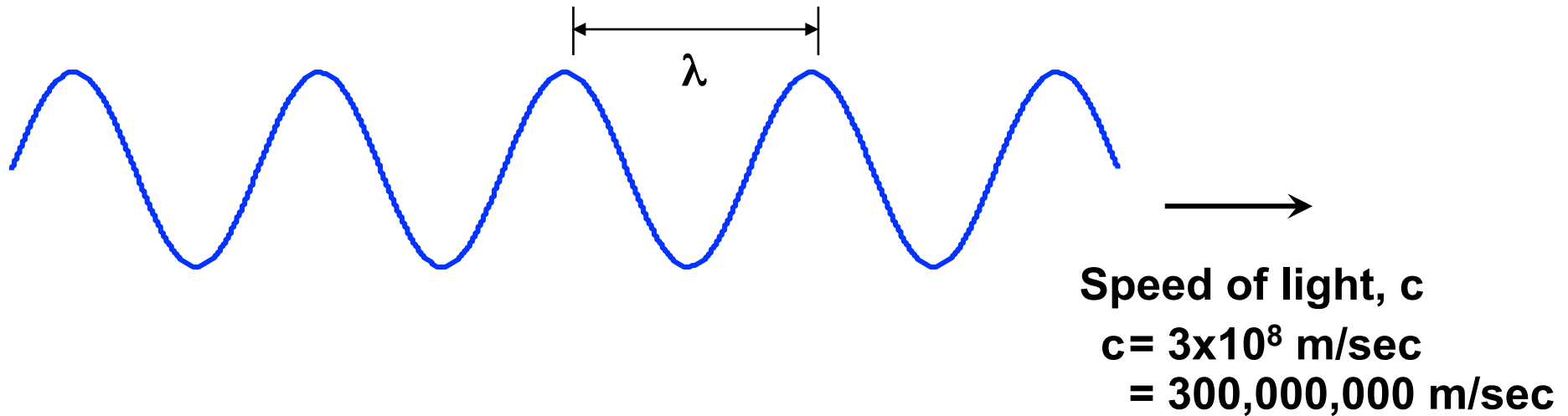


# Outline - Radar Basics

- Electromagnetic spectrum
- • Radio Waves and Propagation
- Radar fundamentals
  - Radar equation
  - Range Resolution and pulsed radars
- Doppler

# Properties of Waves

## Relationship Between Frequency and Wavelength



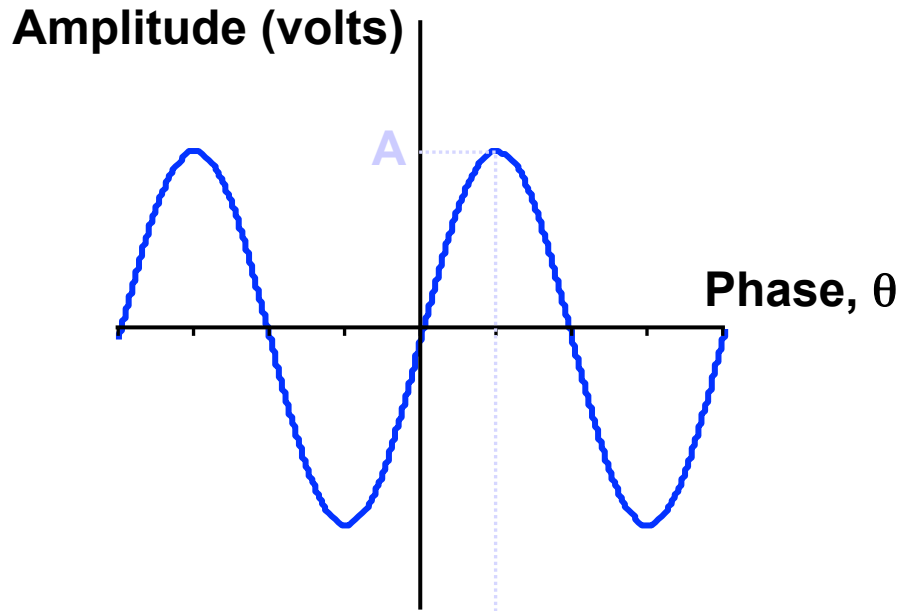
$$\text{Frequency (1/s)} = \frac{\text{Speed of light (m/s)}}{\text{Wavelength } \lambda \text{ (m)}}$$

Examples:

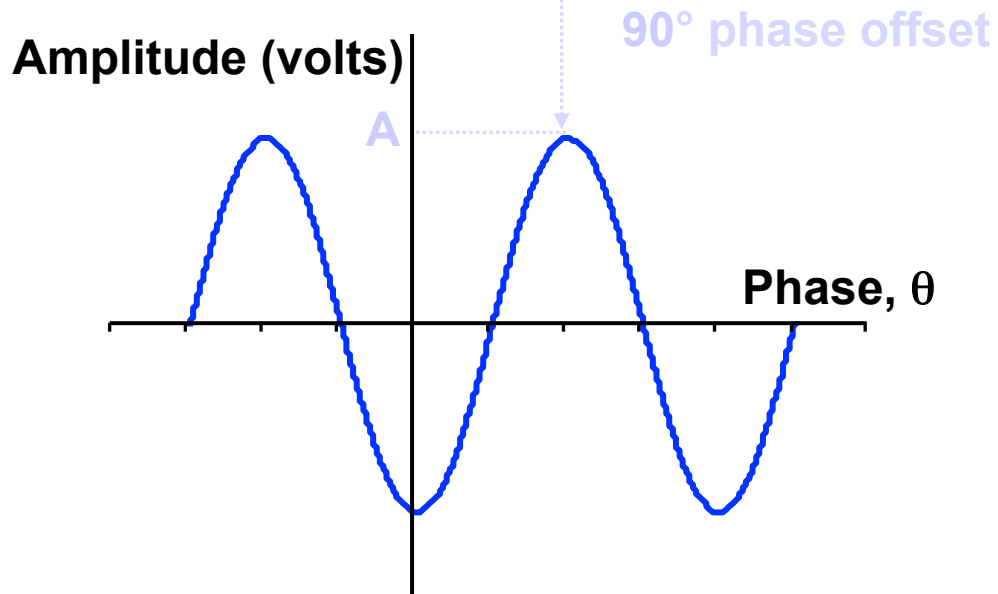
<u>Frequency</u>	<u>Wavelength</u>
100 MHz	3 m
1 GHz	30 cm
3 GHz	10 cm
10 GHz	3 cm

# Properties of Waves

## Phase and Amplitude



$$A \sin(\theta)$$

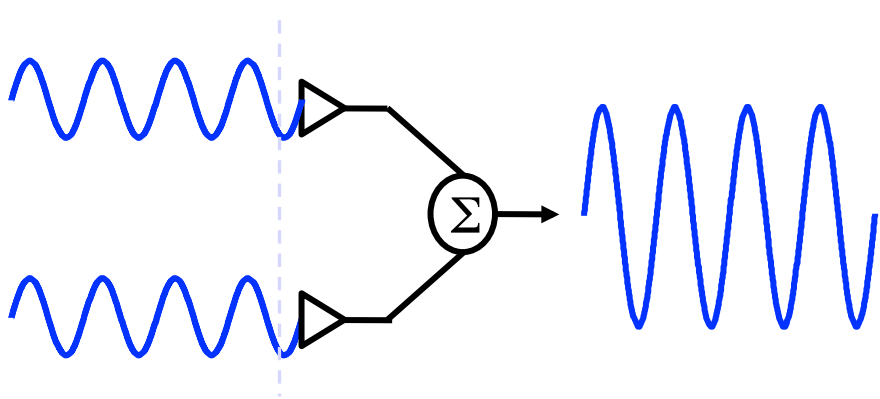


$$A \sin(\theta - 90^\circ)$$

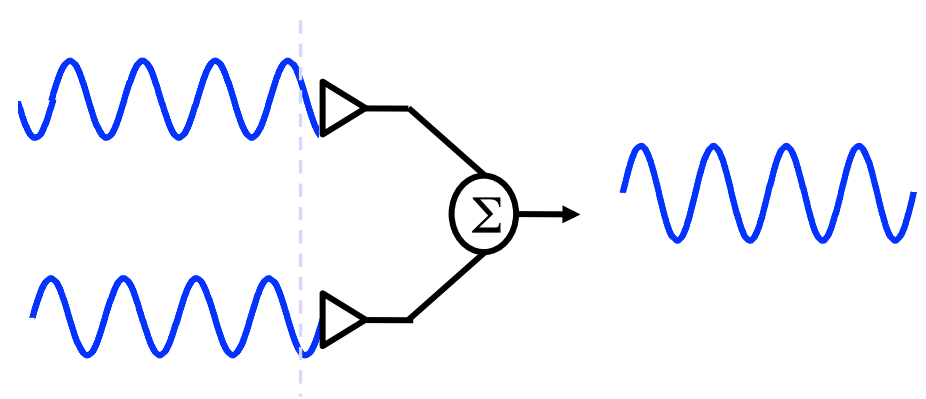


# Properties of Waves

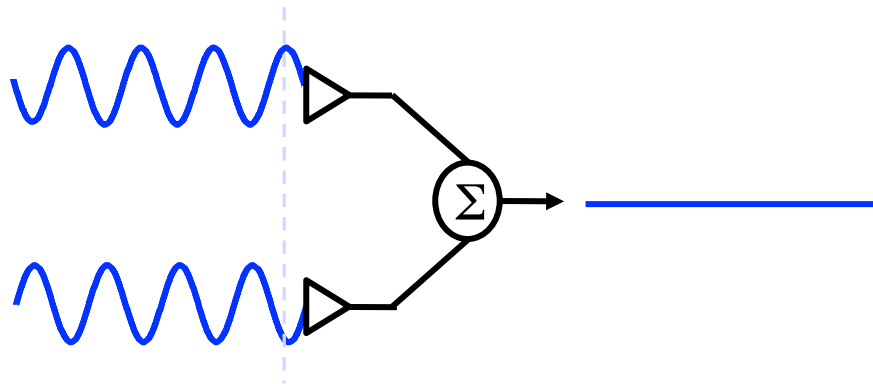
## Constructive vs. Destructive Addition



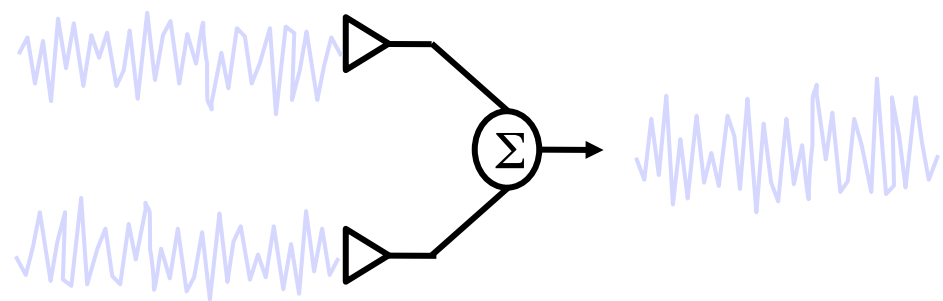
**Constructive**  
(in phase)



**Partially Constructive**  
(somewhat out of phase)



**Destructive**  
(180° out of phase)



**Non-coherent signals**  
(noise)

# Radio Waves

$$y(x, t) = A \cos(\omega t - kx + \phi_0)$$

Angular Frequency

$$\omega = 2\pi f = 2\pi/T$$

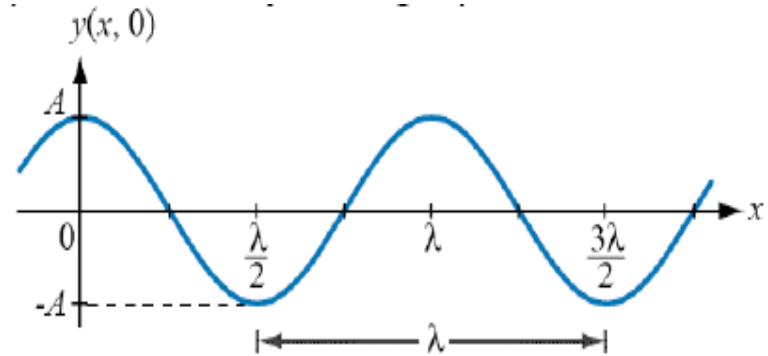
Wavenumber

$$k = 2\pi/\lambda$$

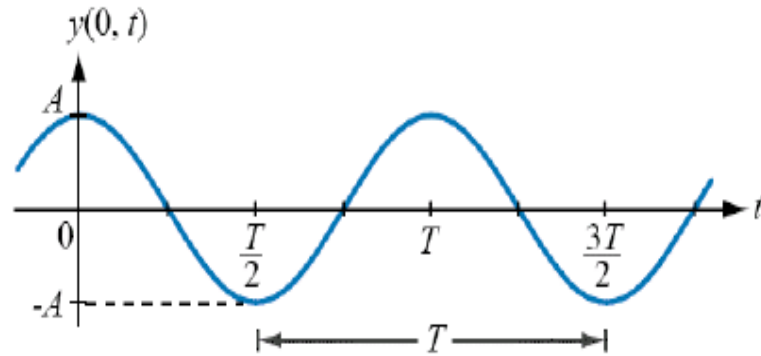
Wave phase velocity

$$c = f\lambda = \omega/k = 3 \times 10^8 \text{ m/s}$$

$$\text{Frequency (1/s)} = \frac{\text{Speed of light (m/s)}}{\text{Wavelength } \lambda \text{ (m)}}$$



(a)  $y(x, t)$  versus  $x$  at  $t = 0$



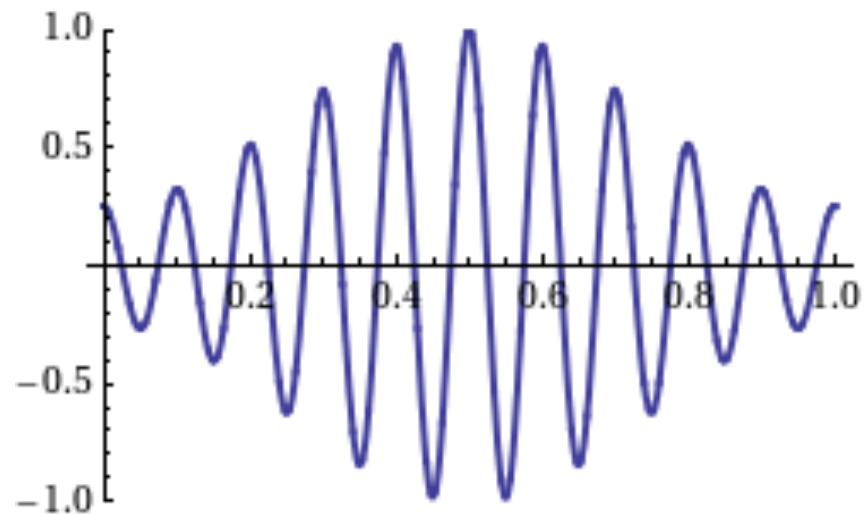
(b)  $y(x, t)$  versus  $t$  at  $x = 0$

