Data Analysis and Fitting 1

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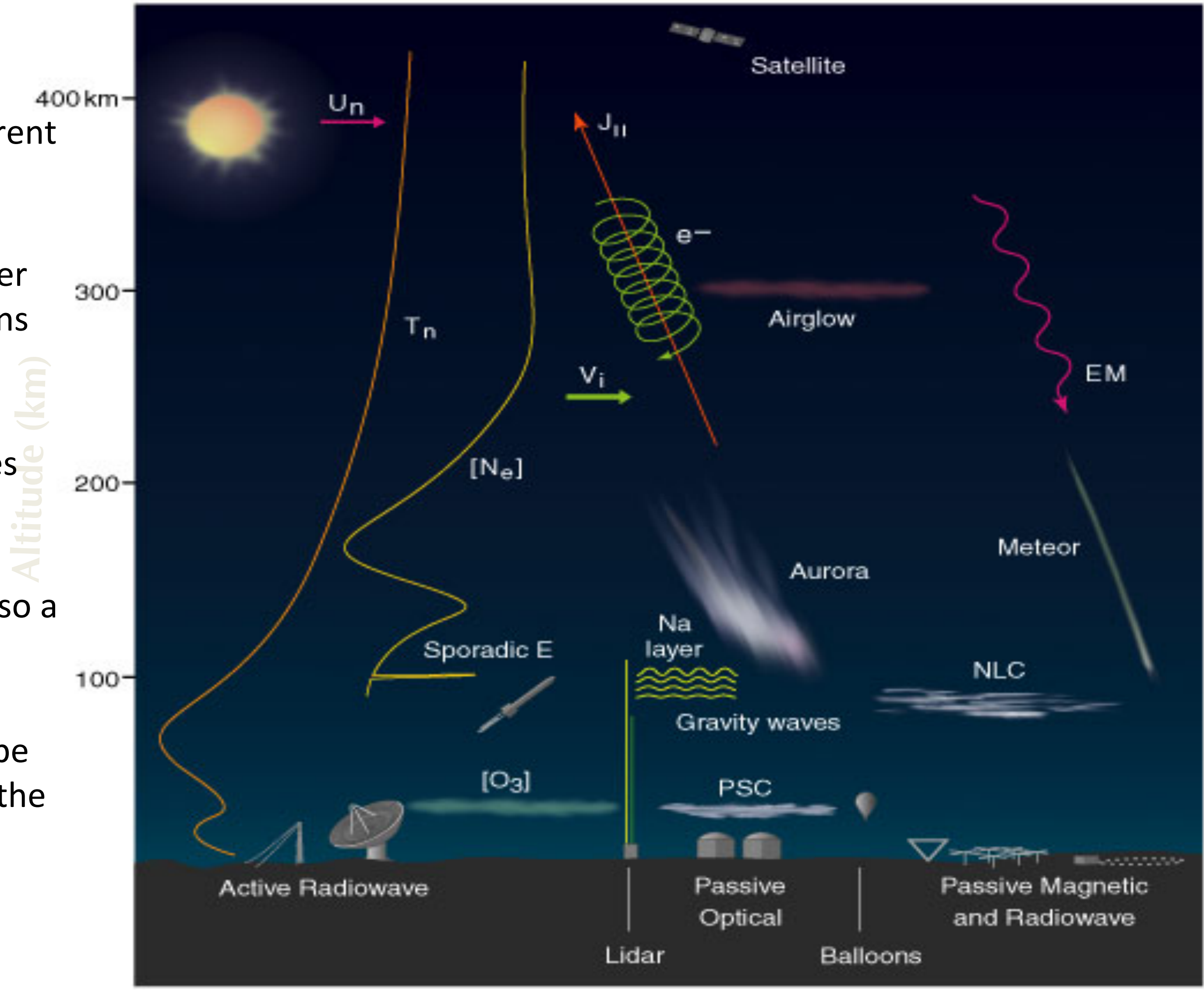
What we want to measure: plasma and neutral state

- Region probed by Incoherent Scatter Radars (ISRs) ~80-1000+ km
- Ionized region of the upper atmosphere (free electrons and ions) - Quasineutral ionized gas (plasma)
- Incoming solar EUV causes atmospheric constituents (N_2, O_2, O) to ionize
- Particle precipitation is also a major ionizing process at high latitudes
- Neutral atmosphere can be probed via influences on the plasma



ISR-Measurable Parameters

BASIC PARAMETERS

Ne, Te, Ti, Vi, ν_{in}, ion composition

ELECTRODYNAMIC PARAMETERS

E, σ_H and Σ_H, σ_P and Σ_P, $J_\perp$ and $J_{||}$

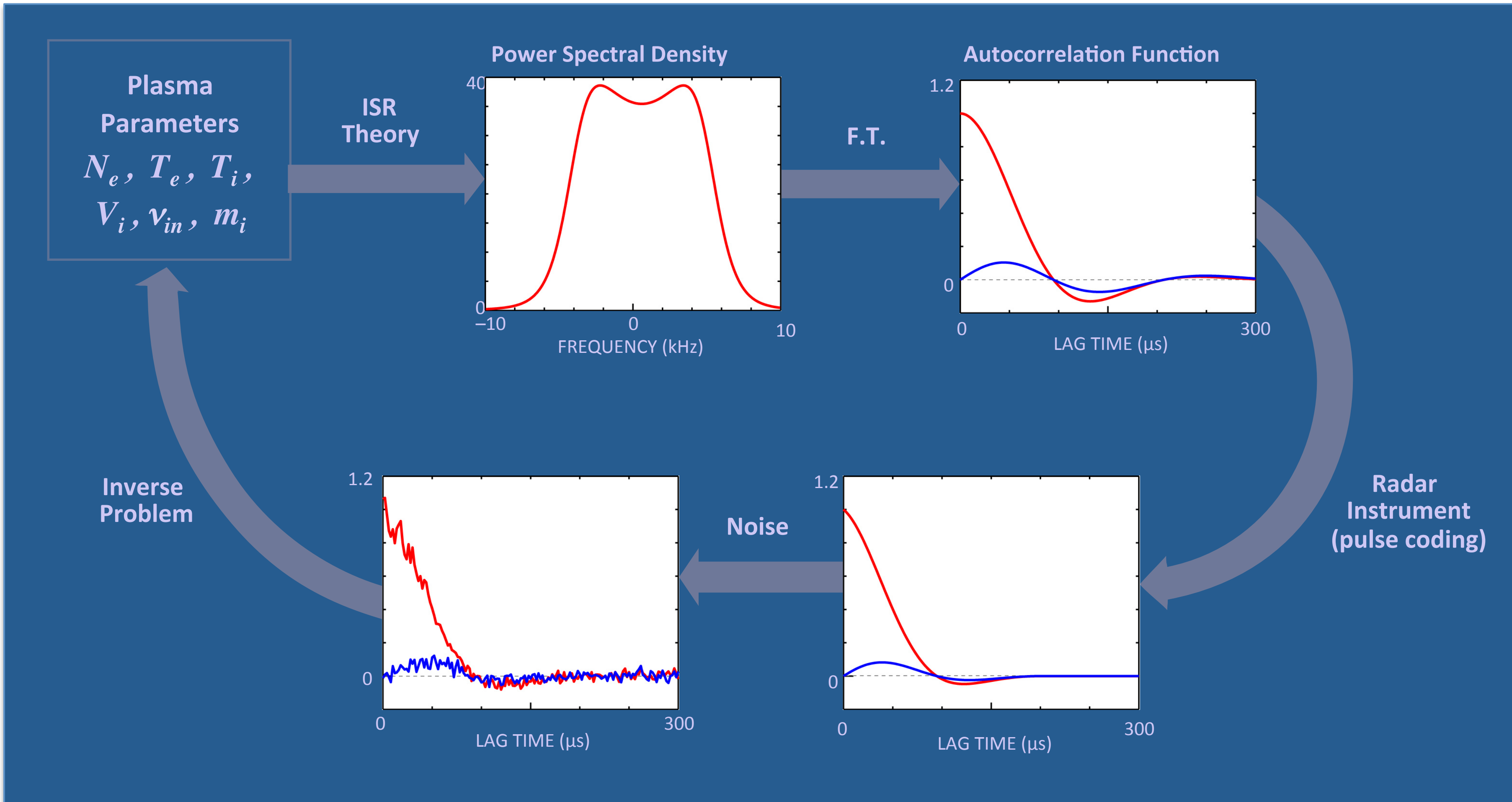
NEUTRAL PARAMETERS

U_{merid}, U, T_{inf}

ENERGY DEPOSITION

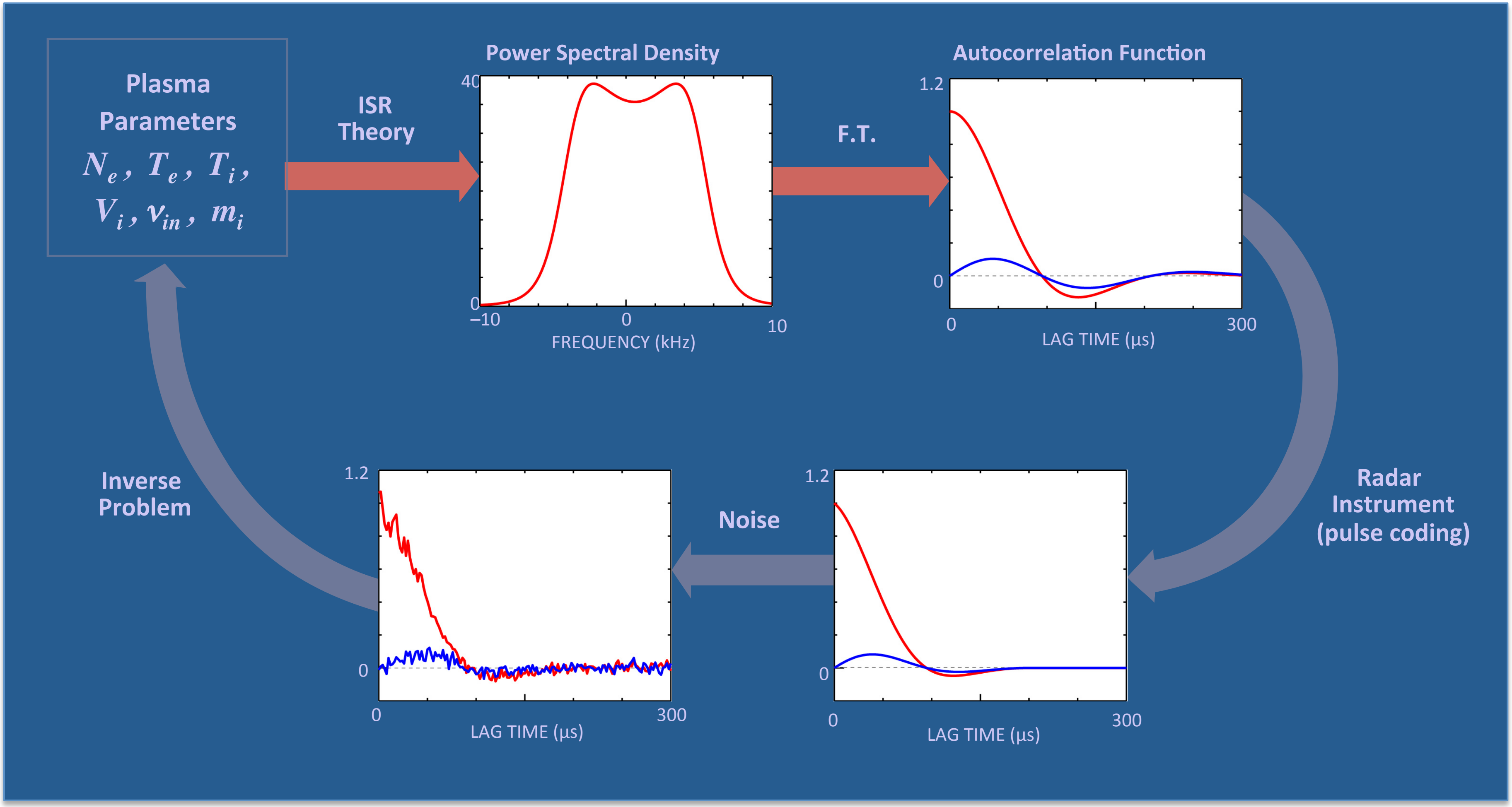
f(E)