# Phase Velocity, Group Velocity, Index of Refraction

$$v_p = \frac{\omega}{k}$$

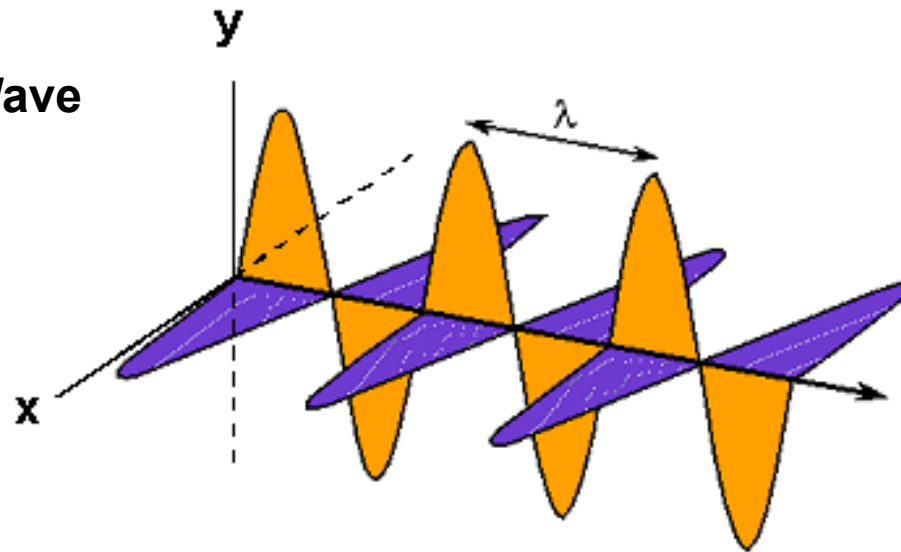
$$v_g \equiv \frac{\partial \omega}{\partial k}$$

$$n = \frac{c}{v_p}$$



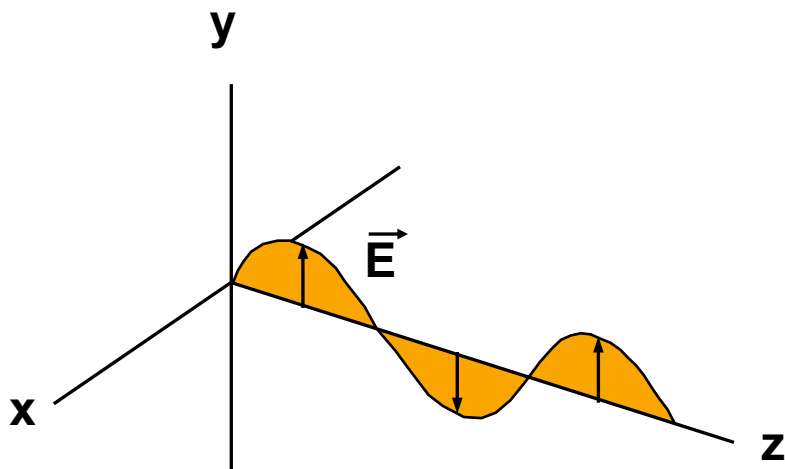
# Polarization

Electromagnetic Wave

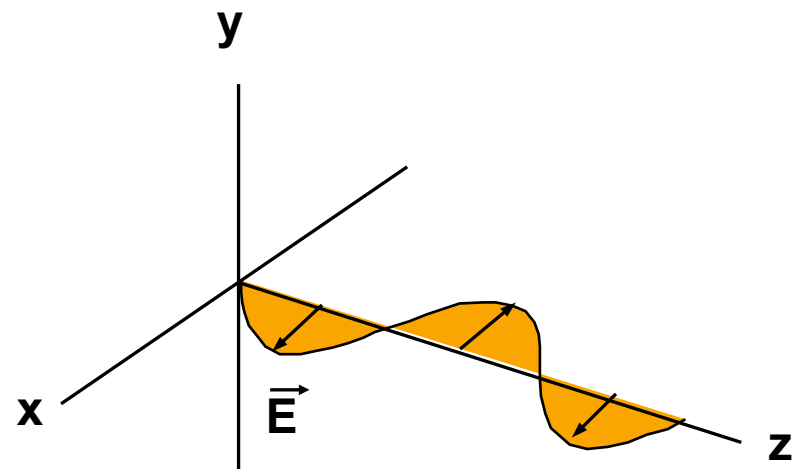


-  Electric Field
-  Magnetic Field

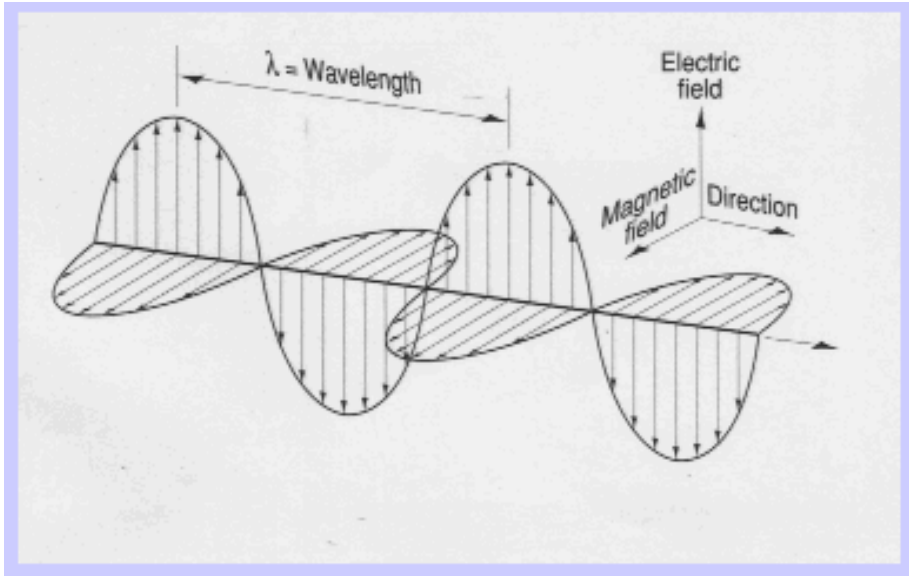
Vertical Polarization



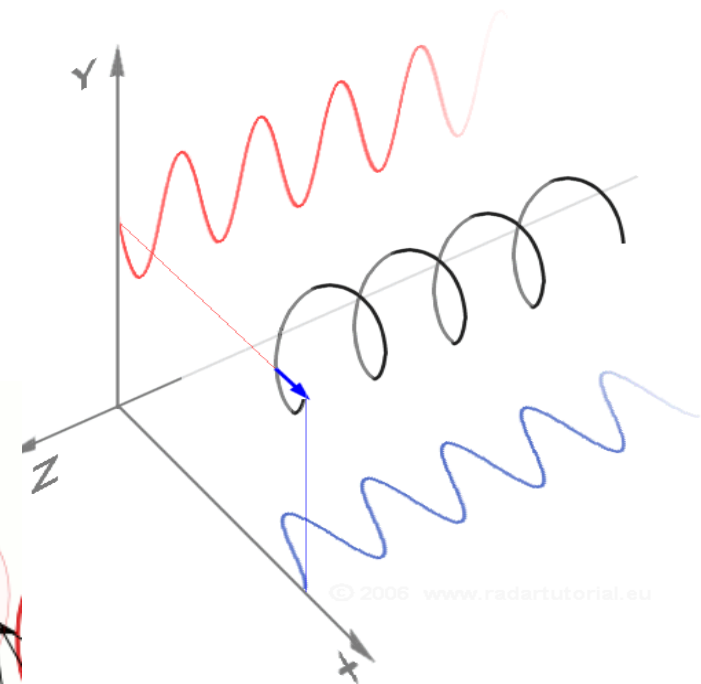
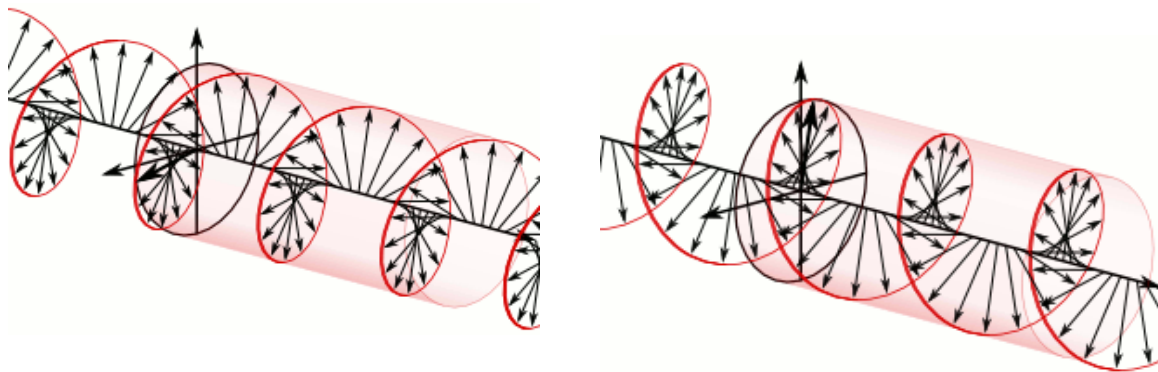
Horizontal Polarization



# TEM Waves: *Transverse electromagnetic (TEM) modes* neither electric nor magnetic field in the direction of propagation



Electromagnetic waves in free space propagate in TEM mode

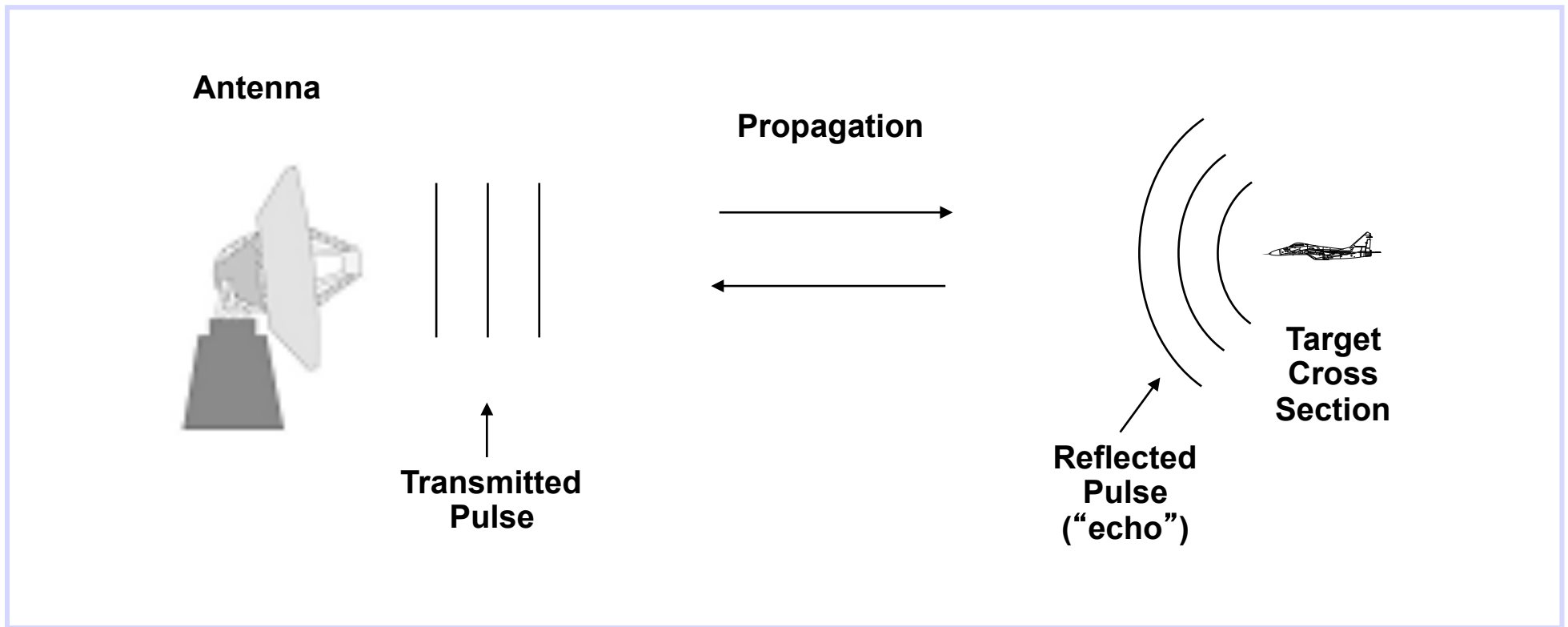


# Outline - Radar Basics

- Electromagnetic spectrum
- Radio Waves and Propagation
- Radar fundamentals
  - Radar equation
  - Range Resolution and pulsed radars
- Doppler

# RADAR

## RA**D**io **D**etection And **R**anging



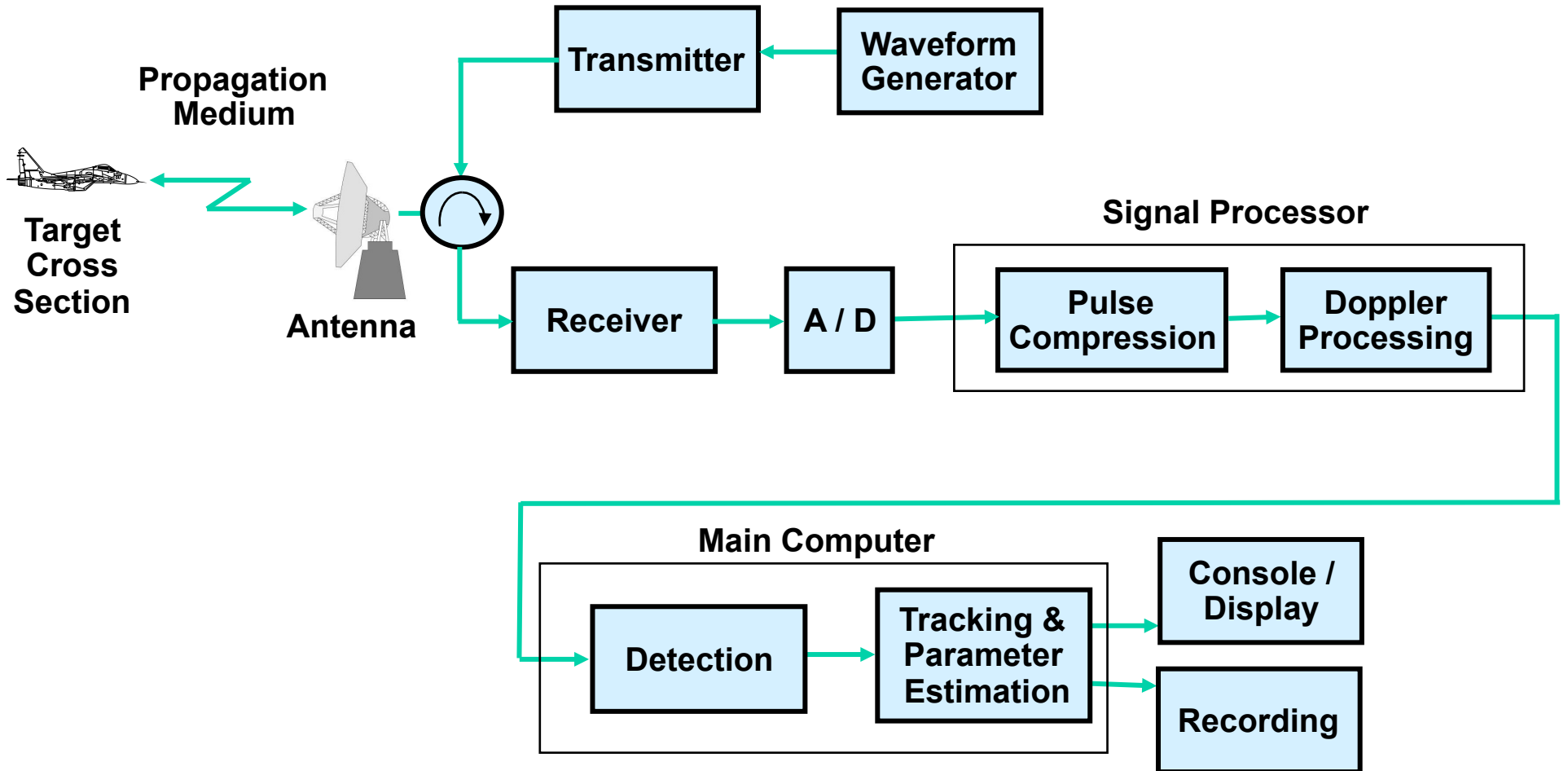
### Radar observables:

- Target range
- Target angles (azimuth & elevation)
- Target size (radar cross section)
- Target speed (Doppler)
- Target features (imaging)

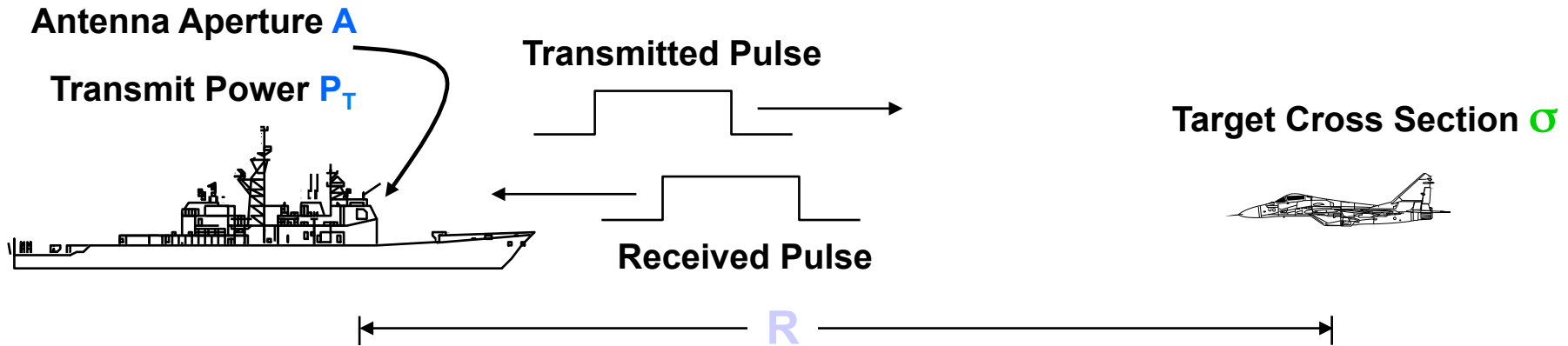
# Demonstrate radar



# Radar Block Diagram



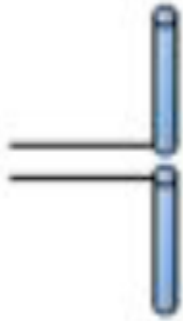
# Radar Range Equation



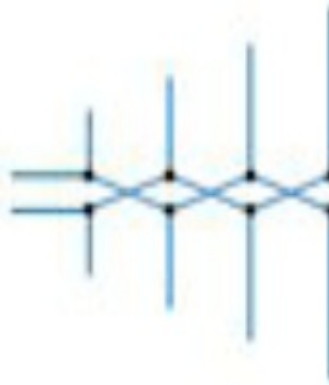
	Transmit Power	Transmit Gain	Spread Factor	Losses	Target RCS	Spread Factor	Receive Aperture	Dwell Time
Received Signal Energy	$[ P_T ]$	$\left[ \frac{4\pi A}{\lambda^2} \right]$	$\left[ \frac{1}{4\pi R^2} \right]$	$\left[ \frac{1}{L} \right]$	$[ \sigma ]$	$\left[ \frac{1}{4\pi R^2} \right]$	$[ A ]$	$[ \tau ]$

# Antennas

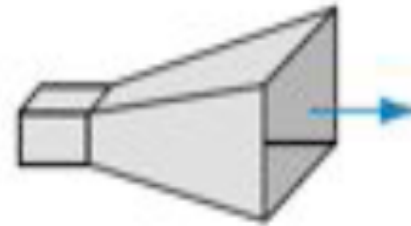
Most basic form of antennas – a wire element with a time varying current flowing in it



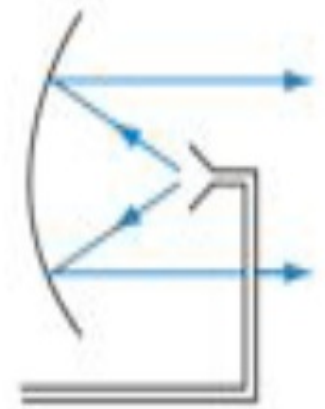
Dipole antenna



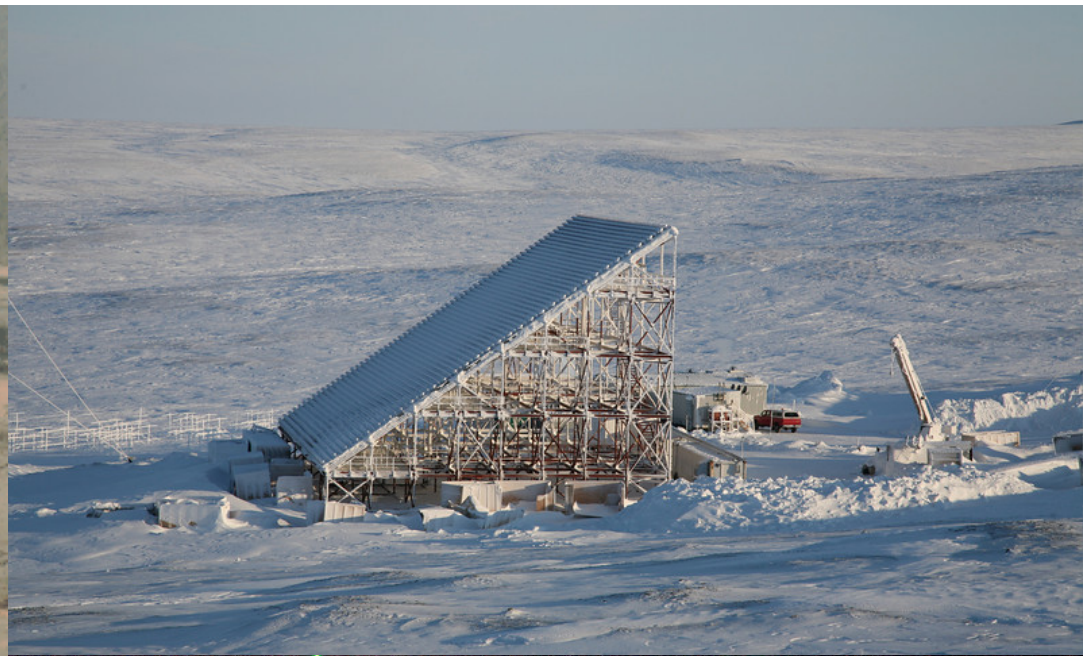
Log Periodic



Horn antenna

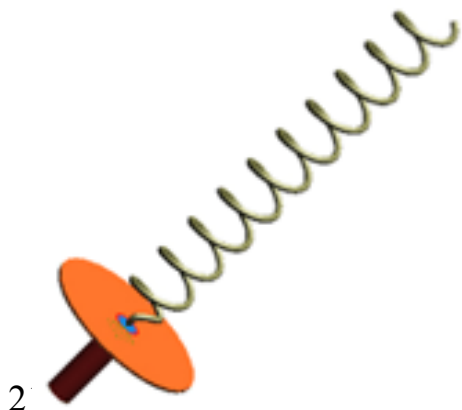


Parabolic dish  
Reflector antenna

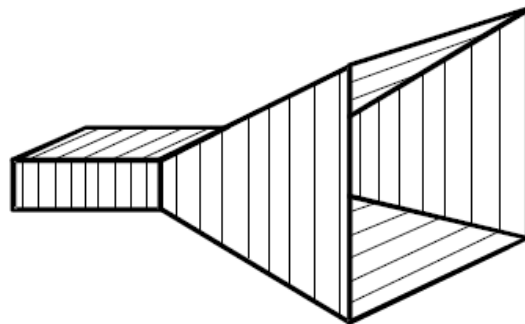


# Antennas

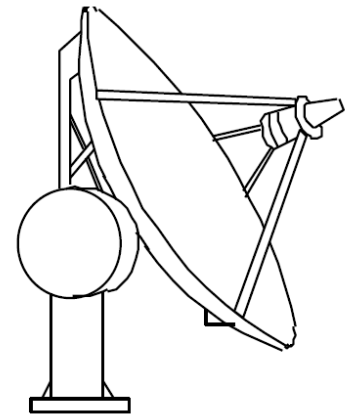
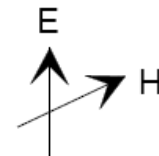
- Four primary functions of an antenna for radar applications
  - Impedance transformation (free-space intrinsic impedance to transmission-line characteristic impedance)
  - Propagation-mode adapter (free-space fields to guided waves)
  - Spatial filter (radiation pattern – direction-dependent sensitivity)
  - Polarization filter (polarization-dependent sensitivity)



2 Helical antenna



Horn antenna



Parabolic reflector antenna

# Impedance transformer

• Intrinsic impedance of free-space,  $\eta_0 \equiv E/H$  is

$$\eta_0 = \sqrt{\mu_0/\epsilon_0} = 120 \pi \cong 376.7 \Omega$$

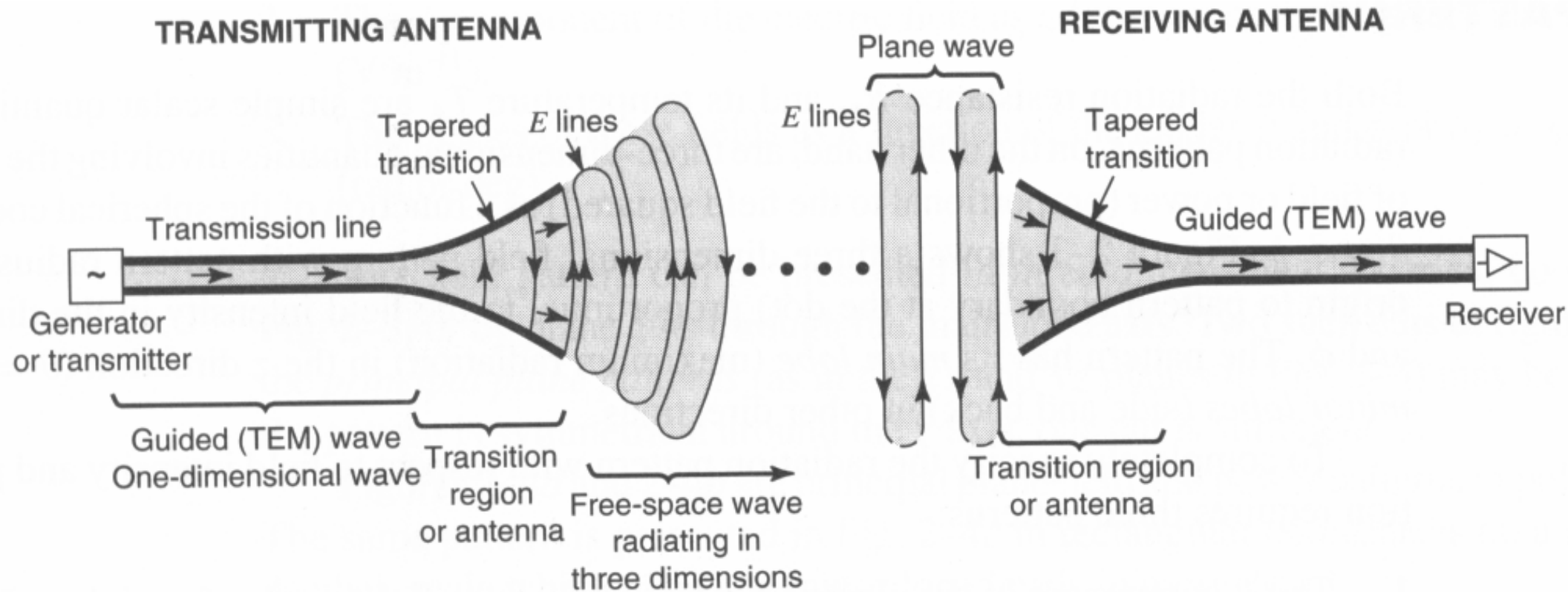
• Characteristic impedance of transmission line,  $Z_0 = V/I$

• A typical value for  $Z_0$  is  $50 \Omega$ .

• Clearly there is an impedance mismatch that must be addressed by the antenna.

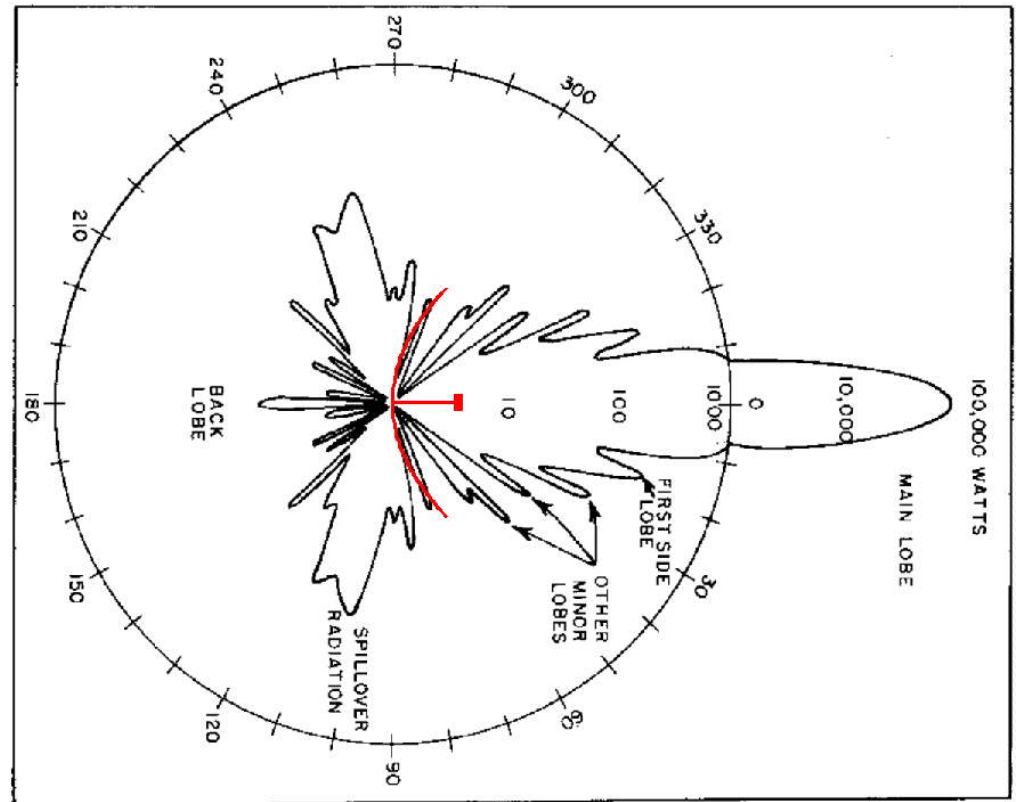
# Propagation-mode adapter

.During both transmission and receive operations the antenna must provide the transition between these two propagation modes.