Incoherent Scatter Radar Data Fitting
Basic Parameters



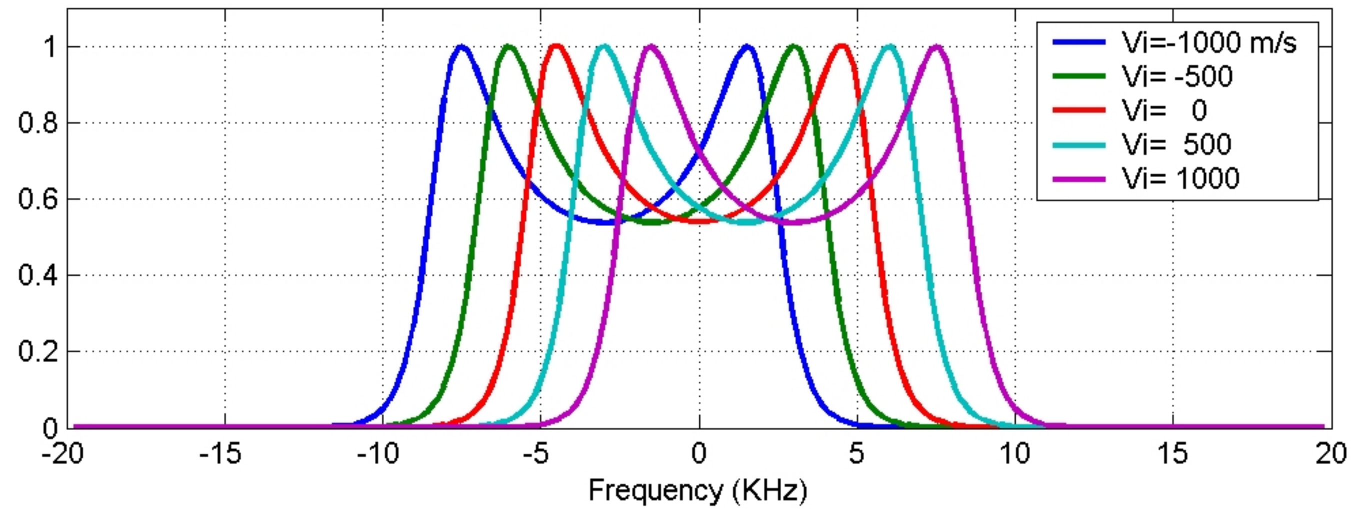
v3

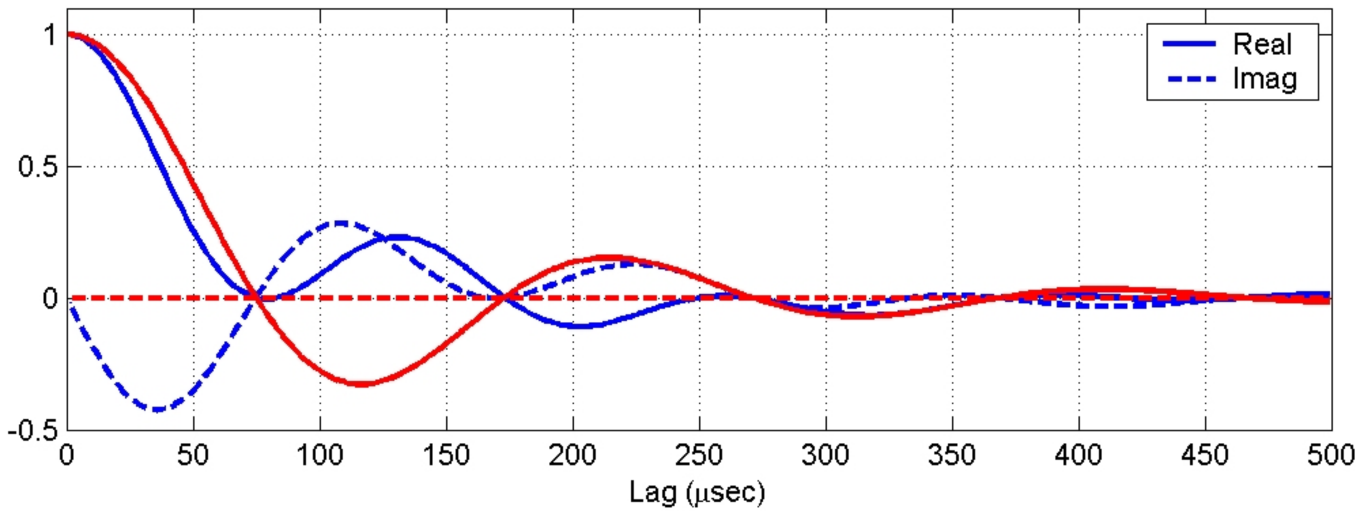
Incoherent Scatter Radar Data Fitting



v3

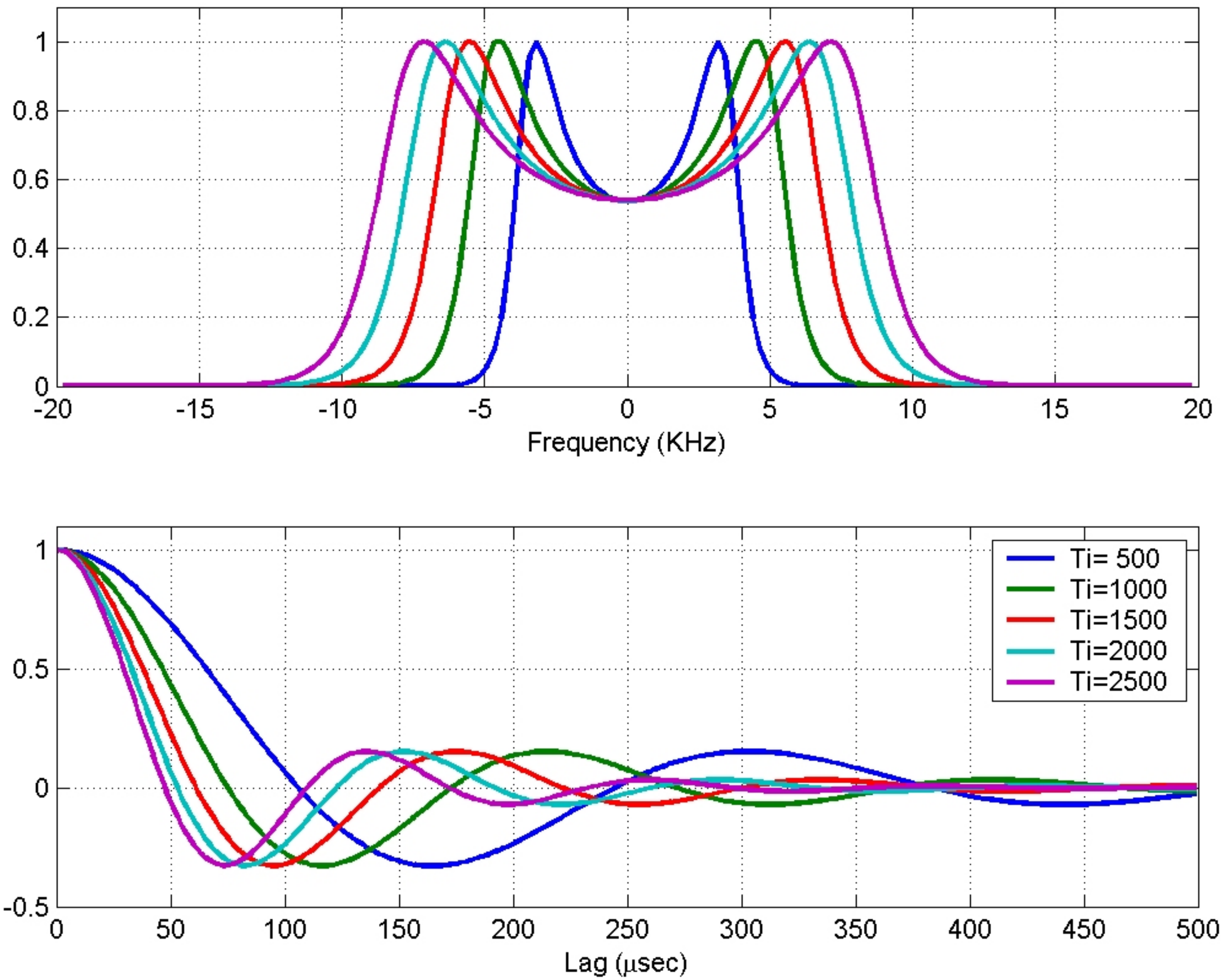
Ion Velocity





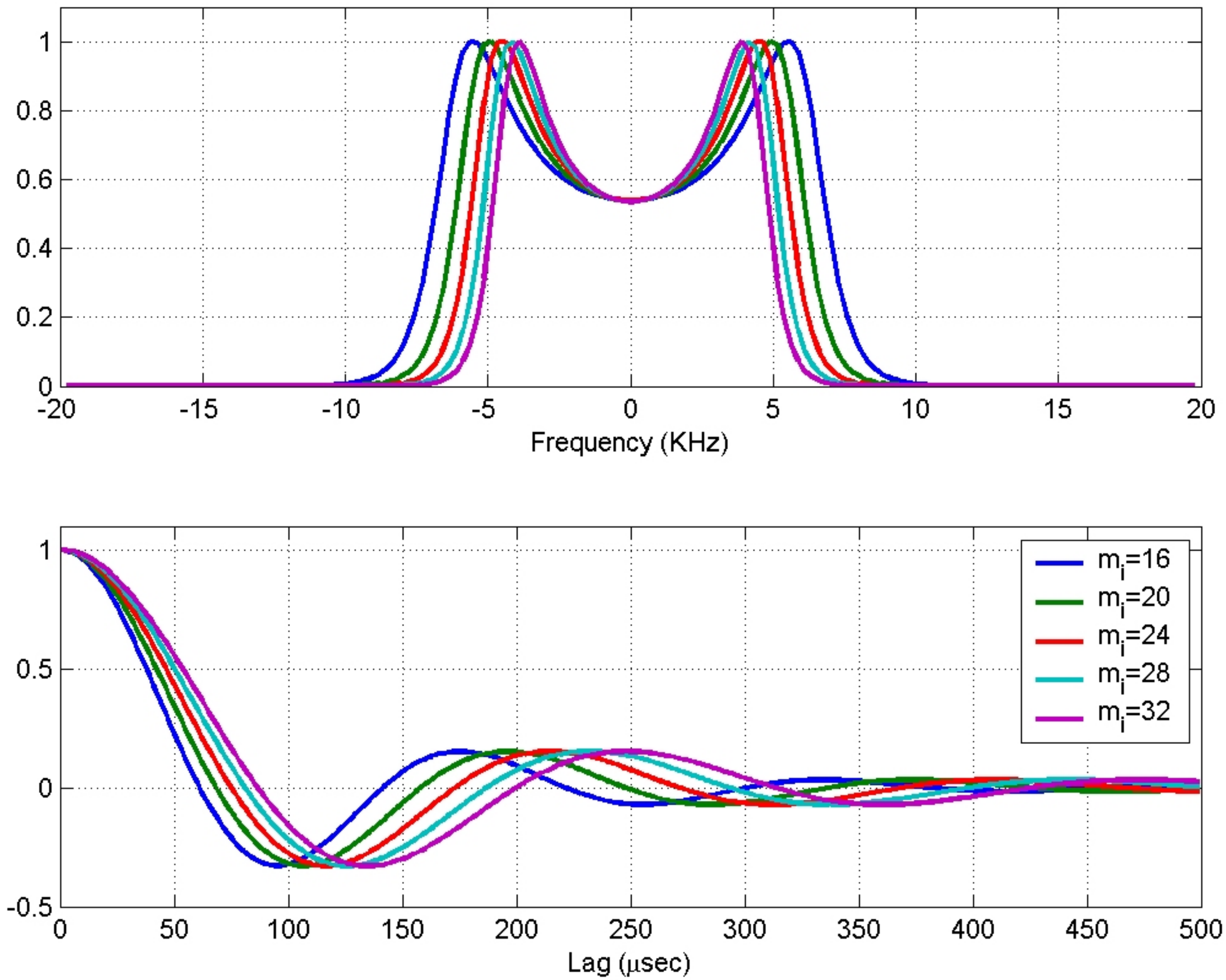
Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1000 K
Te: 2000 K
Comp: 100% O^+
ν_{in}: 10^{-6} KHz