# Spatial filter

Antennas have the property of being more sensitive in one direction than in another which provides the ability to spatially filter signals from its environment.

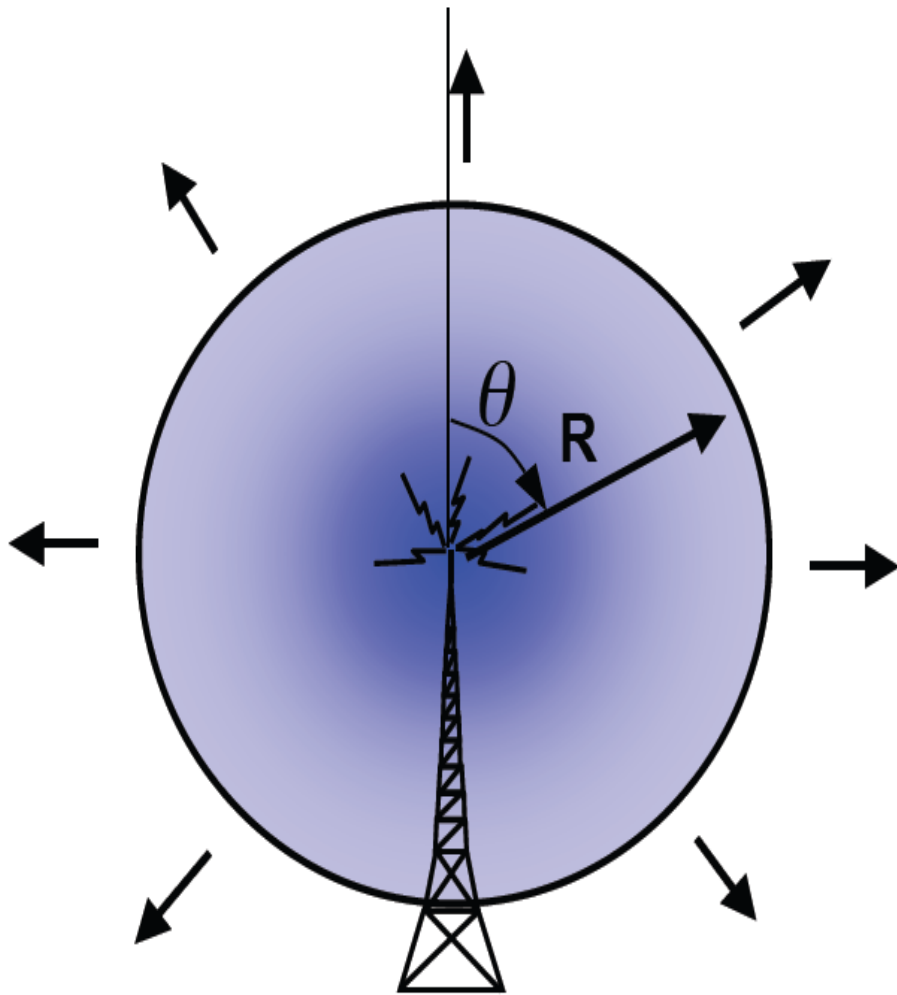


Radiation pattern of directive antenna.

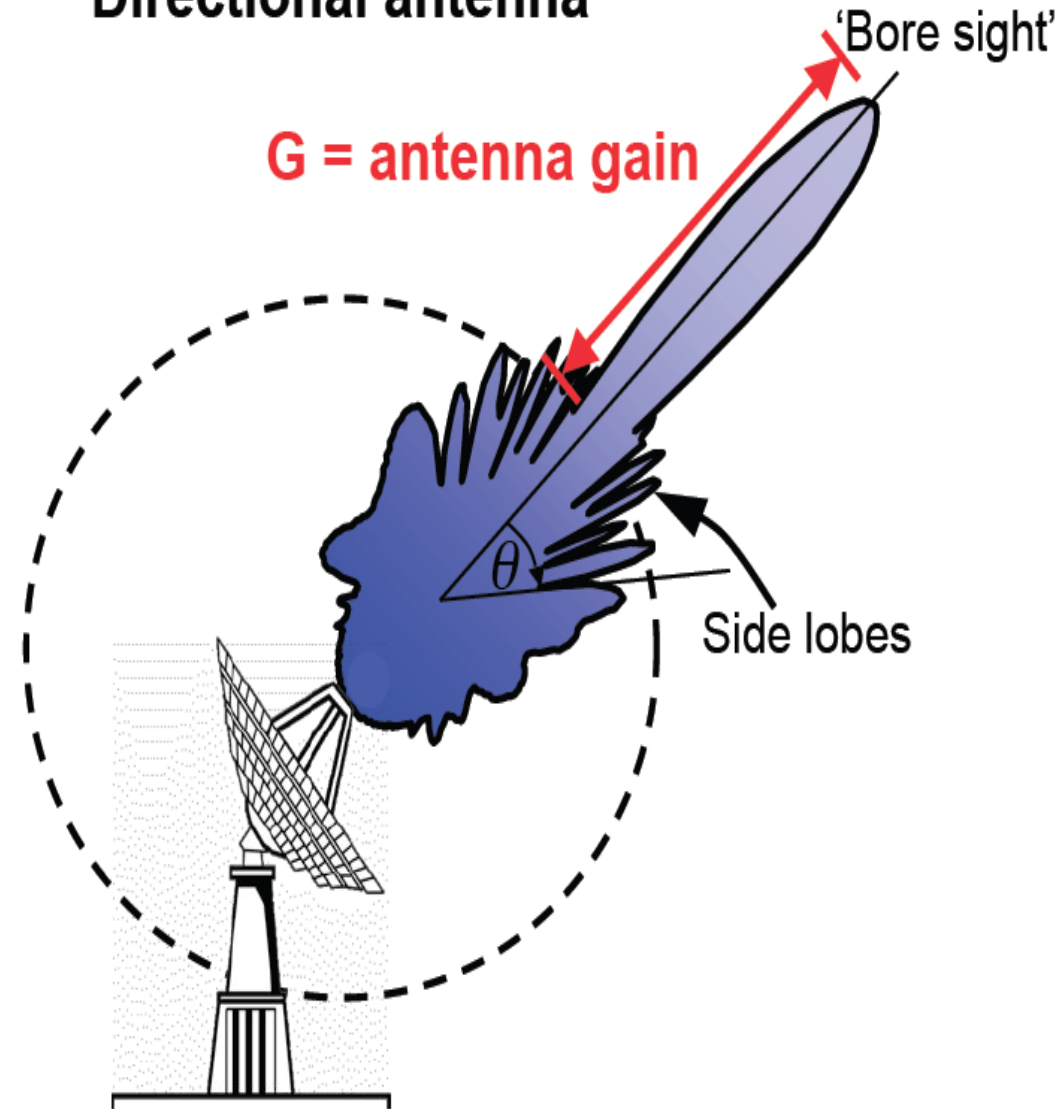


# Radiation Pattern – Antenna Gain

Isotropic antenna



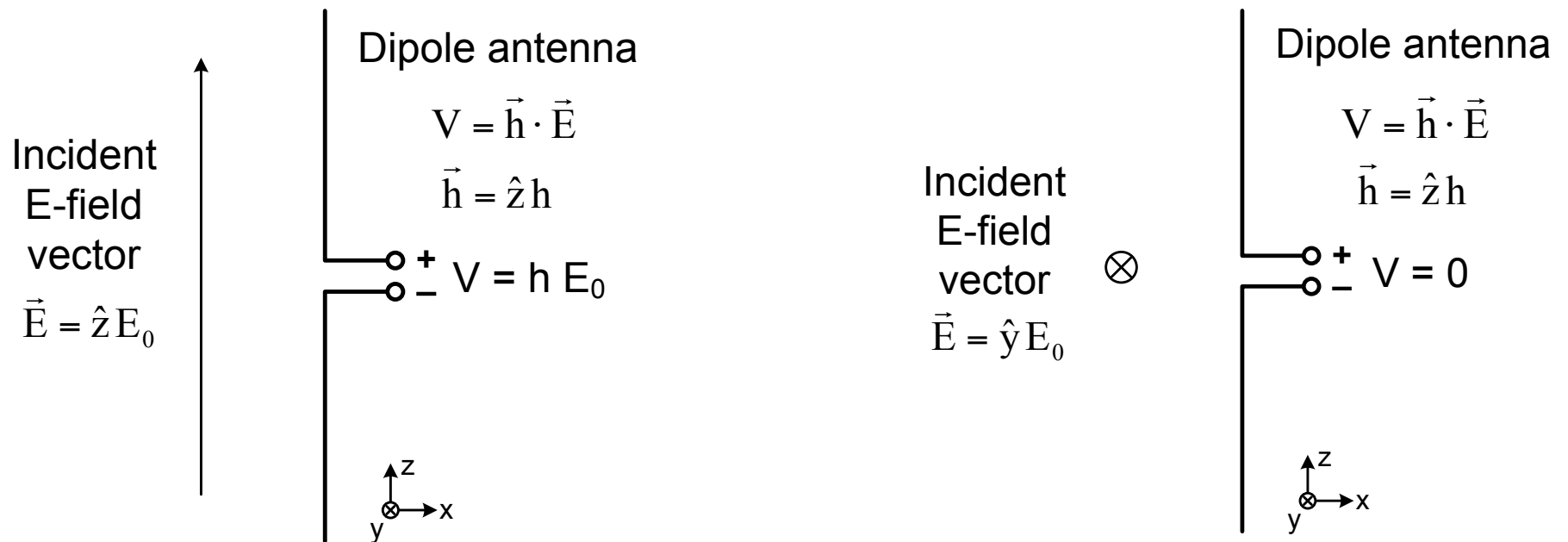
Directional antenna



Write equation for Gain on  
blackboard

# Polarization filter

Antennas have the property of being more sensitive to one polarization than another which provides the ability to filter signals based on its polarization.



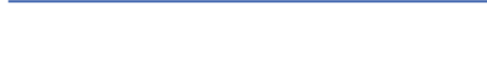
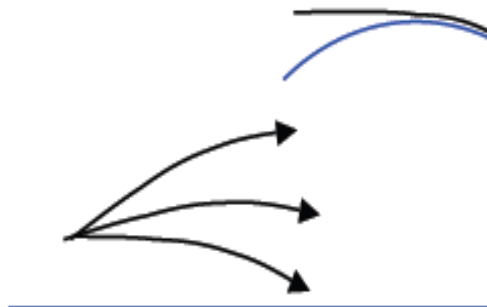
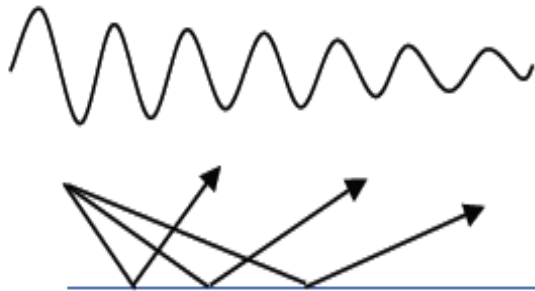
In this example,  $h$  is the antenna's effective height whose units are expressed in meters.

# Propagation Medium - Losses

Radio waves are affected by the medium they propagate in. Effects dependent on the refractive index of the medium and wave frequency

Radio waves are also reflected off of the surface

- Atmospheric attenuation
- Reflection off of earth's surface
- Over-the-horizon diffraction
- Atmospheric refraction



Attenuation usually measured in dB

$$\text{SNR dB} = 10 \log_{10} \frac{\text{signal power}}{\text{noise power}}$$

Example:

<u>Factor of:</u>	<u>Scientific Notation</u>	<u>dB</u>
10	$10^1$	10
100	$10^2$	20
1000	$10^3$	30
⋮		
1,000,000	$10^6$	60

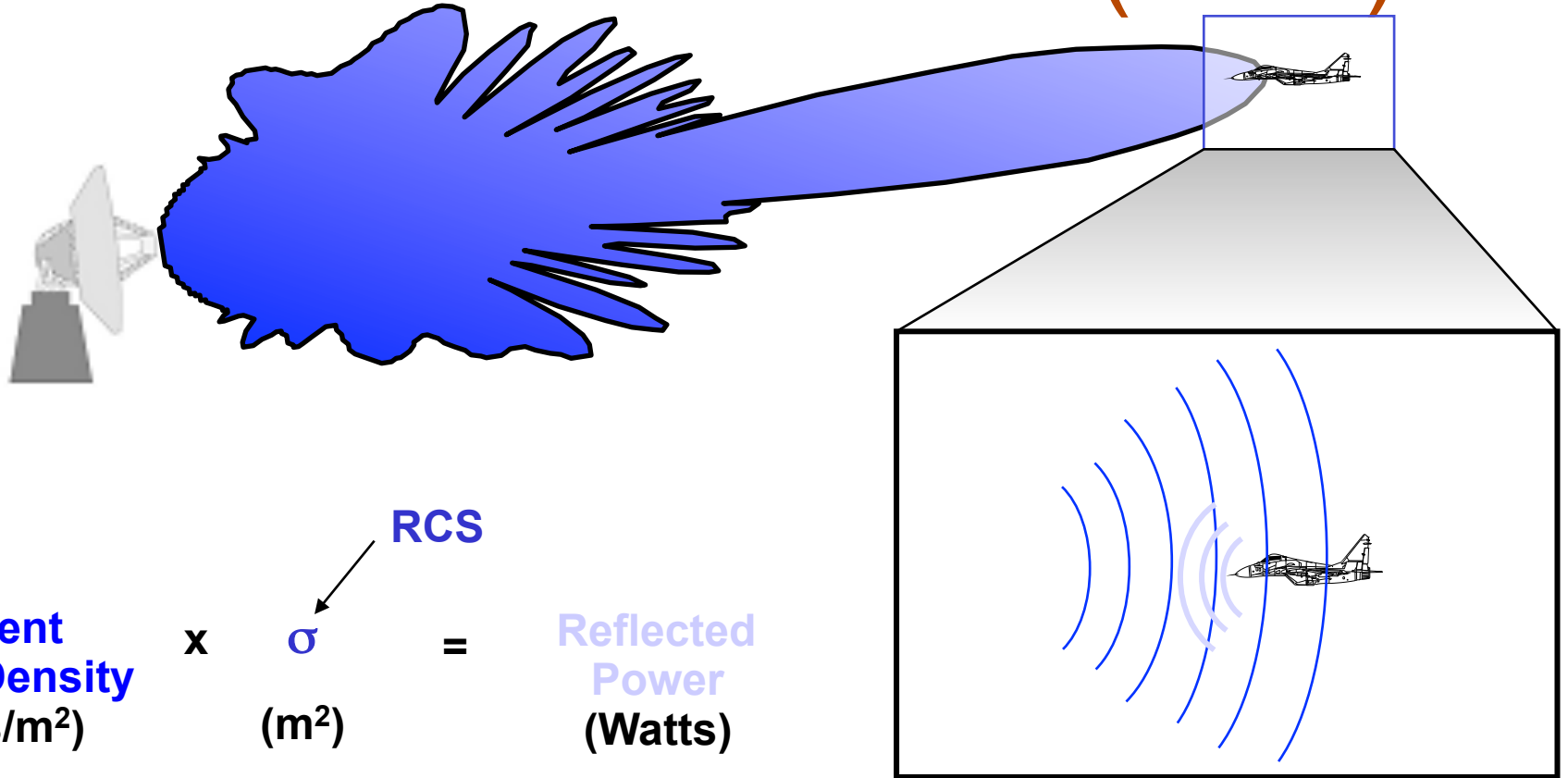
# Radar equation

$$\begin{array}{l} \text{Received Signal} \\ \text{Energy} \end{array} = \begin{array}{ccccccc} \text{Transmit} & \text{Transmit} & \text{Spread} & \text{Losses} & \text{Target} & \text{Spread} & \text{Receive} & \text{Dwell} \\ \text{Power} & \text{Gain} & \text{Factor} & & \text{RCS} & \text{Factor} & \text{Aperture} & \text{Time} \\ [ P_T ] & \left[ \frac{4\pi A}{\lambda^2} \right] & \left[ \frac{1}{4\pi R^2} \right] & \left[ \frac{1}{L} \right] & [ \sigma ] & \left[ \frac{1}{4\pi R^2} \right] & [ A ] & [ \tau ] \end{array}$$

Radar cross section tells us about the target properties

It is the effective target cross section as seen by the radar

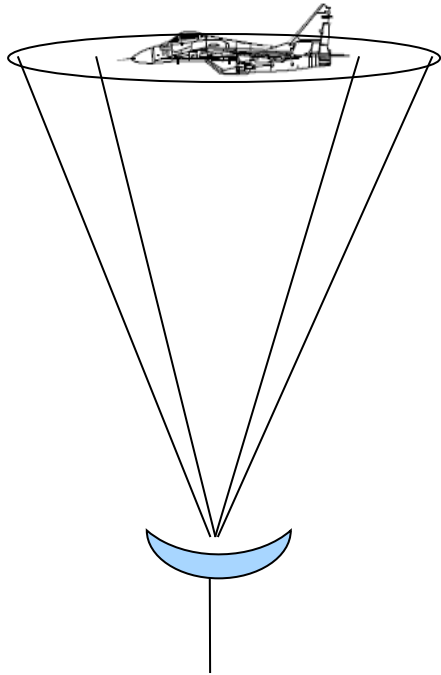
# Radar Cross Section (RCS)



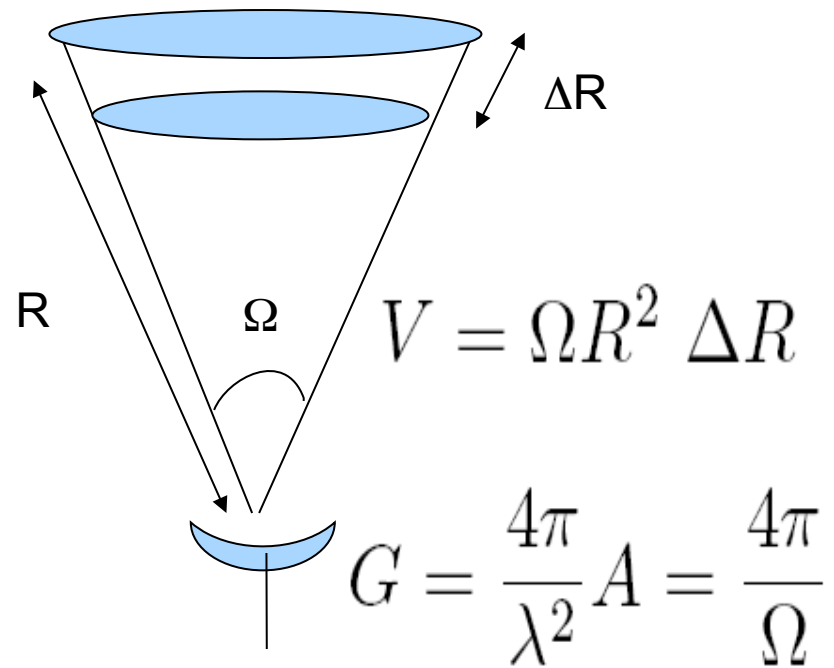
Radar Cross Section (RCS, or  $\sigma$ ) is the effective cross-sectional area of the target as seen by the radar

measured in m<sup>2</sup>, or dBm<sup>2</sup>

# Hard targets vs. Soft targets



vs.



$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$



$$P_r = \frac{P_t A \sigma_v \Delta R}{4\pi R^2}$$

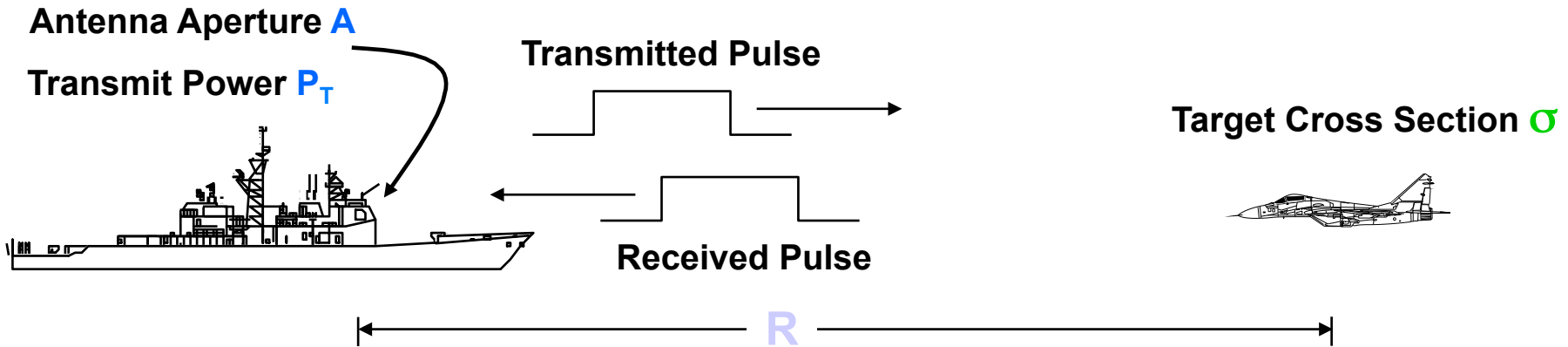


# Volume scattering - Ionosphere

- Volume scattering cross section  $\sigma_v$  has area/volume units
- Signal is proportional to range resolution
- What about the ionosphere ?
- Cross section of a single electron =  $10^{-28} \text{ m}^2$
- Cross section of a bunch of electrons in a  $10 \text{ km}^3$  volume in the ionosphere assuming electron density =  $10^{12} /\text{m}^3$ , is  $10^{10} \times 10^{12} \times 10^{-28} = 10^{-6} \text{ m}^2$  !!)
- CAN be measured by an incoherent scatter radar, which is why we are here.

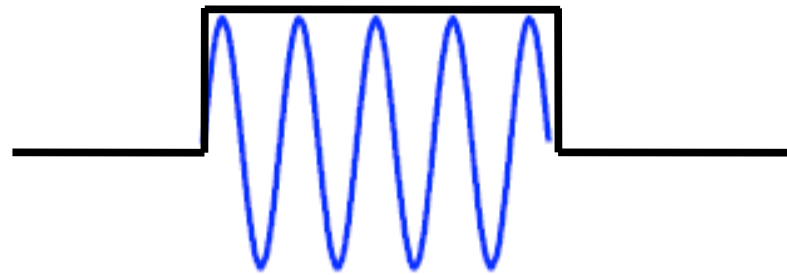


# Radar Range Equation



	Transmit Power	Transmit Gain	Spread Factor	Losses	Target RCS	Spread Factor	Receive Aperture	Dwell Time
Received Signal Energy =	$[ P_T ]$	$\left[ \frac{4\pi A}{\lambda^2} \right]$	$\left[ \frac{1}{4\pi R^2} \right]$	$\left[ \frac{1}{L} \right]$	$[ \sigma ]$	$\left[ \frac{1}{4\pi R^2} \right]$	$[ A ]$	$[ \tau ]$

# What the radar transmits: Pulses and waves

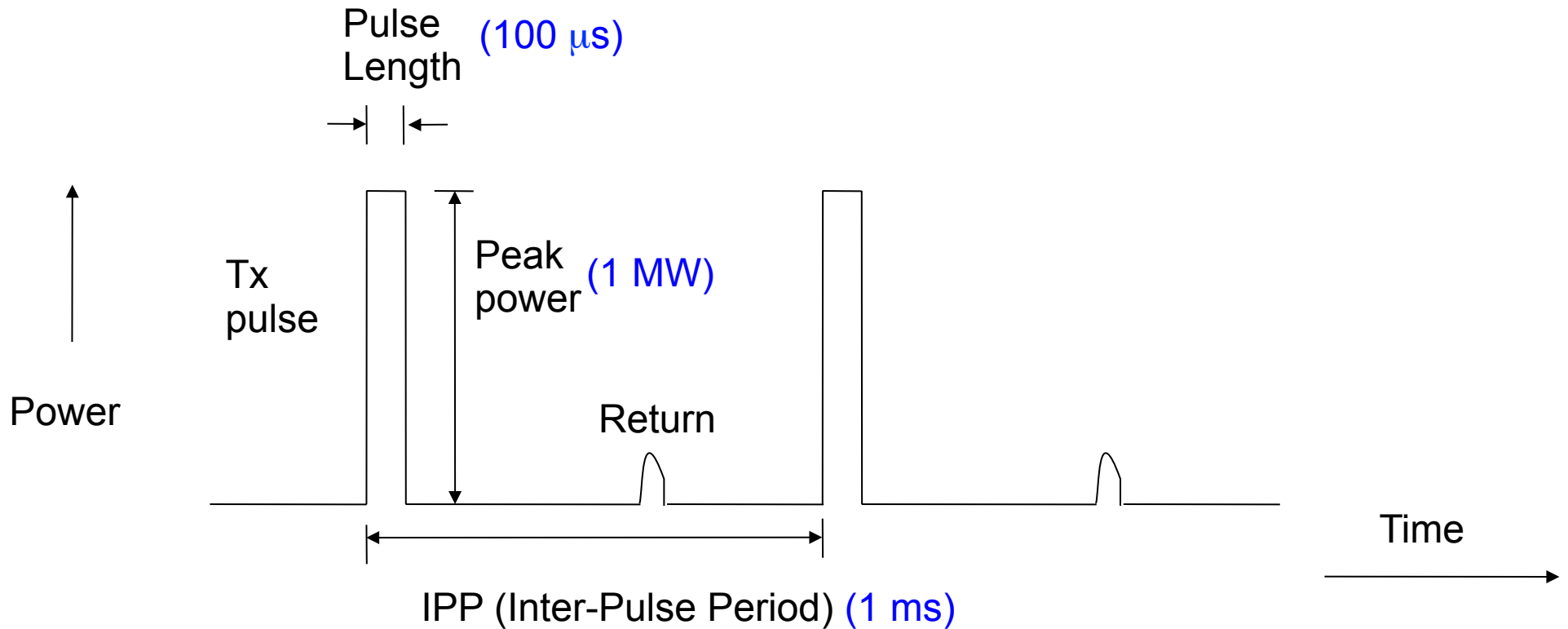


Cycles in a pulse.

PFISR frequency = 449 MHz  
Long pulse length = 480  $\mu$ s  
# of cycles = 215520 !

Radar waveforms  
modulate the waves with  
on-off sequence

# Pulsed Radar



Duty cycle = Pulse Length/IPP (10%)  
Average power = Peak power x Duty cycle (100 kW)  
PRF (Pulse Repetition Frequency) = 1/IPP (1 kHz)

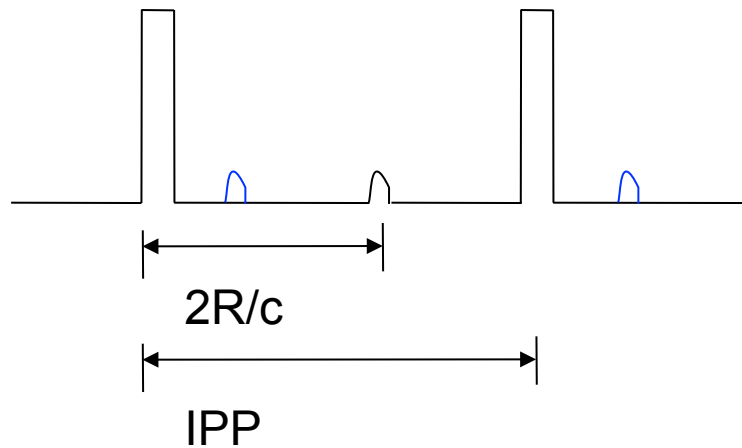
Duty cycle for a CW (continuous wave) radar 100%

# Range Resolution

Range resolution is set by pulse length

Pulse length =  $\tau_p$  , Range resolution =  $c\tau_p/2$  for a single target.

Maximum unambiguous range



$$\text{MUR} = c \cdot \text{IPP} / 2$$

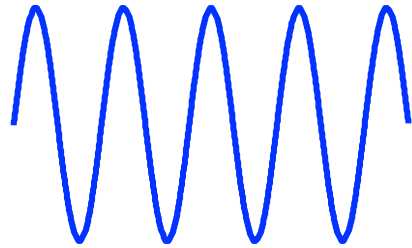
# Pulse duration vs. Range resolution

Pulse Duration	Range Resolution
0.1 nsec	1.5 cm
1.0 nsec	15 cm
10 nsec	1.5 m
100 nsec	15 m
1 $\mu$ sec	150 m
10 $\mu$ sec	1.5 km
100 $\mu$ sec	15 km
1 msec	150 km

**What is a typical F region ISR pulselength?**

# Radar Waveforms

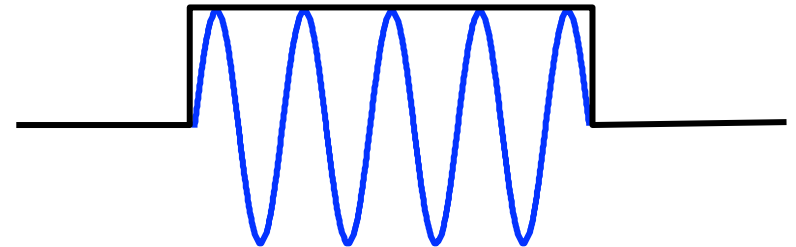
What do radars transmit?