Ion Temperature



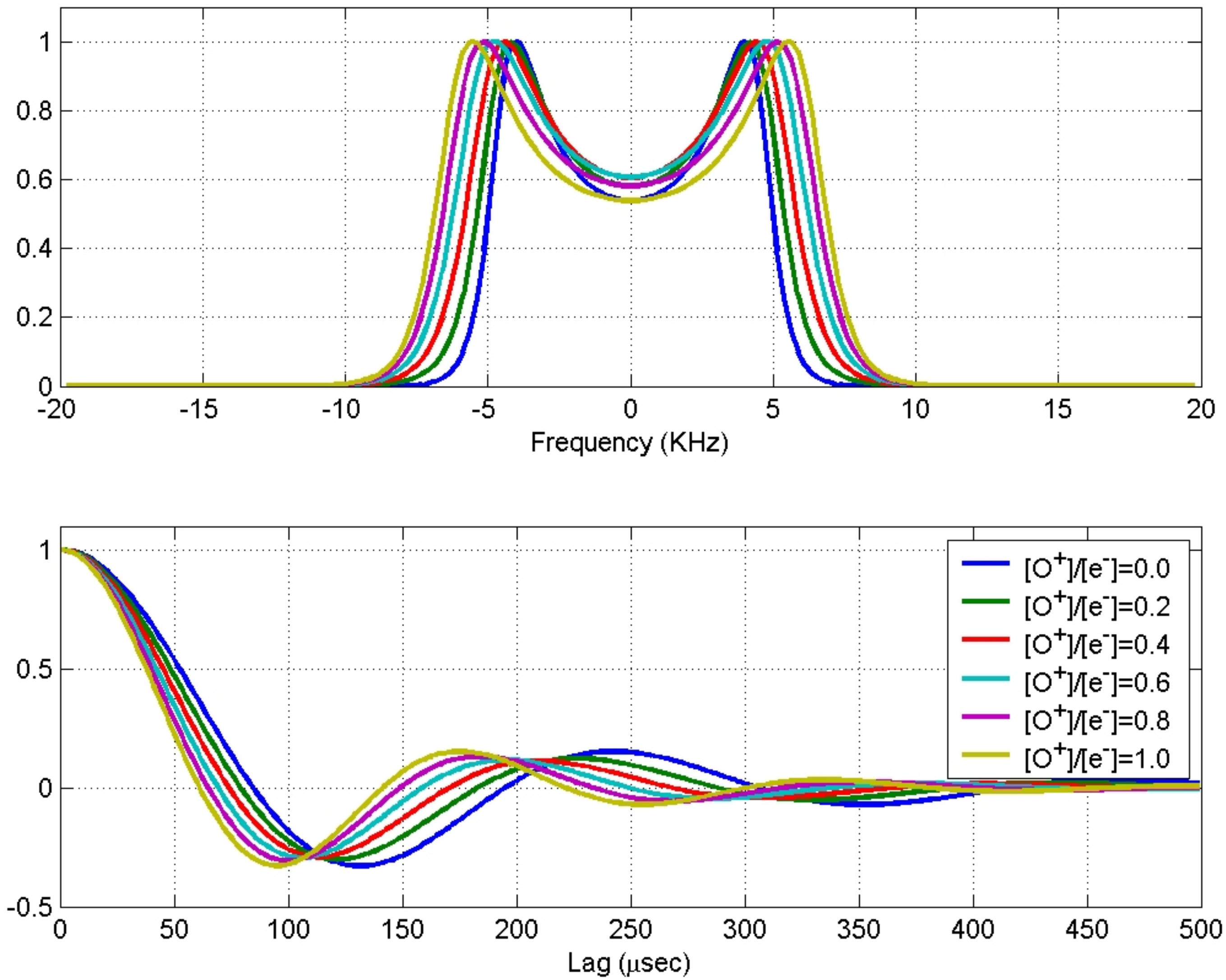
Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Te: 2*Ti
Comp: 100% O^{+}
ν_{in}: 10^{-6} KHz

Ion Mass



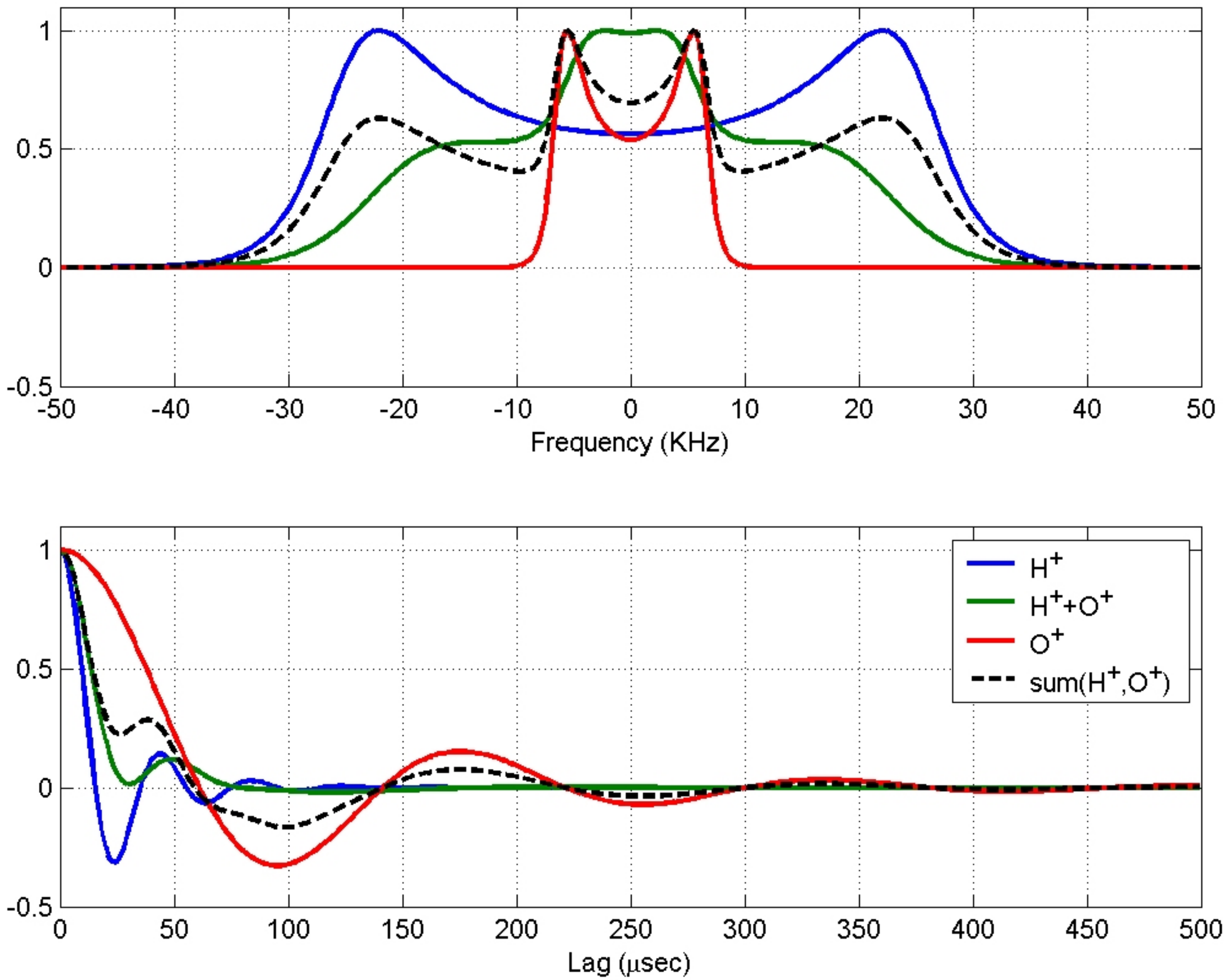
<u>Parameters</u>
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1500 K
Te: 3000 K
ν_{in}: 10^{-6} KHz

Ion Composition (O^+ vs. NO^+)



<u>Parameters</u>
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1500 K
Te: 3000 K
ν_{in}: 10^{-6} KHz

Ion Composition (O^+ vs. H^+)