Waves?



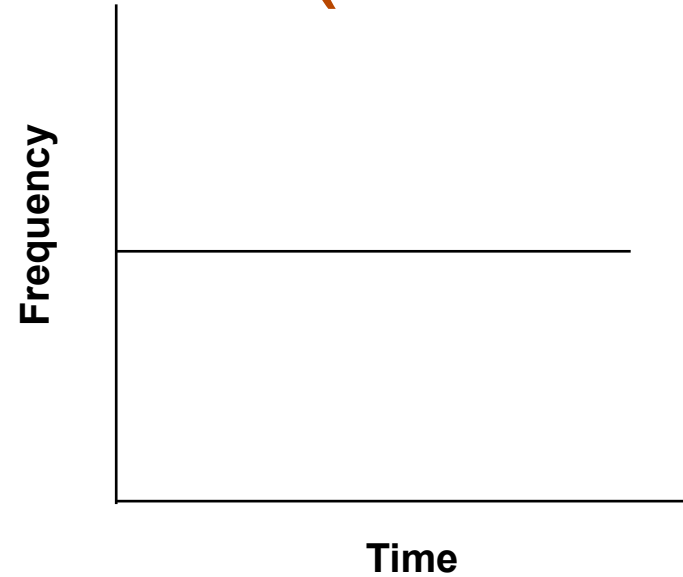
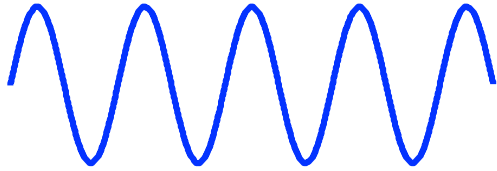
or Pulses?



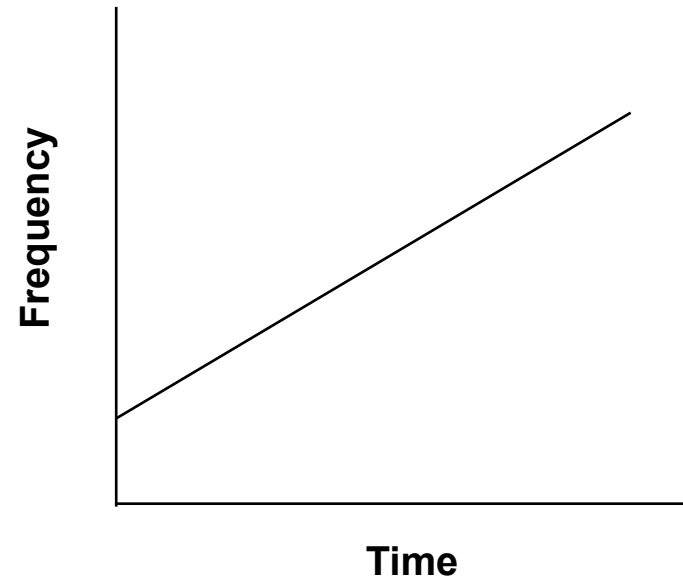
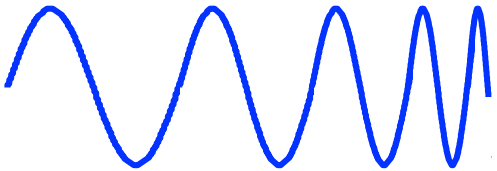
Waves, modulated  
by "on-off" action of  
pulse envelope

# Radar Waveforms (cont' d.)

**Pulse at single frequency**

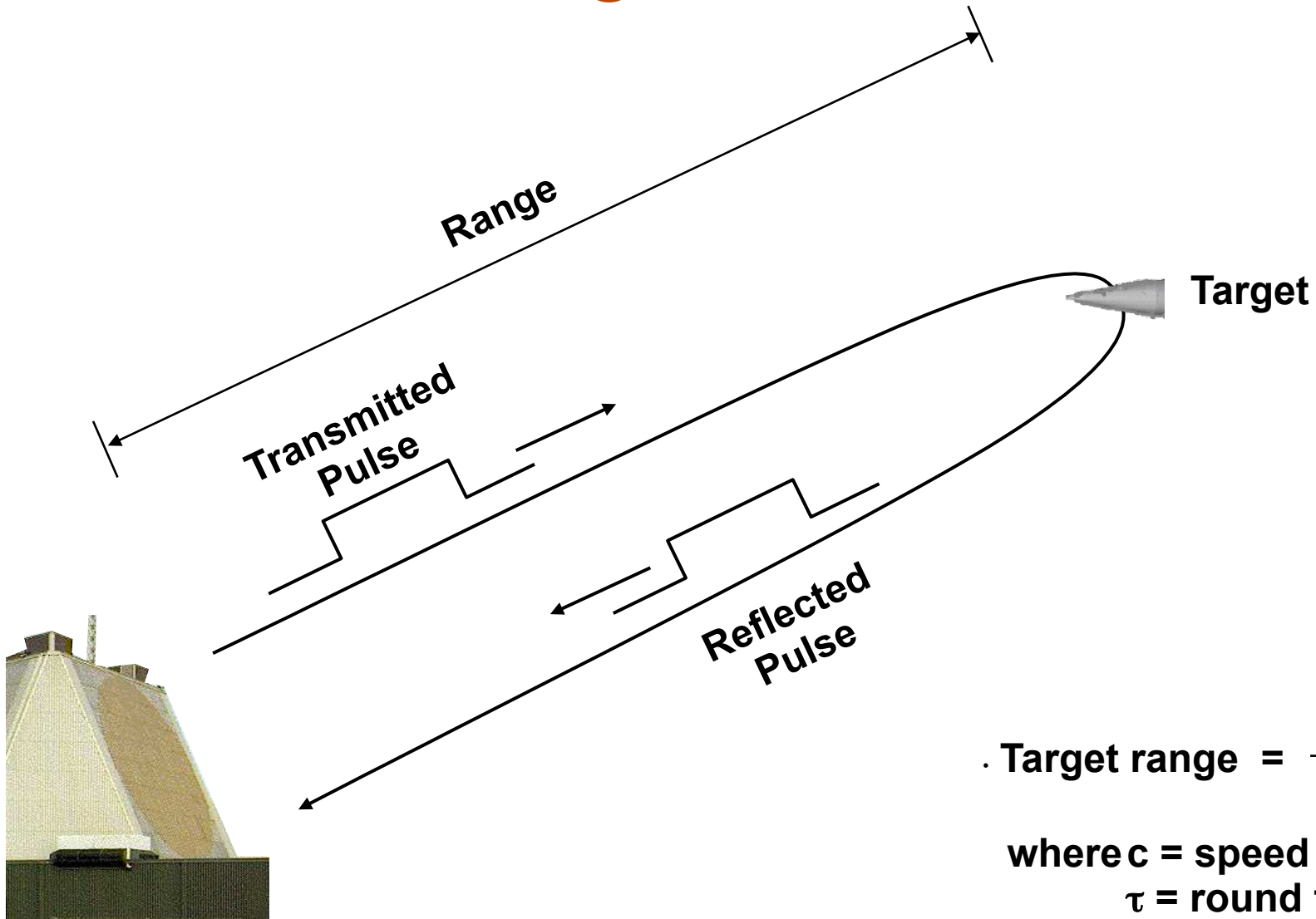


**Pulse with changing frequency**



**Linear  
Frequency-  
Modulated  
(LFM)  
Waveform**

# Radar Range Measurement

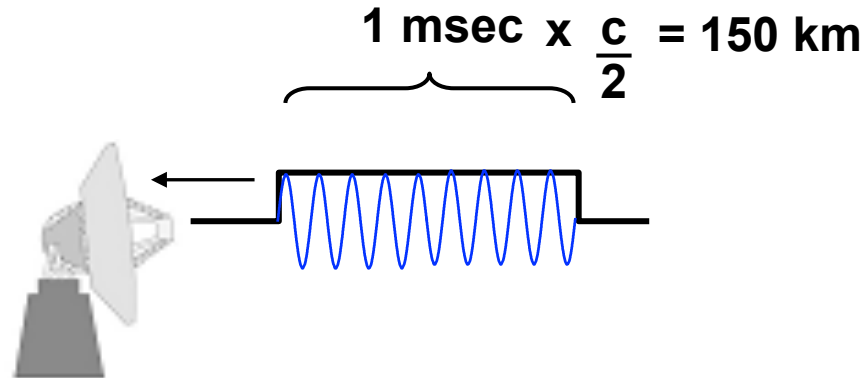




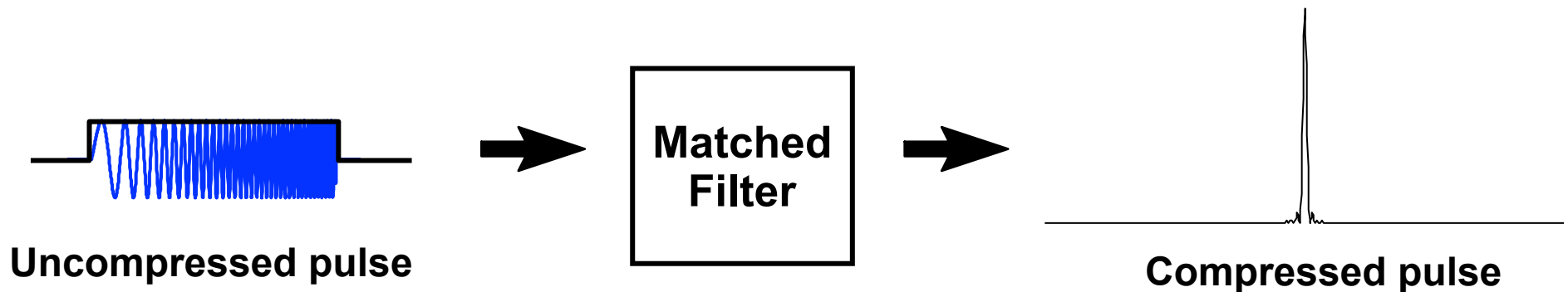
# Signal Processing

## Pulse Compression

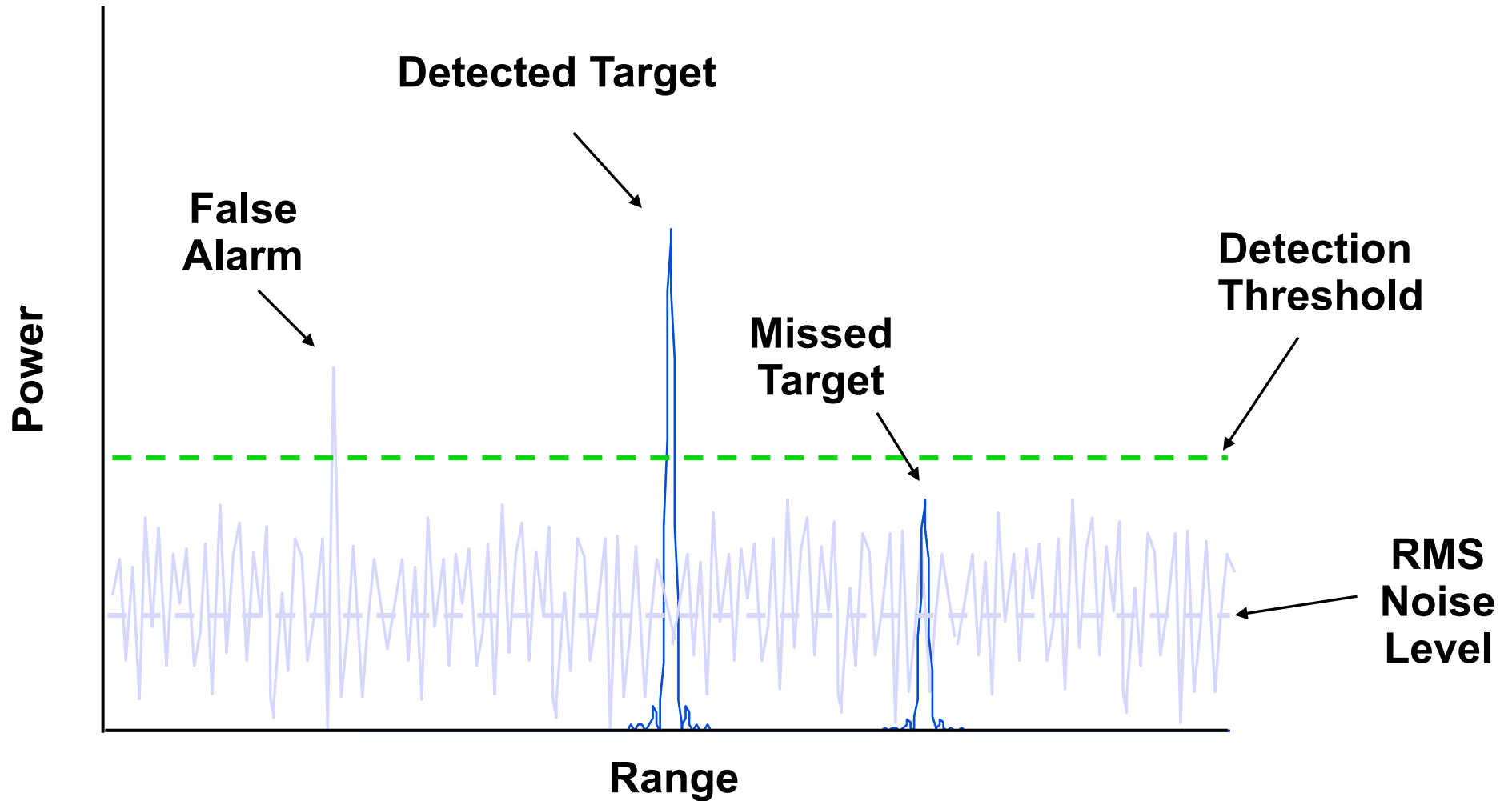
**Problem:** Pulse can be very long; does not allow accurate range measurement



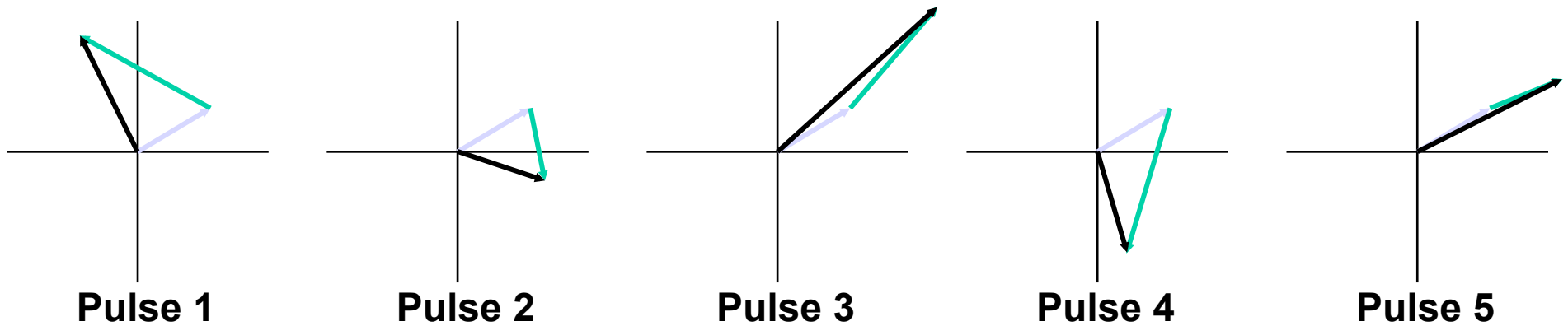
**Solution:** Use pulse with changing frequency and signal process using “matched filter”



# Detection of Signals in Noise



# Coherent Integration

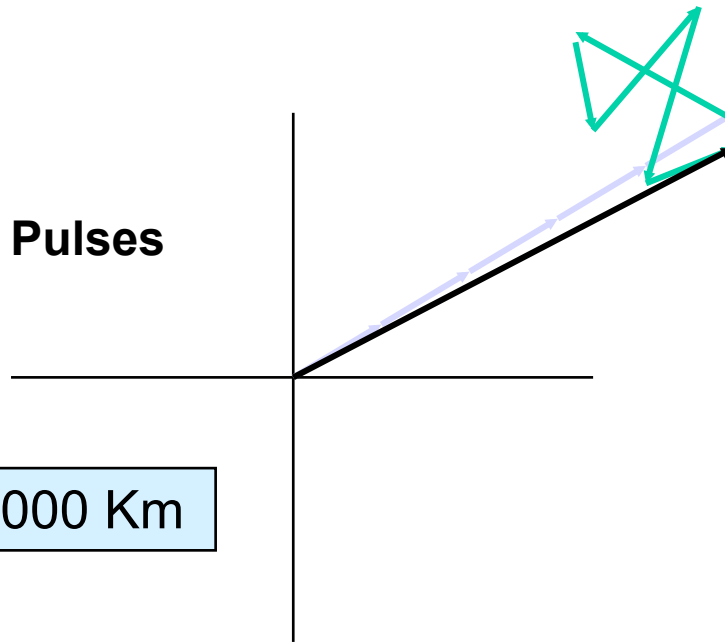


- Coherent target returns
- Noise samples at low SNR

· Resultant signal

---

Coherently Integrated Pulses

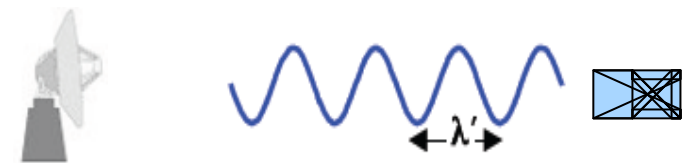
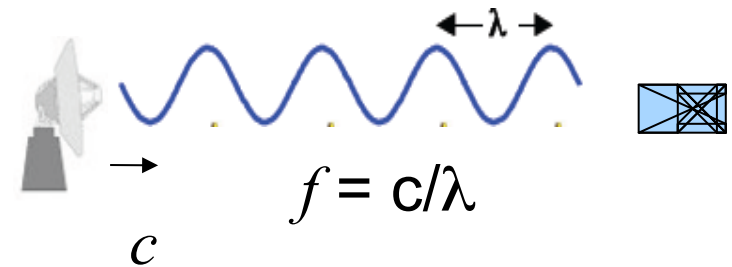
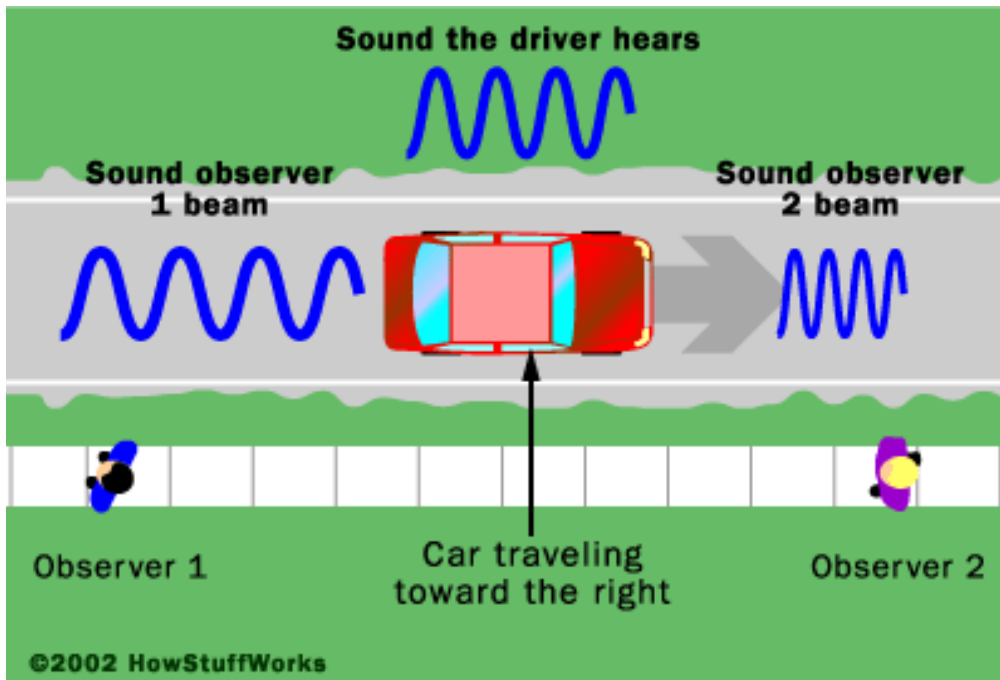


Deep space targets at 30,000 – 40,000 Km

# Outline - Radar Basics

- Electromagnetic spectrum
- Radio Waves and Propagation
- Radar fundamentals
  - Radar equation
  - Range Resolution and pulsed radars
- Doppler and Doppler Radars

# Moving target - Doppler



$$f' = f \pm \frac{2v}{\lambda}$$

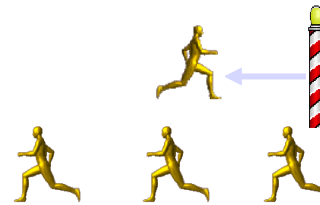
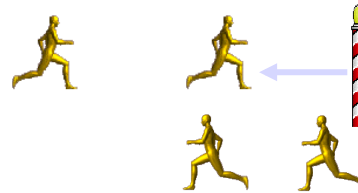
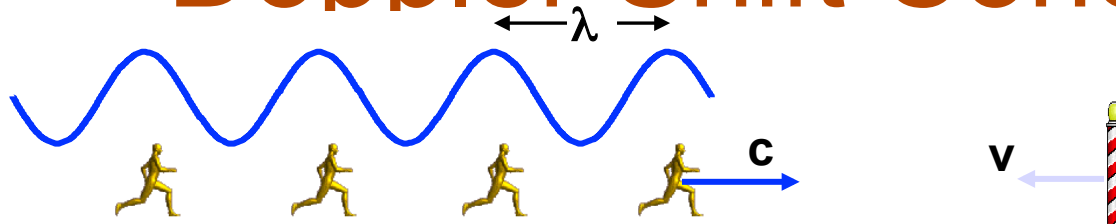
Doppler shift

**Positive** Doppler = target moving **toward** the observer

**Negative** Doppler = target moving **away** from the observer

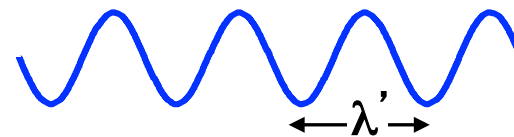
# Doppler Shift Concept

$$f = \frac{c}{\lambda}$$



$$f' = f \pm (2v/\lambda)$$

Doppler shift



# Doppler shift frequency

Tx signal:  $\cos(2\pi f_o t)$

Return from a moving target:  $\cos[2\pi f_o(t + 2R/c)]$

If target is moving with a constant velocity:  $R = R_o + v_o t$

then,

Return:  $\cos[2\pi(f_o + f_o 2v_o/c)t + 2\pi f_o R_o/c]$

Doppler frequency:  
 $-2f_o v_o/c = -2v_o/\lambda_o$

# Resolving Doppler

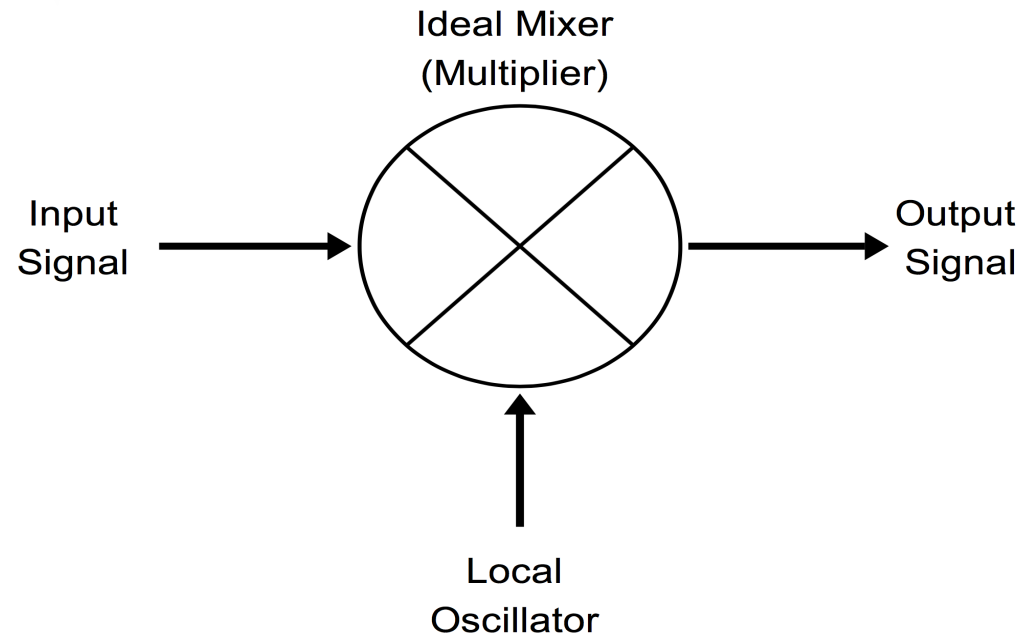
Tx signal:  $\cos(2\pi f_o t)$

Doppler shifted:  $\cos[2\pi(f_o + f_D)t]$

Multiply by  $\cos(2\pi f_o t)$  -> Low pass filter ->  $\cos(2\pi f_D t)$



# Mixing to Baseband



$$\sin u \sin v = \frac{1}{2} [\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2} [\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2} [\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2} [\sin(u + v) - \sin(u - v)]$$

# Resolving Doppler

Tx signal:  $\cos(2\pi f_o t)$

Doppler shifted:  $\cos[2\pi(f_o + f_D)t]$

Multiply by  $\cos(2\pi f_o t)$  -> Low pass filter ->  $\cos(2\pi f_D t)$

BUT, the sign of  $f_D$  is lost (cosine is an even function)

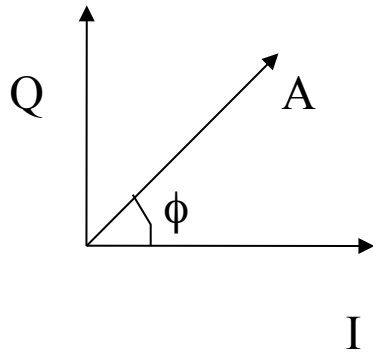
So, instead use

$$\exp(j2\pi f_D t) = \cos(2\pi f_D t) + j\sin(2\pi f_D t)$$

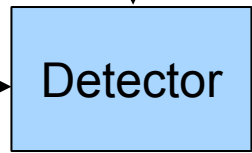
Generate this signal by mixing cos and sin via two oscillators (same frequency, 90° out of phase)

Components are called I (In phase) and Q (Quadrature):  $A\exp(j2\pi f_D t) = I + jQ$

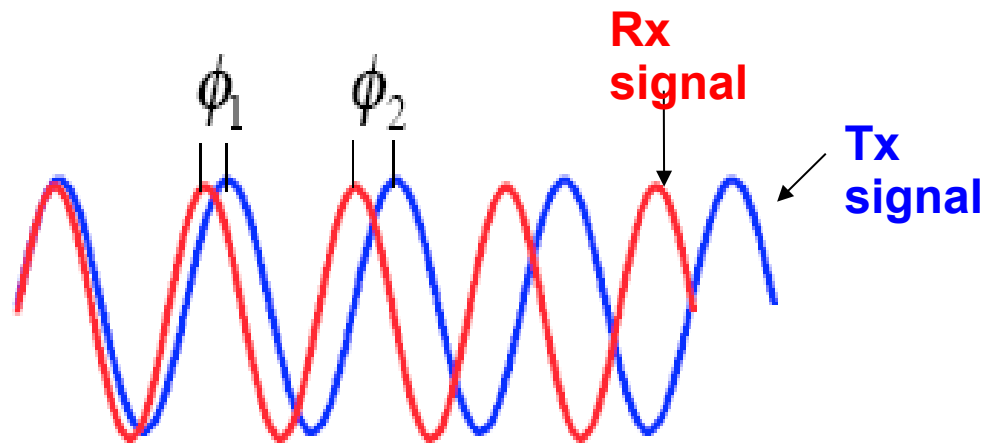
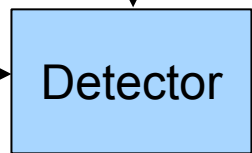
# I/Q Demodulation