Parameters

Freq: 449 MHz

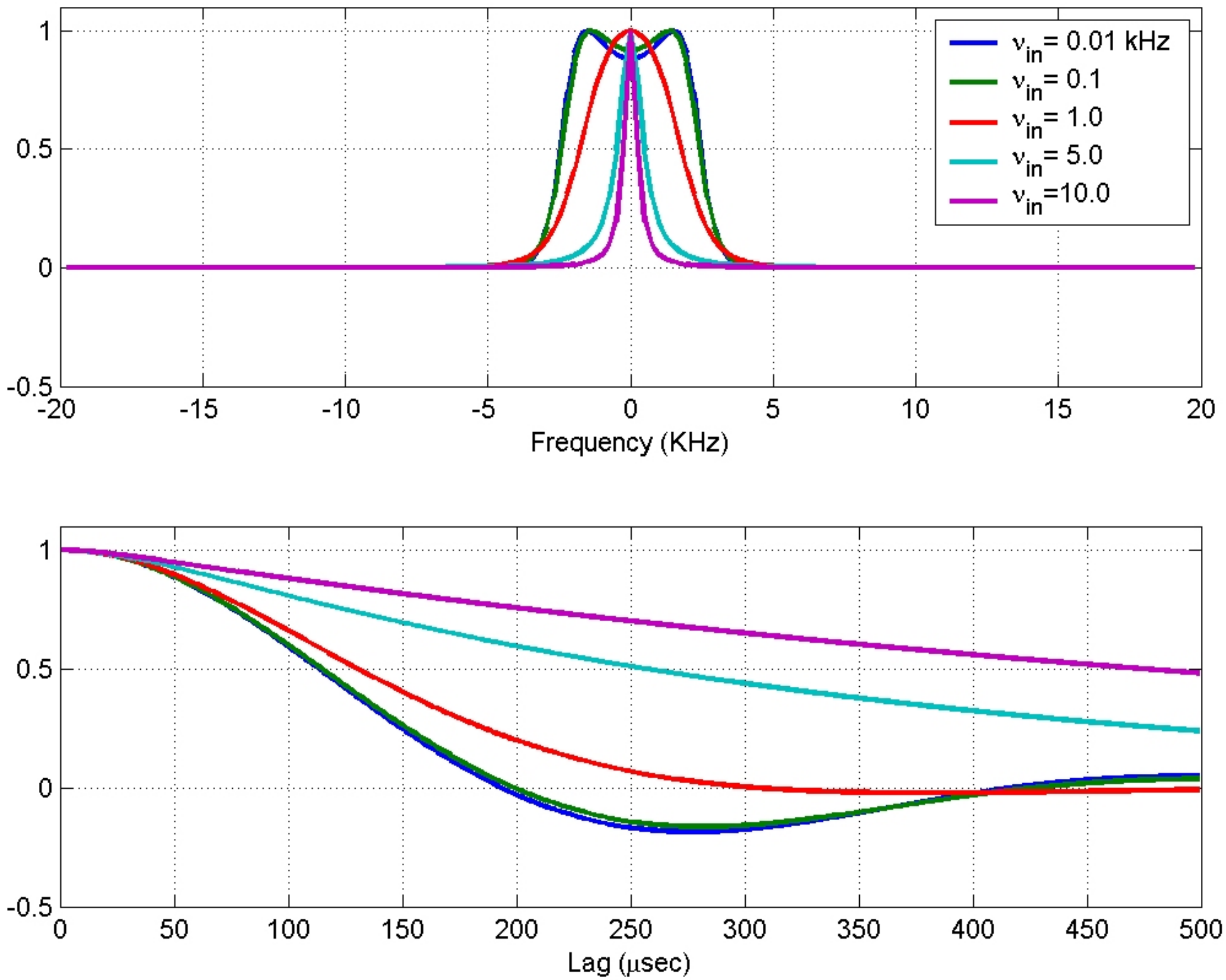
Ne: 10^{12} m^{-3}

Ti: 1500 K

Te: 3000 K

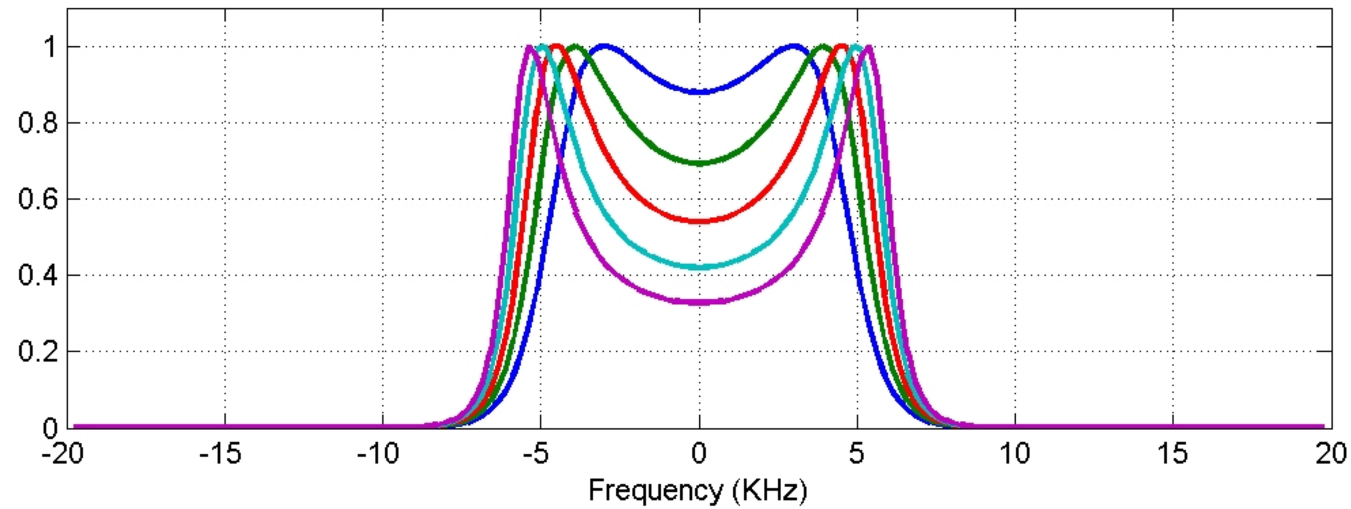
ν_{in}: 10^{-6} KHz

Ion-Neutral Collision Frequency

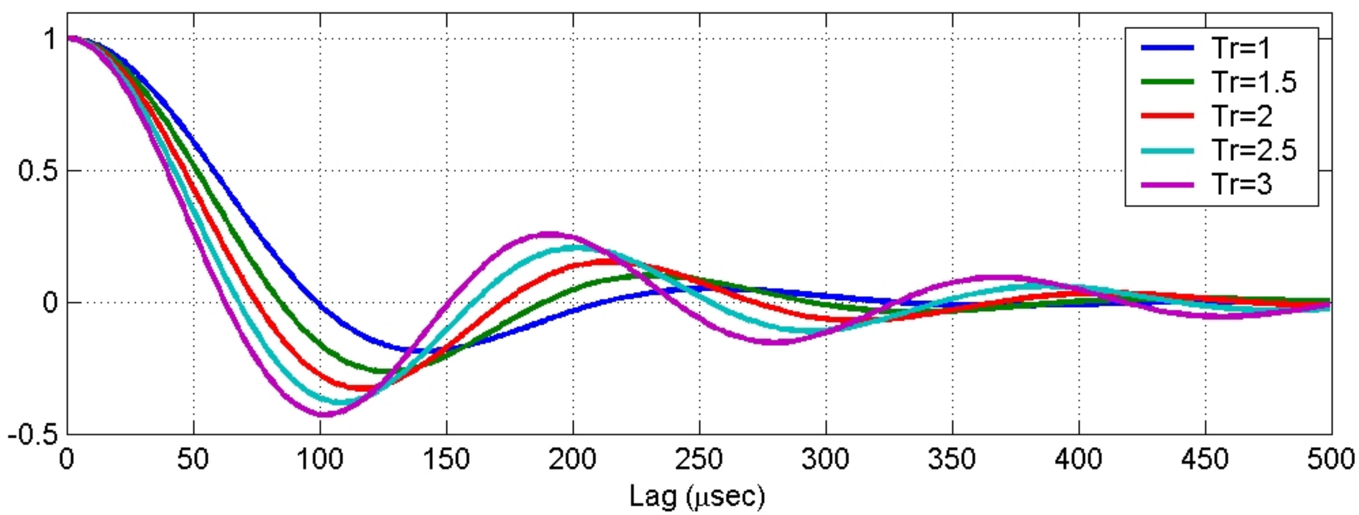


<u>Parameters</u>
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 500 K
Te: 500 K
Comp: 100% NO^{+}

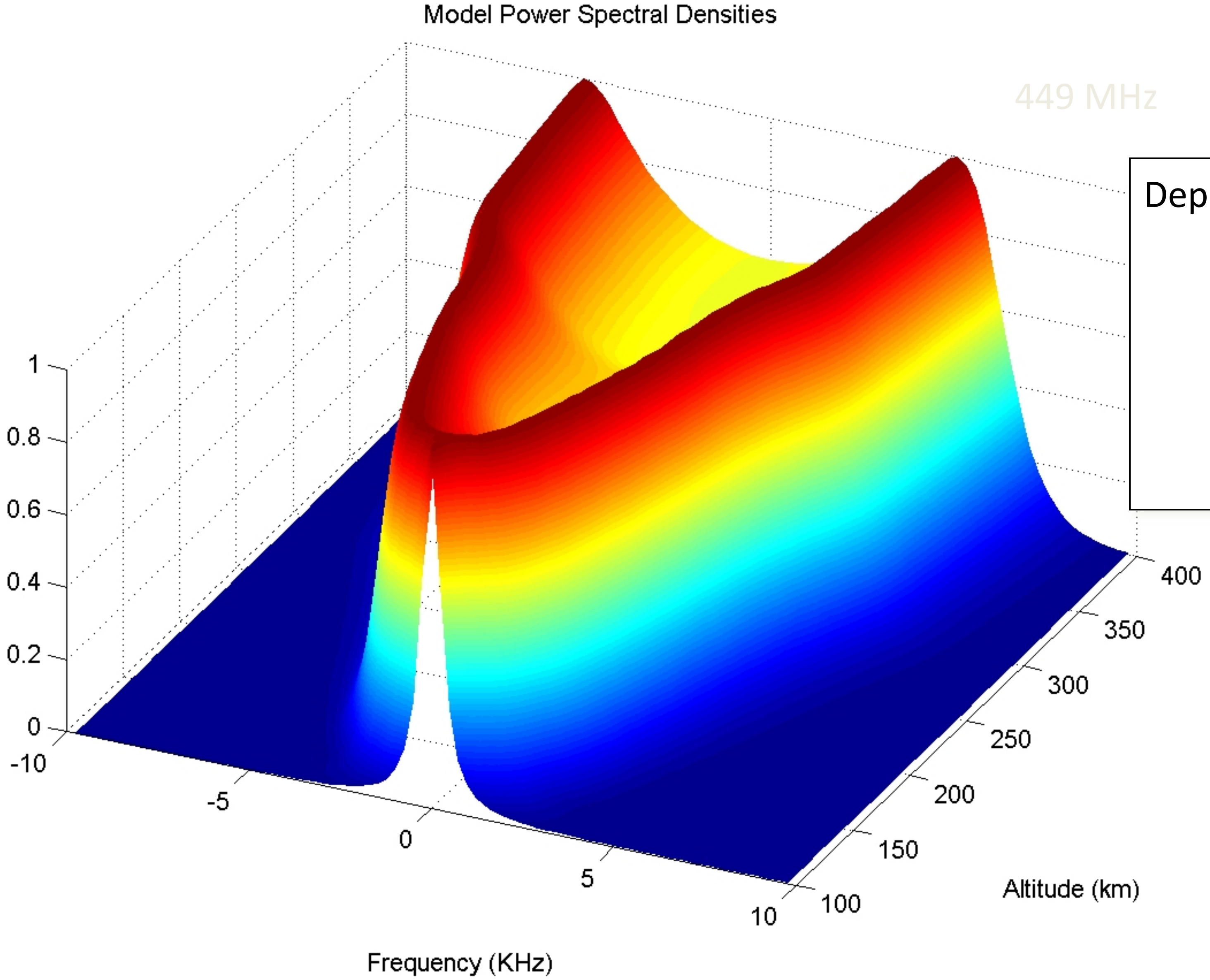
Electron/Ion Temperature Ratio



Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1000 K
Comp: 100% O^{+}
ν_{in}: 10^{-6} KHz

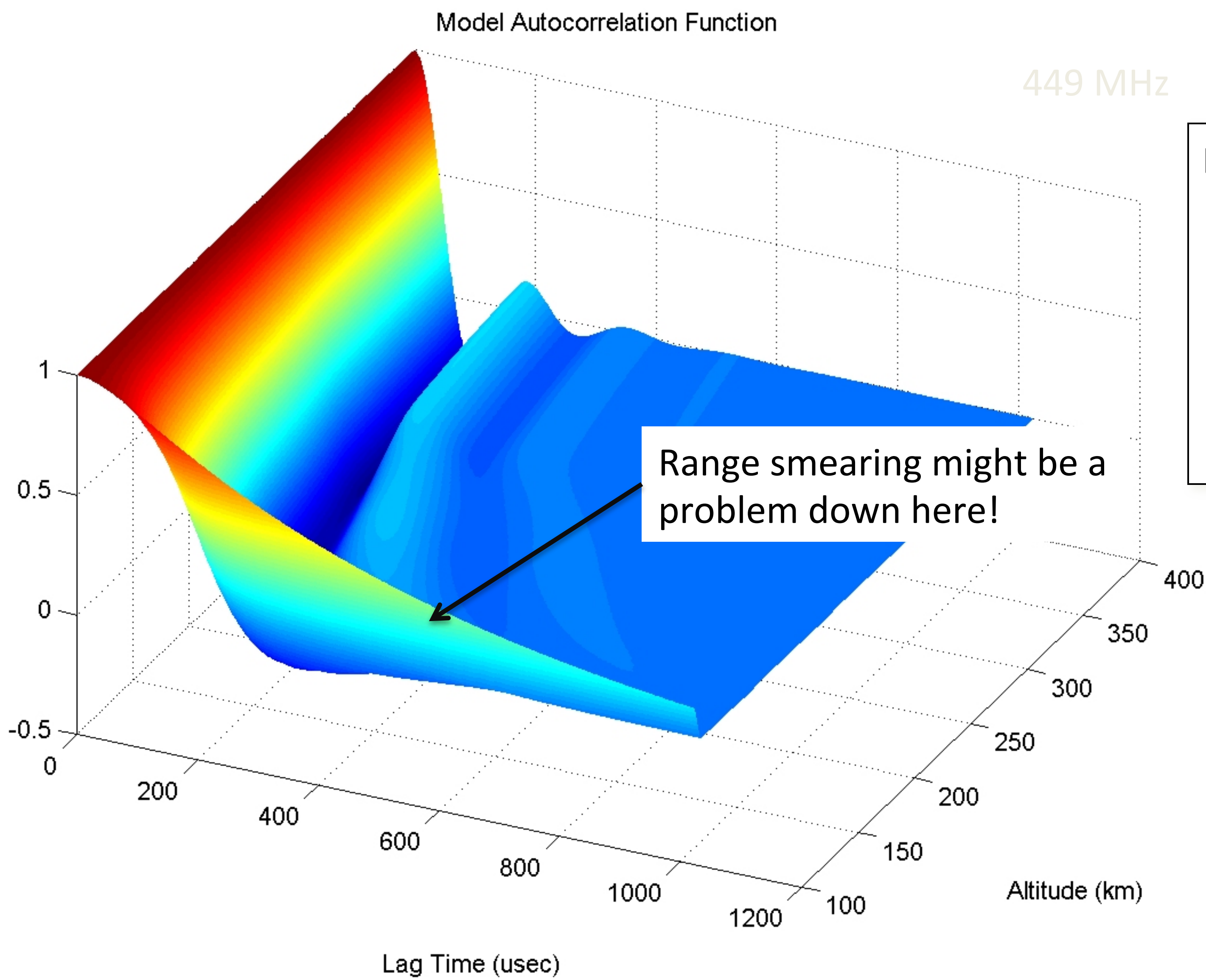


Incoherent Scatter Power Spectra



Dependencies:
N_e
T_e
T_i
V_i
m_i
ν_{in}

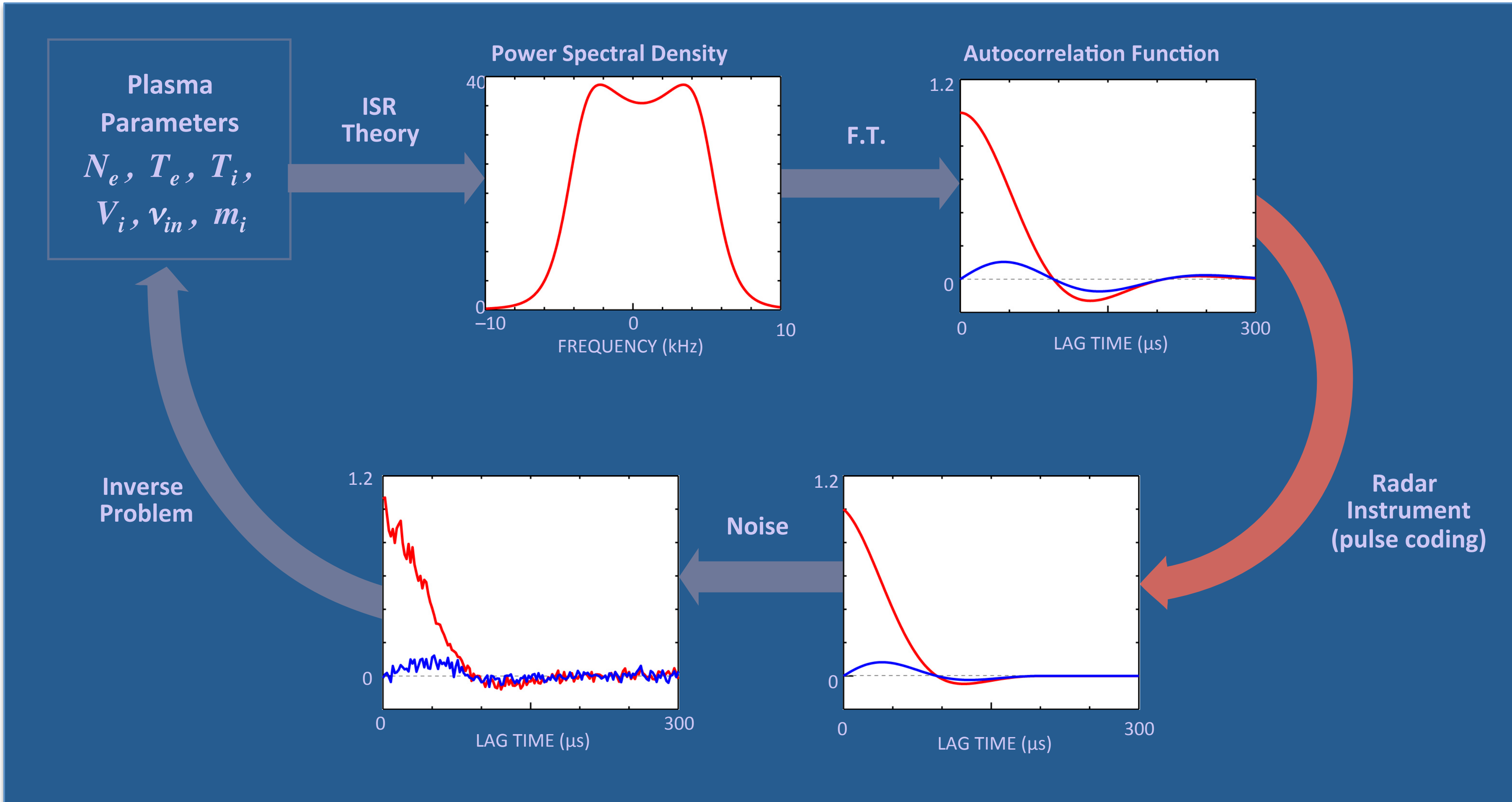
Incoherent Scatter Autocorrelation Functions