Reference signal from synchronizer

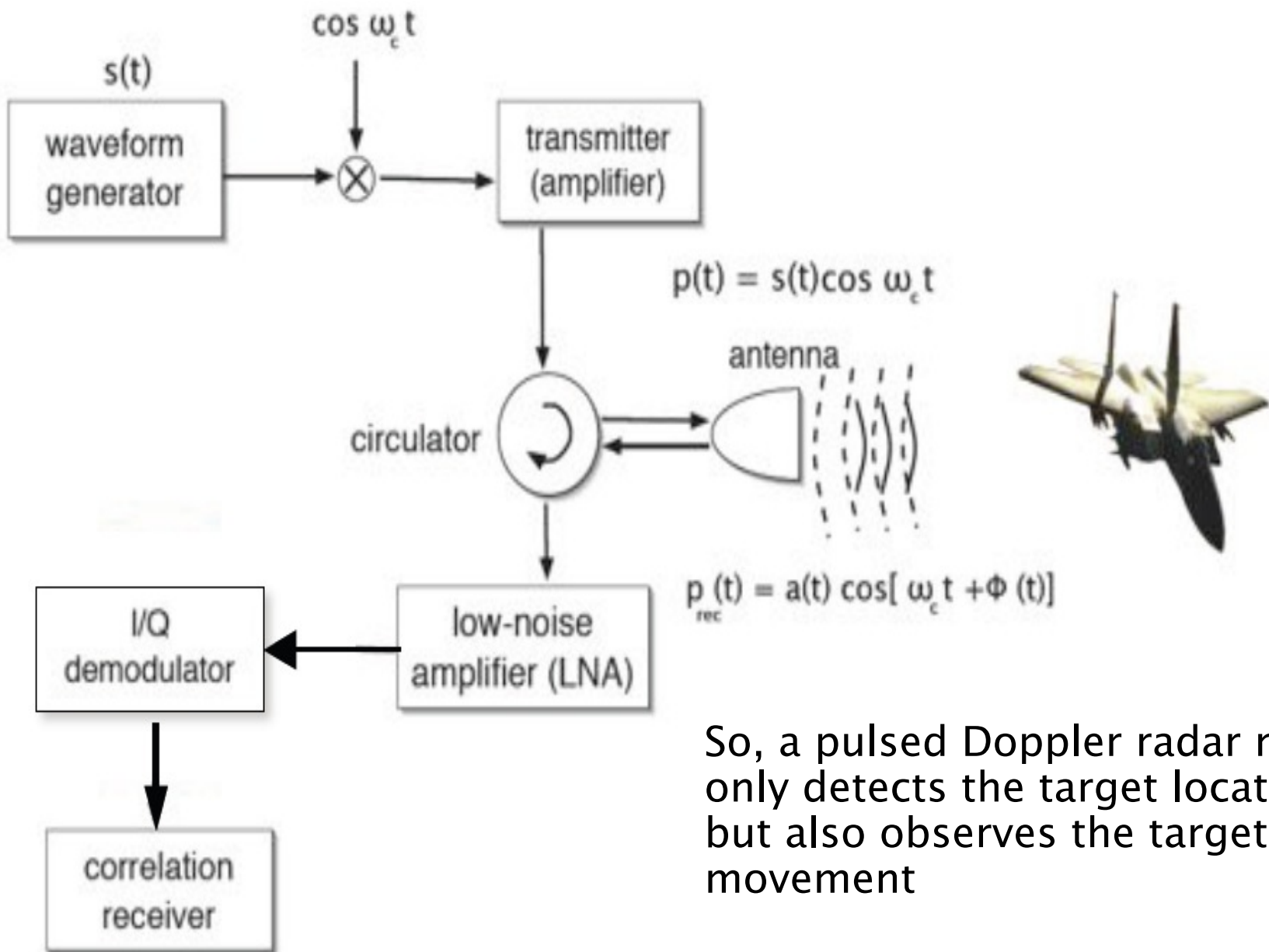


90° out of phase signal



The Doppler frequency shows up as a pulse-to-pulse phase shift

# Pulsed Doppler Radar system



So, a pulsed Doppler radar not only detects the target location, but also observes the target movement

# Determining the Doppler Spectrum

1. Doppler spectrum is measured at a particular range gate (e.g. at  $r = \frac{c\Delta t}{2}$  )

2. Must process a time series of discrete samples of echo  $E_r(t)$  at intervals of the pulse period  $T_r$

3. Analyze the sampled signal using (fast) Fourier Transform methods:

$$E(mT_r) = \frac{1}{M} \sum_{m=0}^{M-1} F(kf_0) \cos[2\pi kf_0 mT_r] \quad \begin{array}{l} M = \# \text{ of samples} \\ f_0 = \text{frequency resolution} \end{array}$$

$$F(kf_0) = \sum_{m=0}^{M-1} E_r(mT_r) \cos[2\pi kf_0 mT_r]$$

4. Frequency components (radial velocities) occur at discrete intervals, with  $M$  intervals separated by intervals of  $1/MT_r = f_D$

# Spectrum analysis (FS)

## Fourier series (FS)

The coefficients  $c(i)$  contain the time domain information and are evaluated as

$$c(n) = \frac{1}{P} \int_{-P/2}^{P/2} y(t) e^{-j2\pi n f_0 t} dt$$

$P$  is the period of the wave

The FS is often expressed in trigonometric form as

$$y(t) = a(0)/2 + \sum_{n=1}^{\infty} a(n) \cos(2\pi n f_0 t) + b(n) \sin(2\pi n f_0 t)$$

$$a(0) = \frac{2}{P} \int_{-P/2}^{P/2} y(t) dt$$

$$a(m) = \frac{2}{P} \int_{-P/2}^{P/2} y(t) \cos(2\pi m f_0 t) dt$$

$$b(m) = \frac{2}{P} \int_{-P/2}^{P/2} y(t) \sin(2\pi m f_0 t) dt$$

$m$  is any integer greater than zero

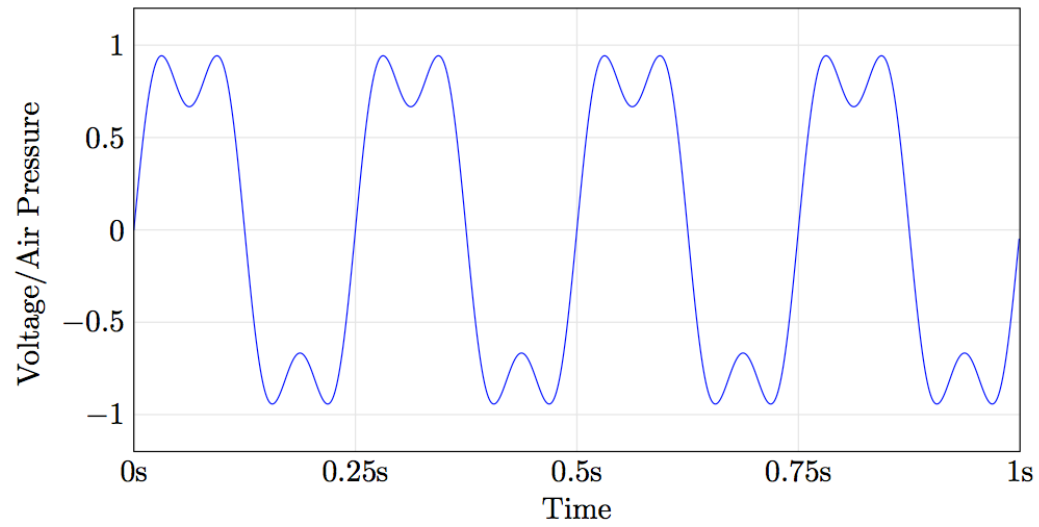


Figure 4: 4Hz + 12Hz Sin Wave.

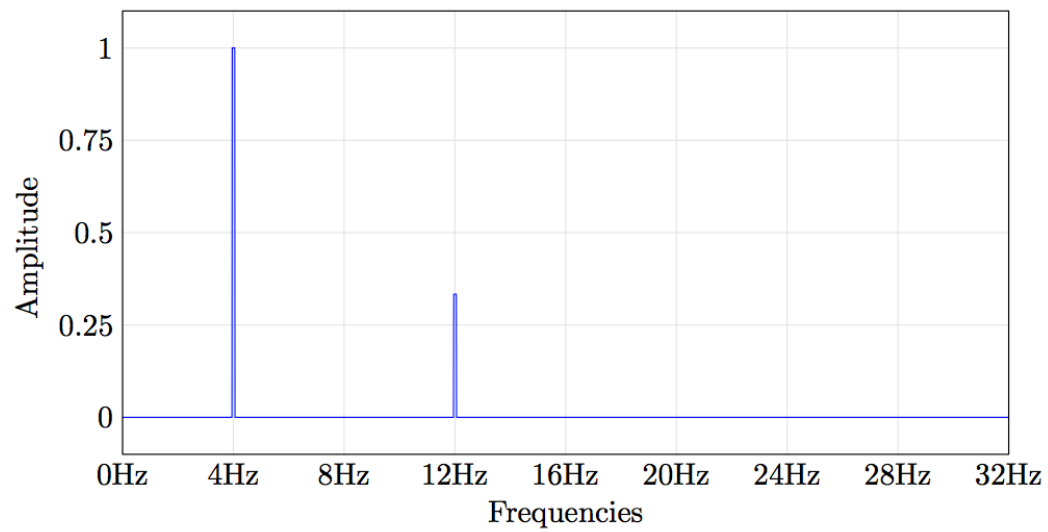
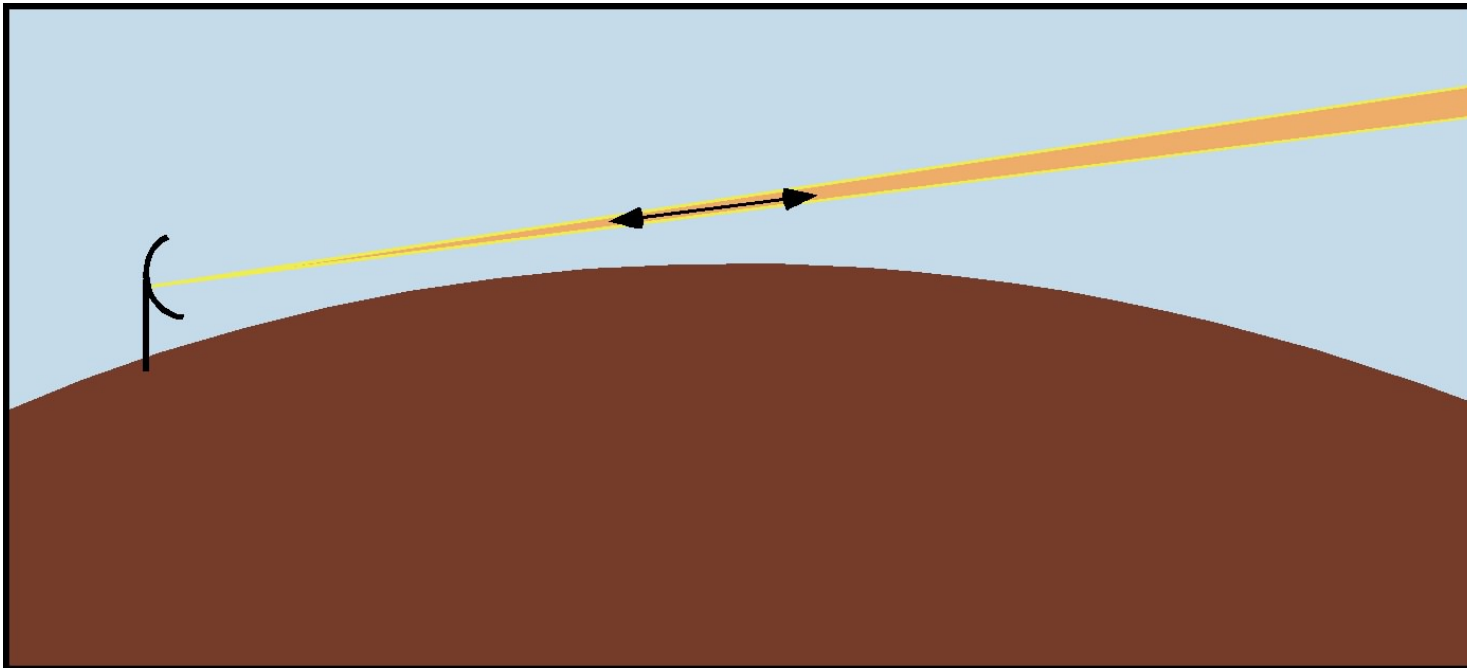


Figure 5: Frequency Domain of 4Hz + 12Hz Sin Waves.

Note that Doppler radars are only sensitive to the radial motion of objects

*Air motion is a three dimensional vector:* A Doppler radar can only measure one of these three components – the motion along the beam toward or away from the radar



**Question: how does a steerable dish like Millstone – or a phased array dish like PFISR – determine vector ion velocities?**



# Sign conventions

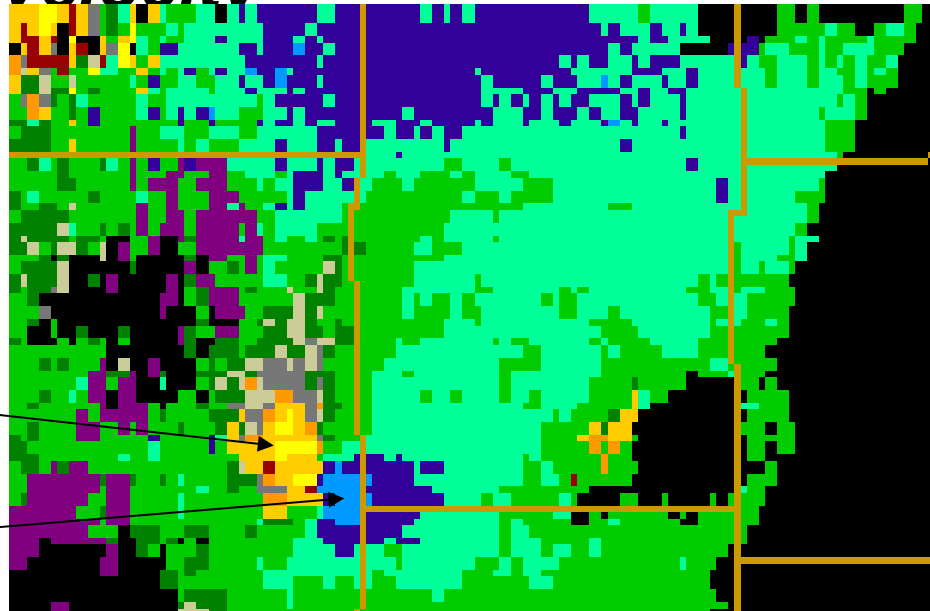
*The Doppler frequency is negative (lower frequency, red shift) for objects receding from the radar*

*The Doppler frequency is positive (higher frequency, blue shift) for objects approaching the radar*

***These “color” shift conventions are typically also used on radar displays of Doppler velocity***

*Red: Receding from radar*

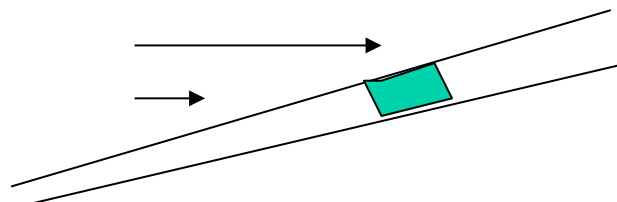
*Blue: Toward radar*



# Real characteristics of a returned signal from a distributed target

Velocity of individual targets in contributing volume vary due to:

1) Wind shear (particularly in the vertical)



2) Turbulence

3) Differential fall velocity (particularly at high elevation angles)

4) Antenna rotation

5) Variation in refraction of microwave wavefronts

# NET RESULT: A series of pulses will measure a spectrum of velocities (Doppler frequencies)

Power per unit velocity interval (db)