Dependencies:

N_e
T_e
T_i
V_i
m_i
ν_{in}

Incoherent Scatter Radar Data Fitting



v3

Ambiguity Functions

- Based on the principle of a 'matched filter'
  - Output of the matched filter maximizes the attainable SNR when both signal and white noise are applied to the input
  - Impulse response is the complex conjugate of the time reversed version of the signal

$$h(t) = s^*(t_M - t)$$

$$H(f) = S^*(f)\exp(-j2\pi f t_M)$$

where

$h(t)$ is the impulse response of the matched filter

$s(t)$ is the signal to be detected

t_M is the measurement time

t, f are time and frequency

Ambiguity Functions

- The ambiguity function is defined as the absolute value of the envelope of the output of a matched filter when the input to the filter is a Doppler shifted version of the original signal

$$\left|X(\tau, f)\right| = \left|\int_{-\infty}^{\infty} u(t) u^*(t-\tau) \exp(j2\pi f t)\, dt\right|$$

$u(t)$ is the complex envelope of the signal

τ is the additional delay

f is the frequency shift (Doppler)

Ambiguity Functions

For $u(t)$ with unit energy

$$|X(\tau, f)| \le |X(0,0)| = 1$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |X(\tau, f)|^2 d\tau df = 1$$

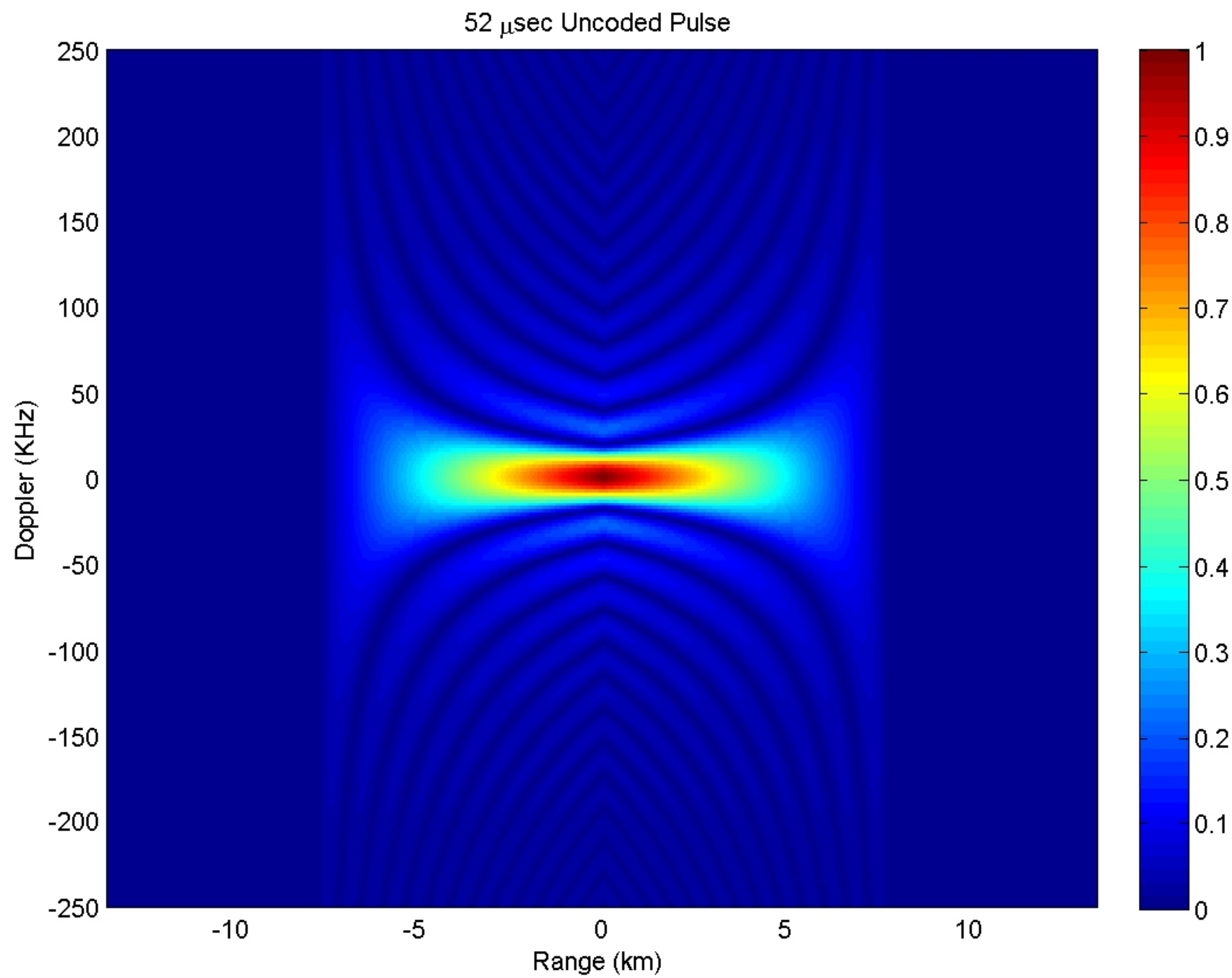
and for all signals

$$|X(-\tau, -f)| = |X(\tau, f)|$$

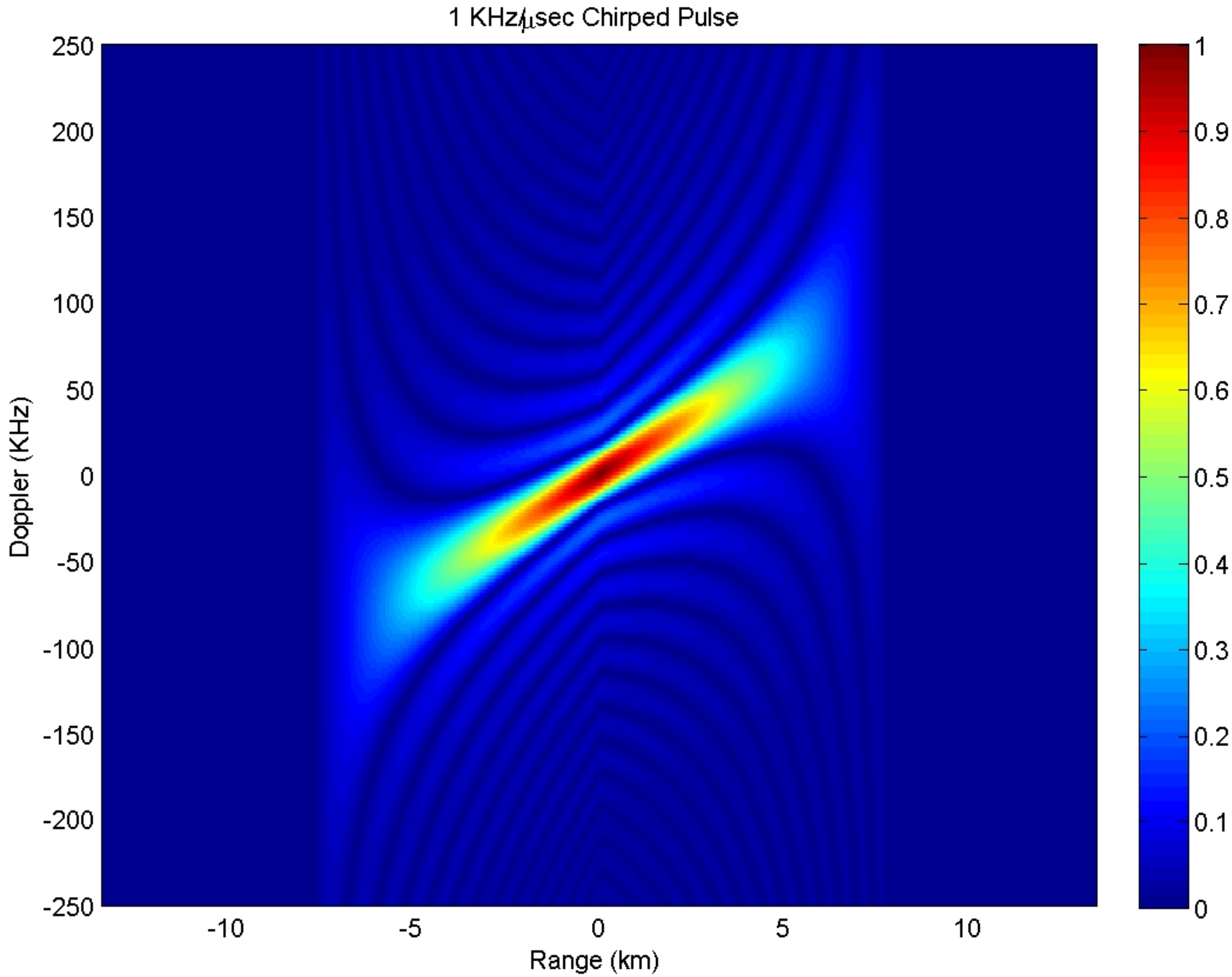
if $u(t) \leftrightarrow |X(\tau, f)|$

then $u(t)\exp(j\pi k t^2) \leftrightarrow |X(\tau, f + k\tau)|$

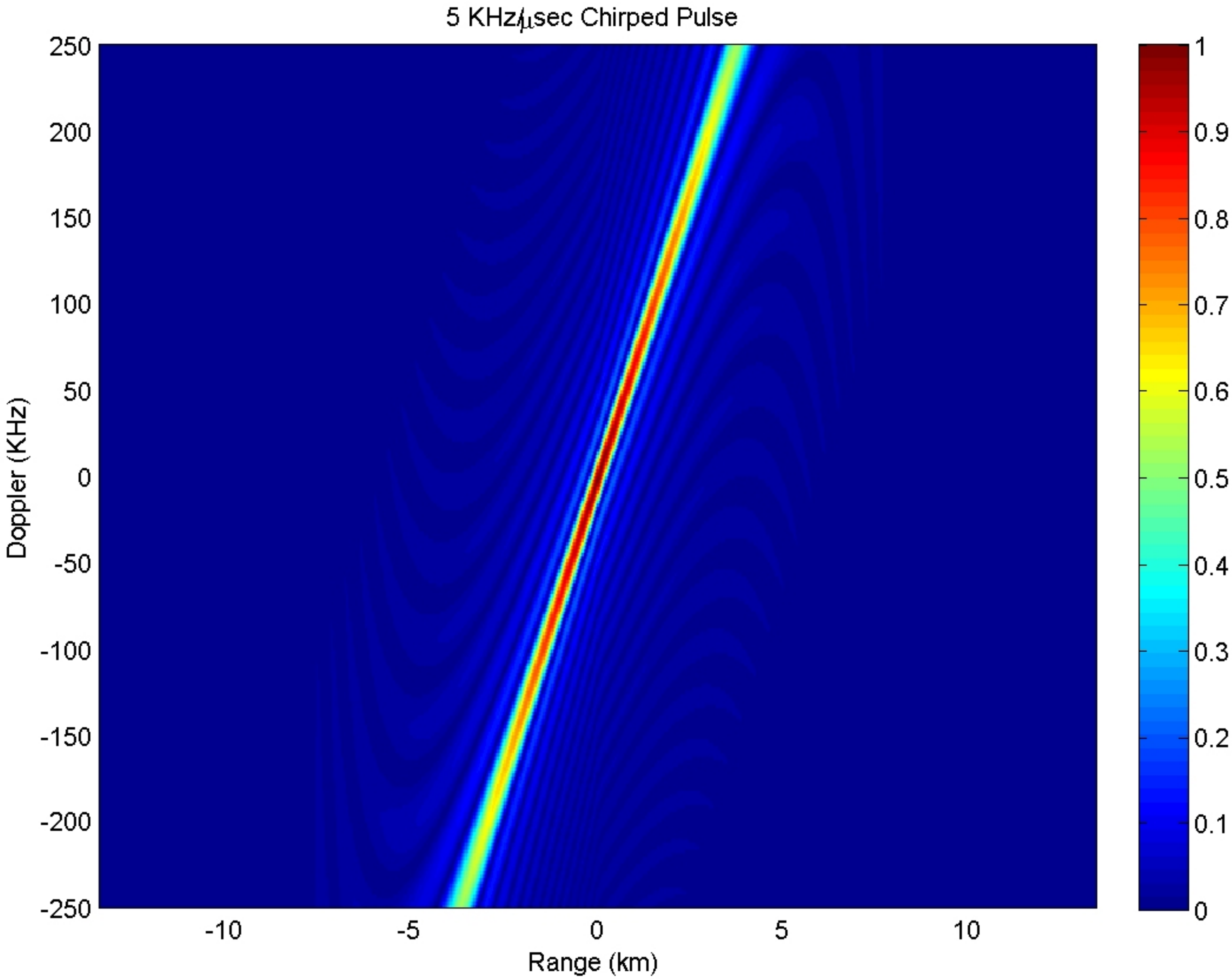
Ambiguity Function



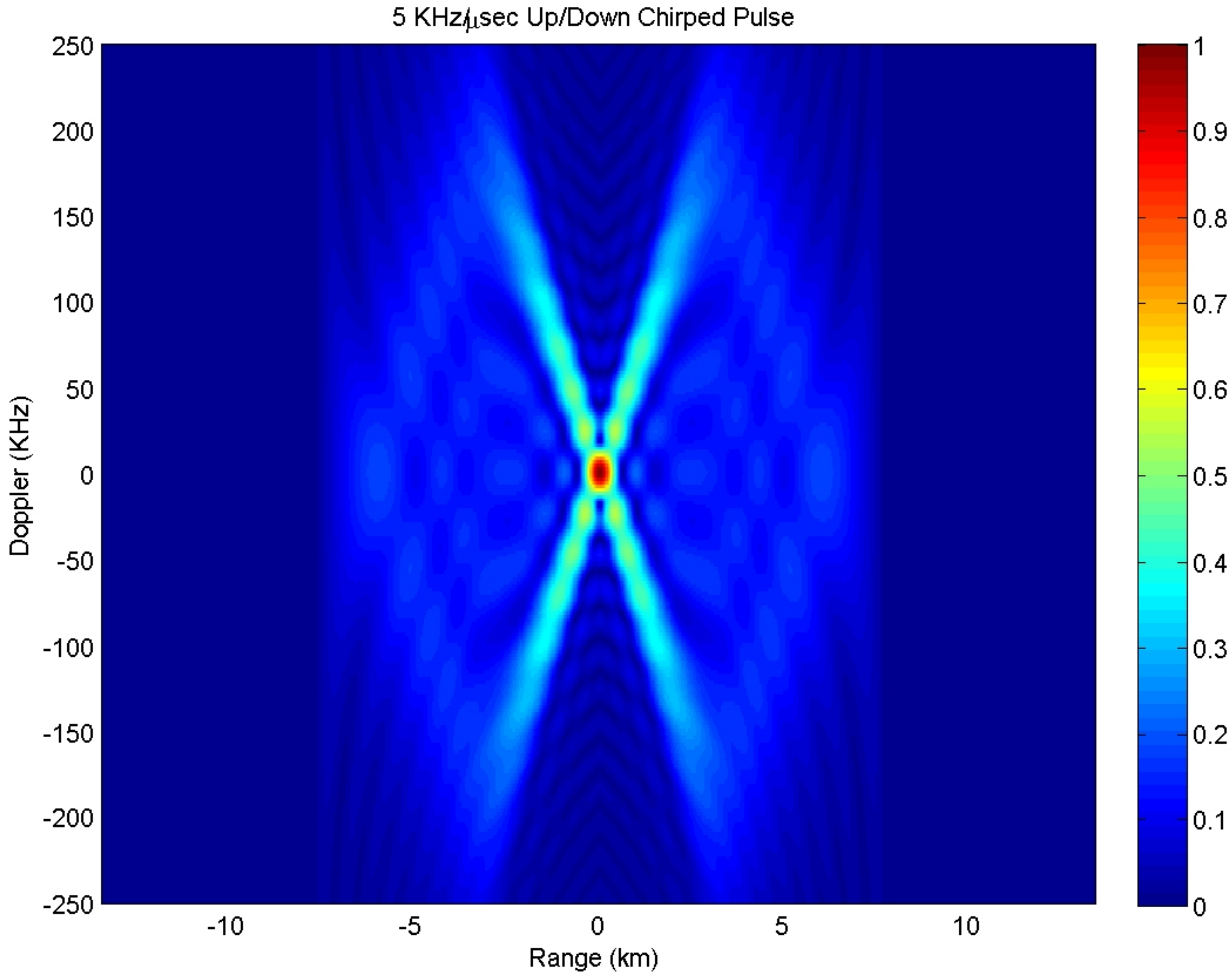
Ambiguity Function



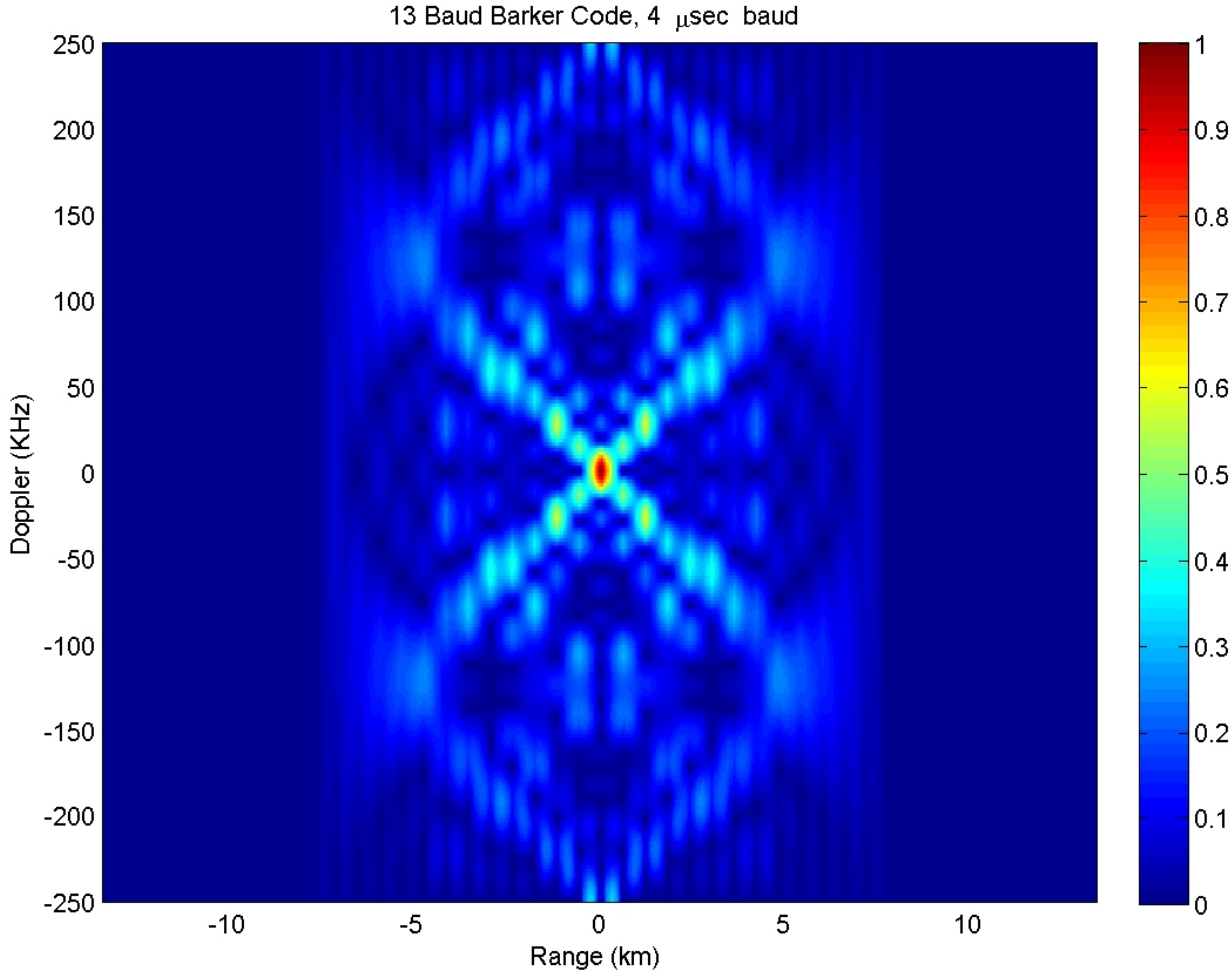
Ambiguity Function



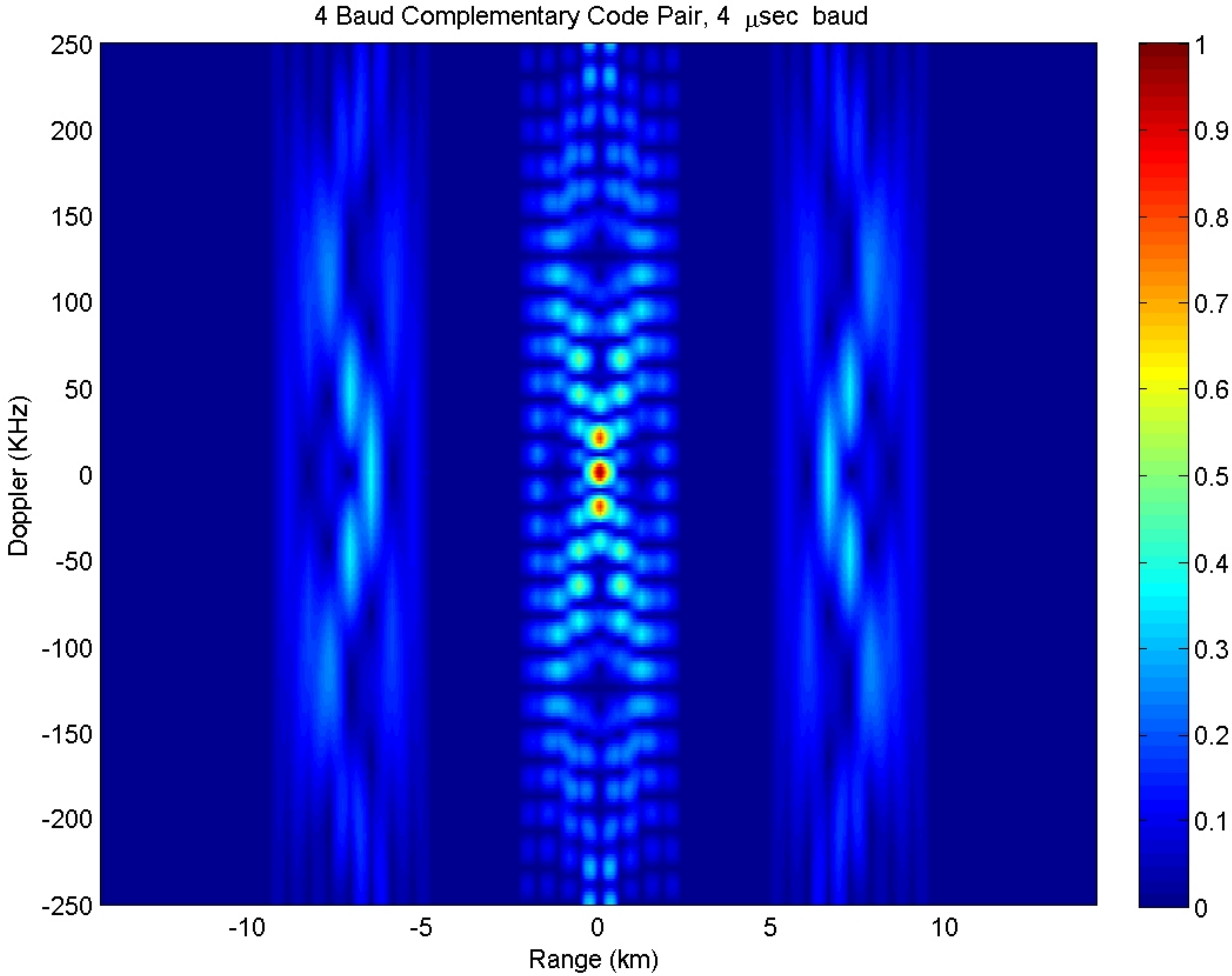
Ambiguity Function



Ambiguity Function

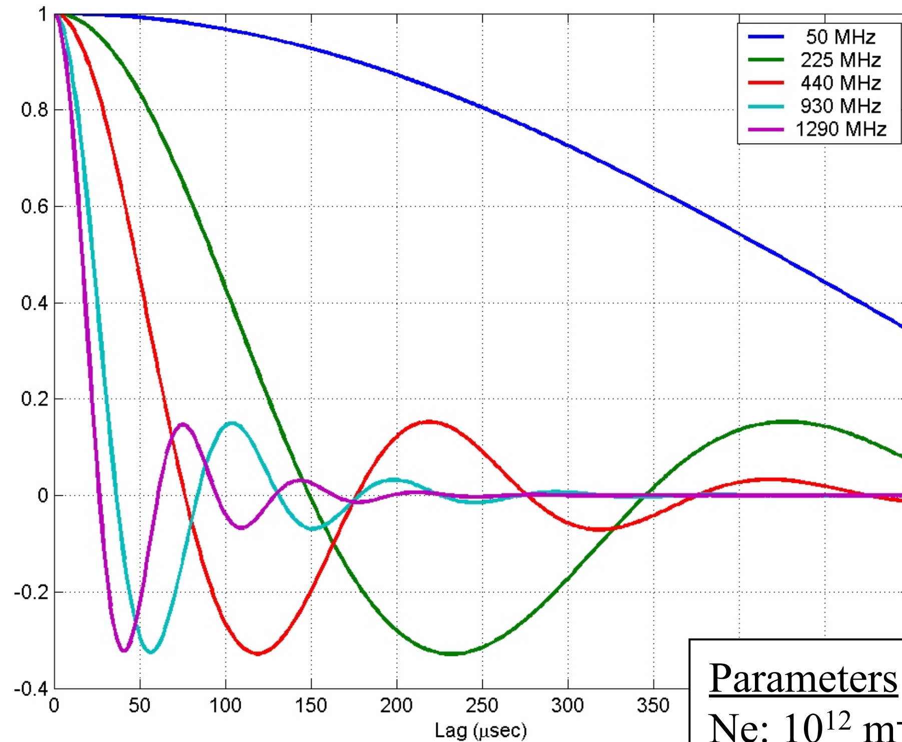


Ambiguity Function



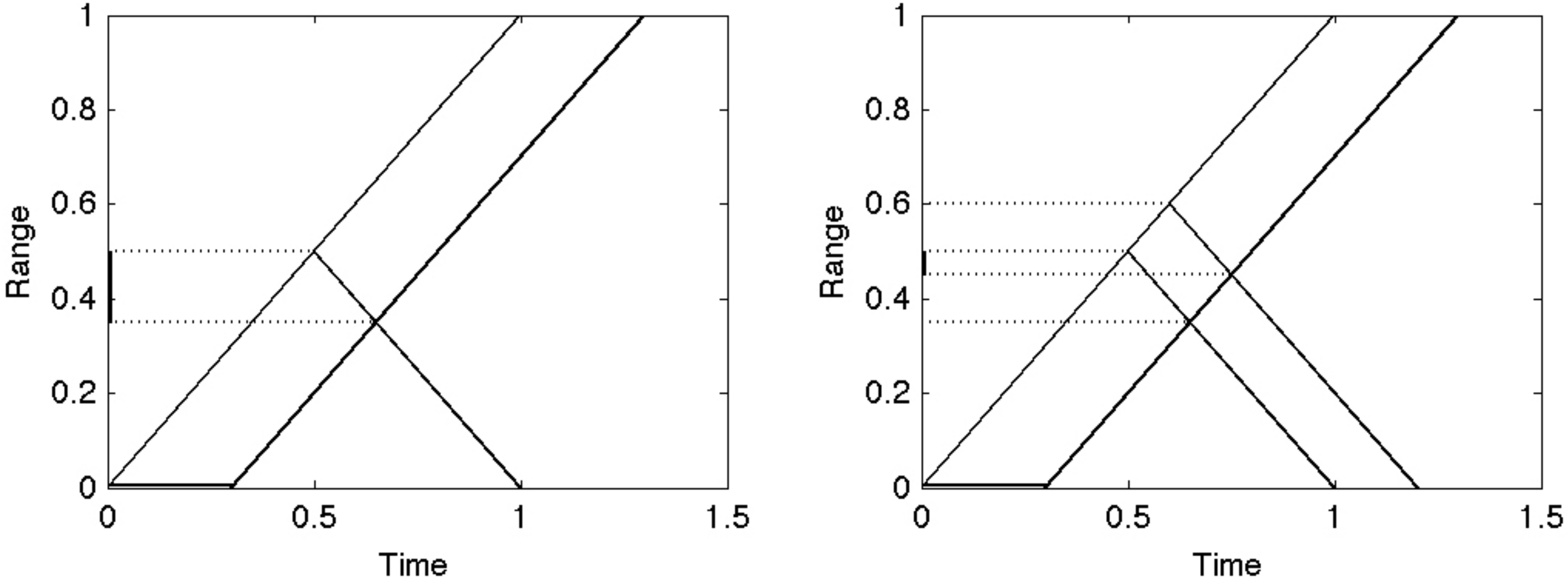
ACF measurements, can we use phase coding?

- Yes, but we must be careful!
- Barker codes, for instance, can be used if the total code length is sufficiently short (less than the correlation time of the medium – Gray and Farley, 1973). This only gives us power (0-lag) information!
- Other classes of modulation are also available that, when incoherently averaged, provide good range resolution at the expense (usually) of increased bandwidth and processing complexity
  - Alternating Codes (Lehtinen and Haggstrom, 1987)
  - Coded Long Pulse (Sulzer, 1986)
  - Compressed Alternating Codes
  - Multipulse (not used much for ISR any more because of the superior performance of other techniques)
  - A good, slightly dated reference for many of these techniques is (Sulzer, 1989)
- Finally, at Arecibo they often have too much SNR and use phase coding to obtain more estimates of the acf.

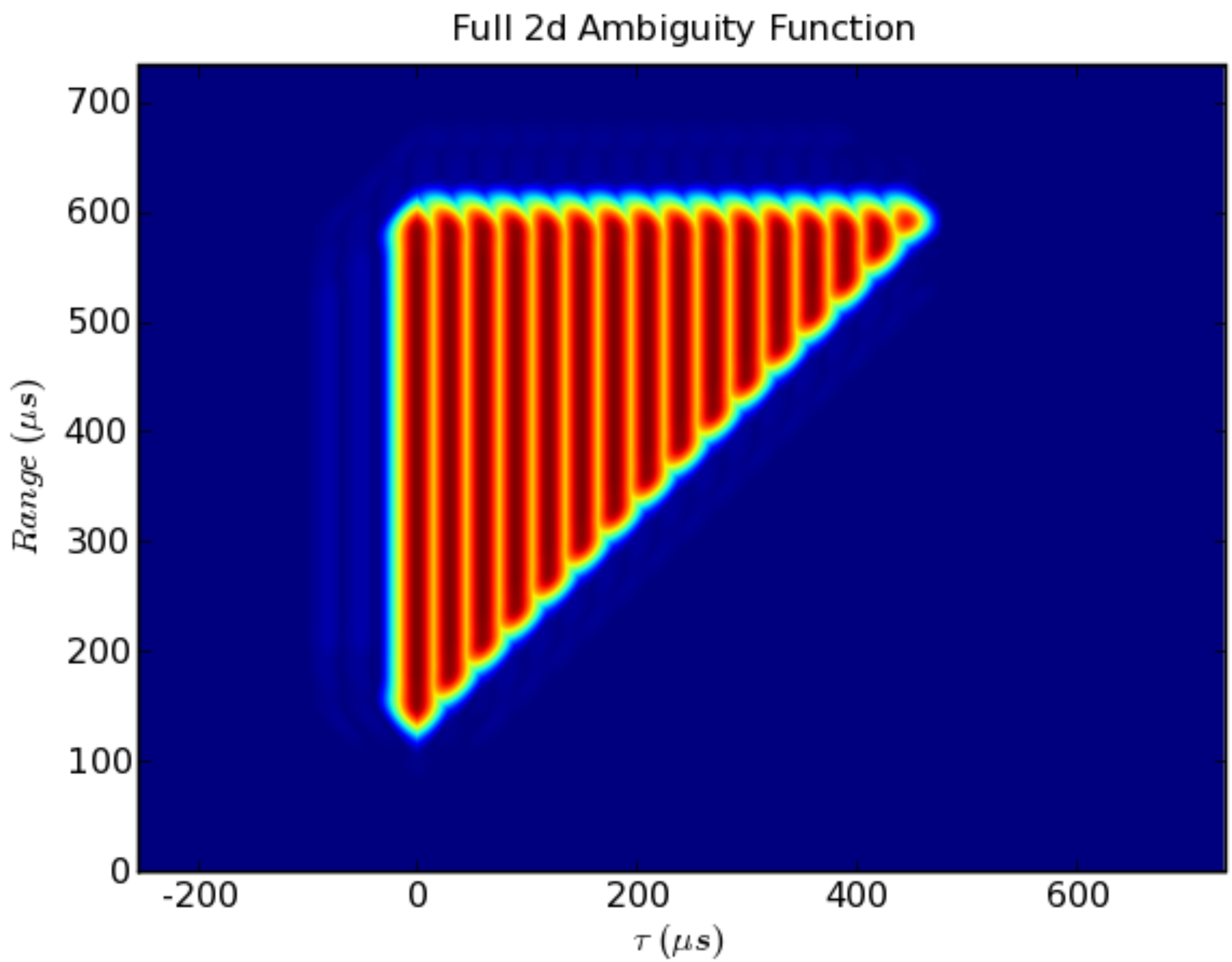


<u>Parameters</u>
Ne: 10^{12} m^{-3}
Ti: 1000 K
Te: 2000 K
Comp: 100% O^{+}
ν_{in}: 10^{-6} KHz

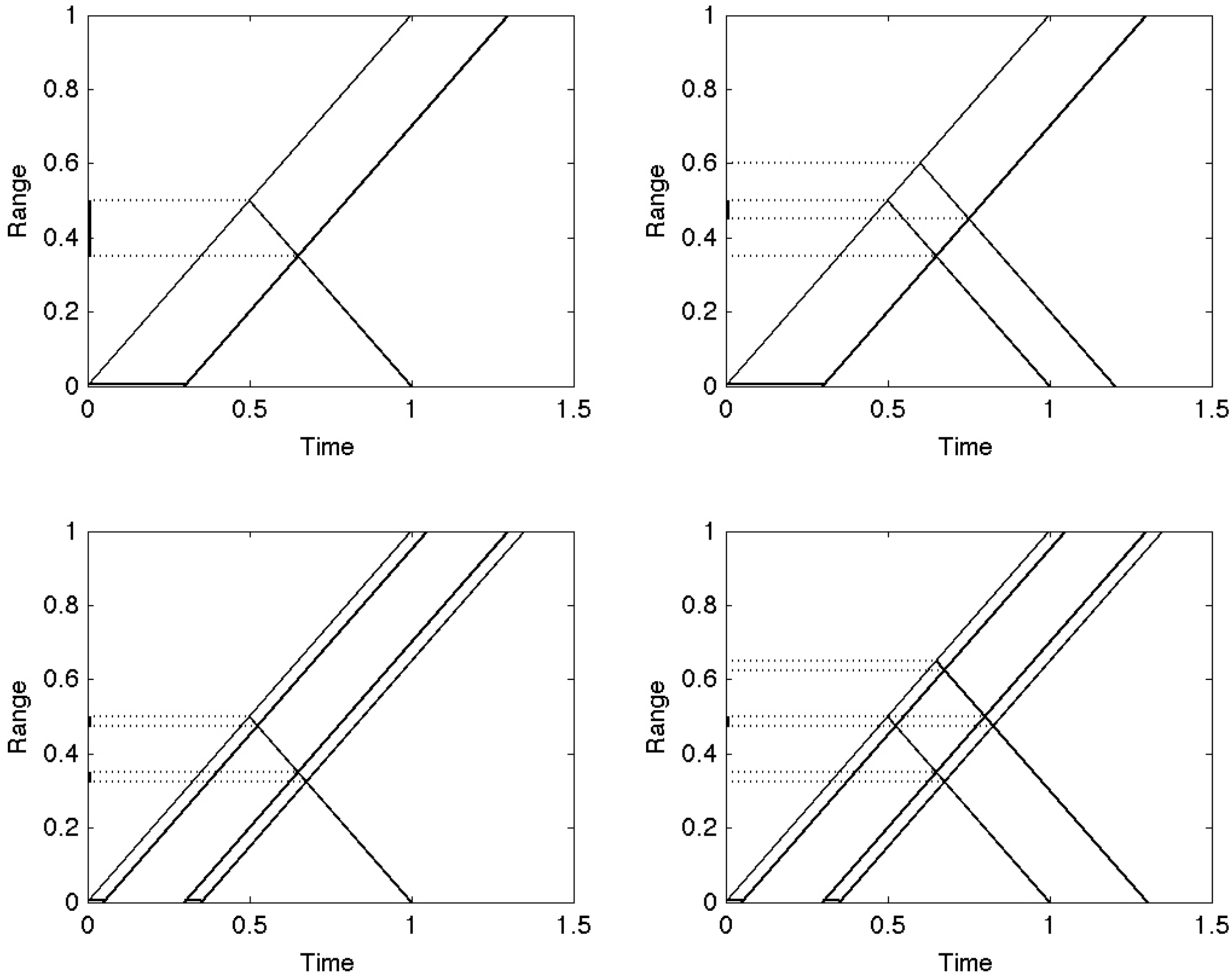
Measuring ACFs



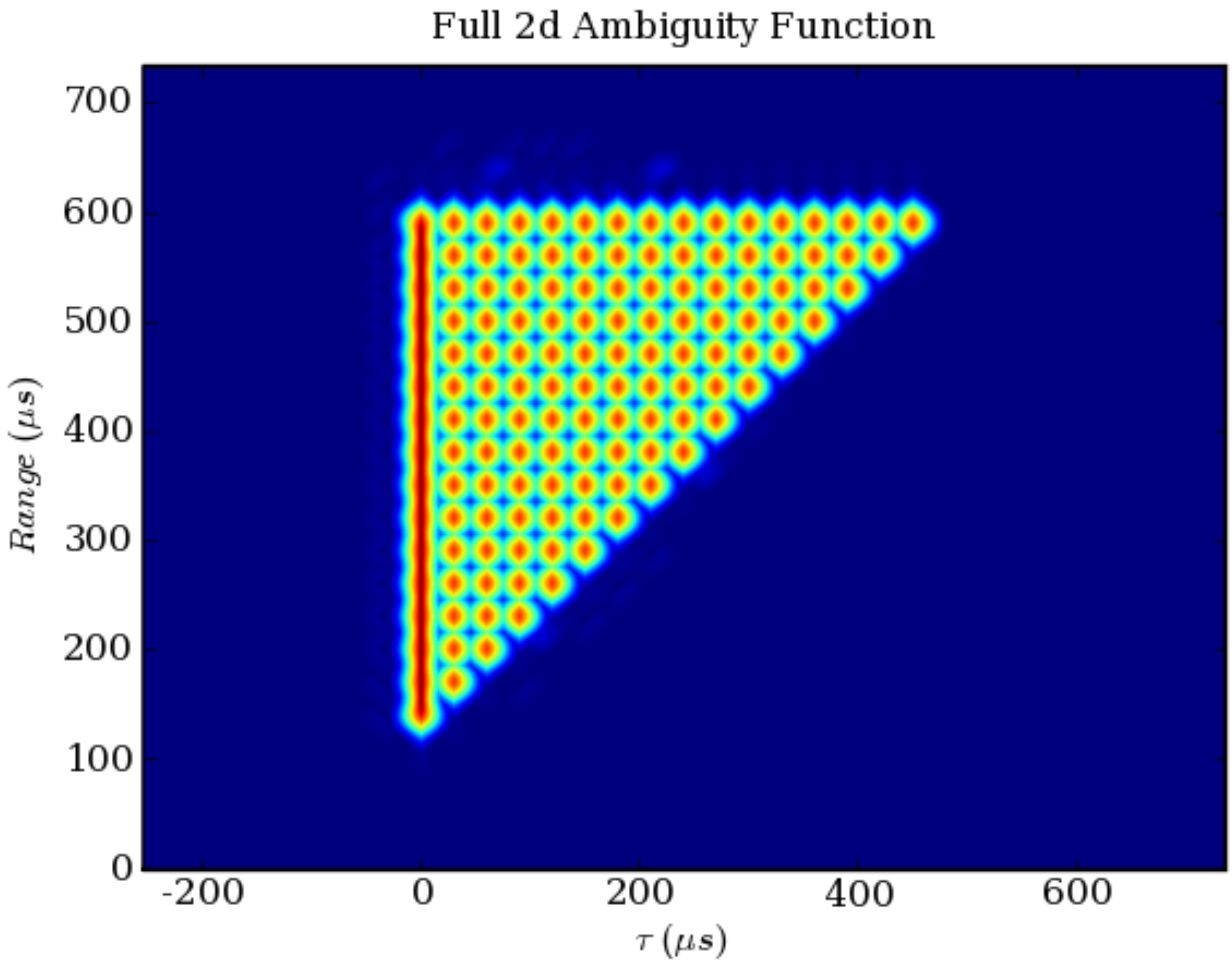
Ambiguity Function
(smearing in range and lag)



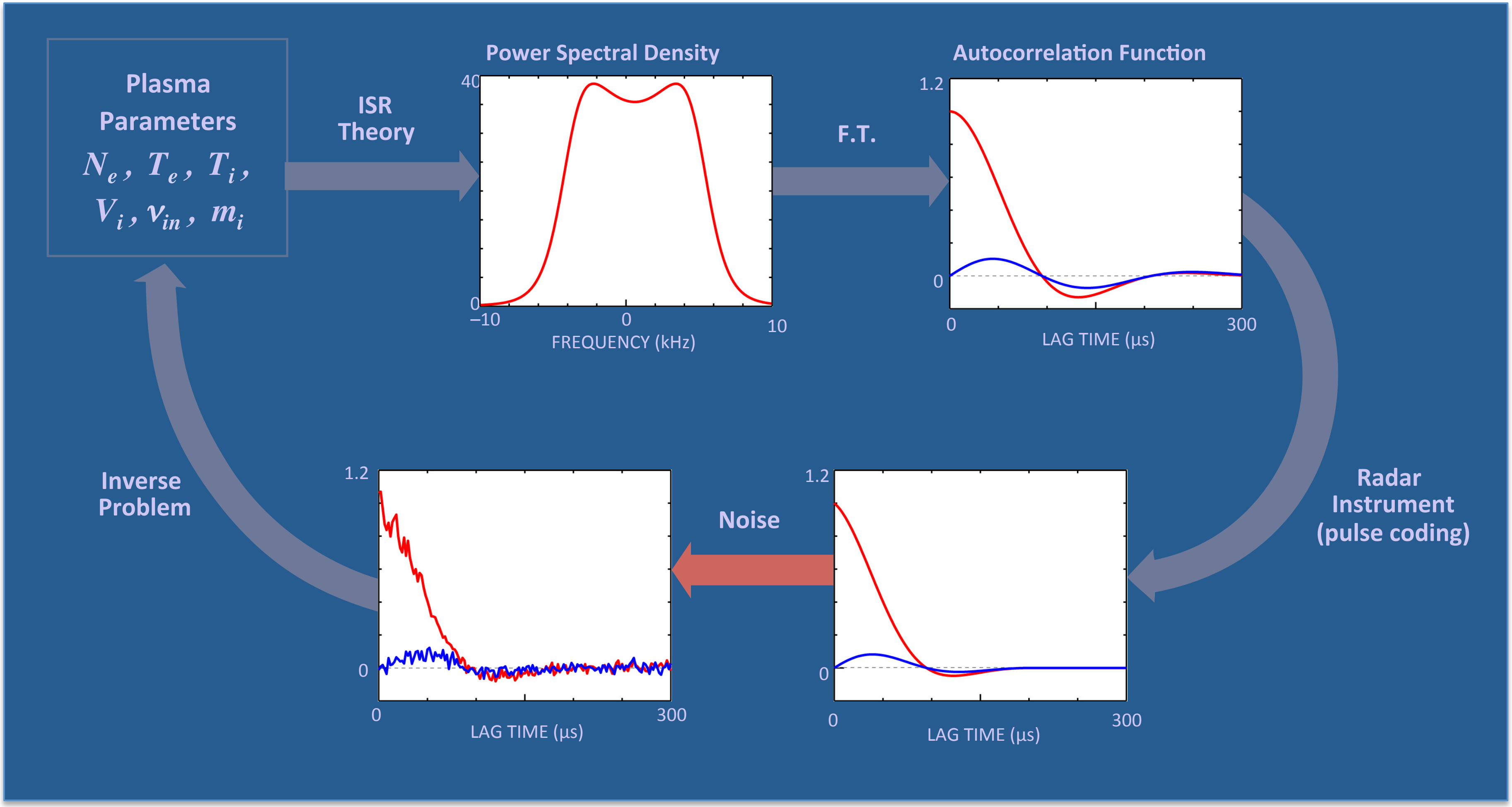
Range Smearing



Ambiguity Function Alternating Code (smearing in range and lag)

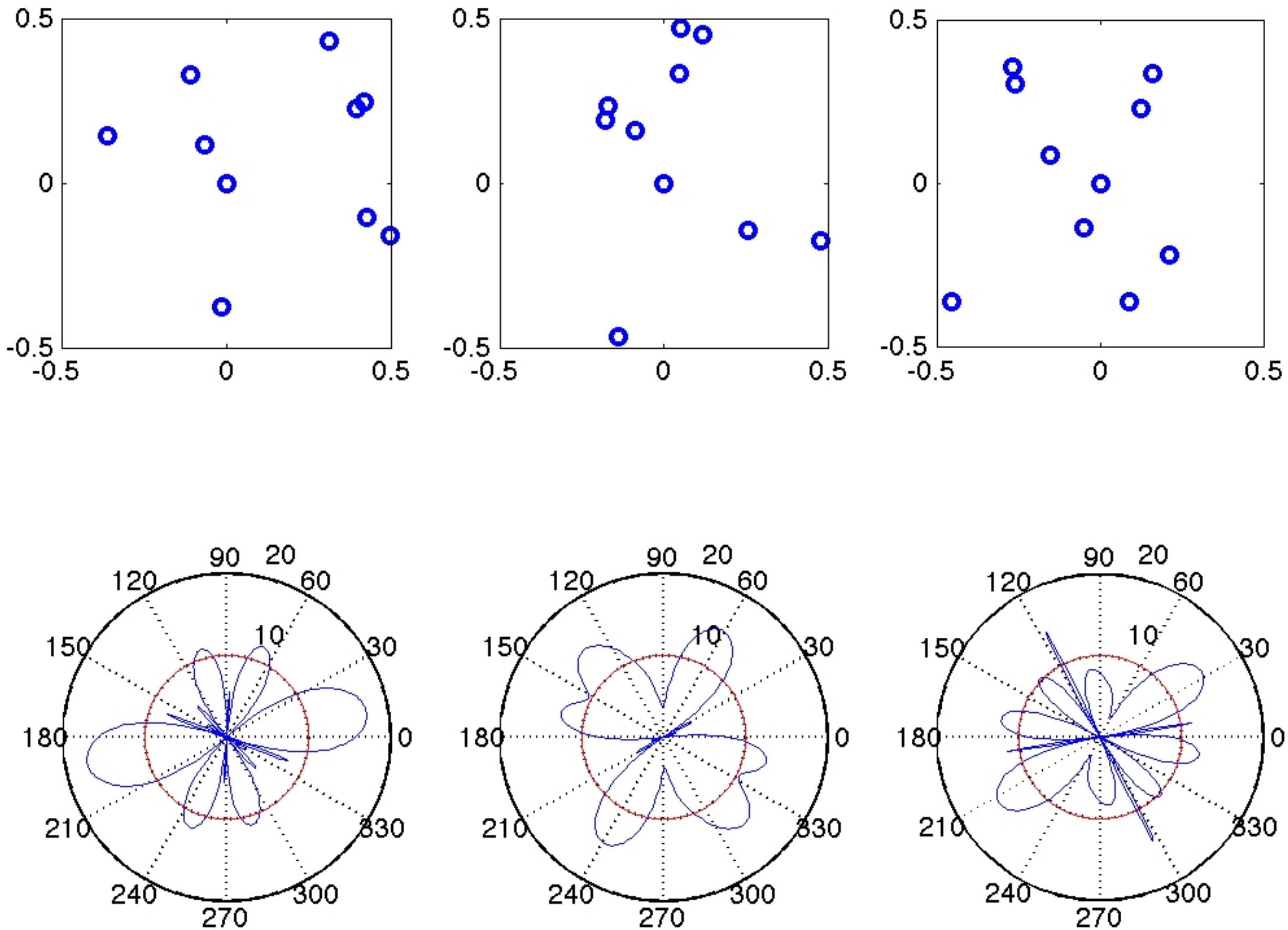


Incoherent Scatter Radar Data Fitting

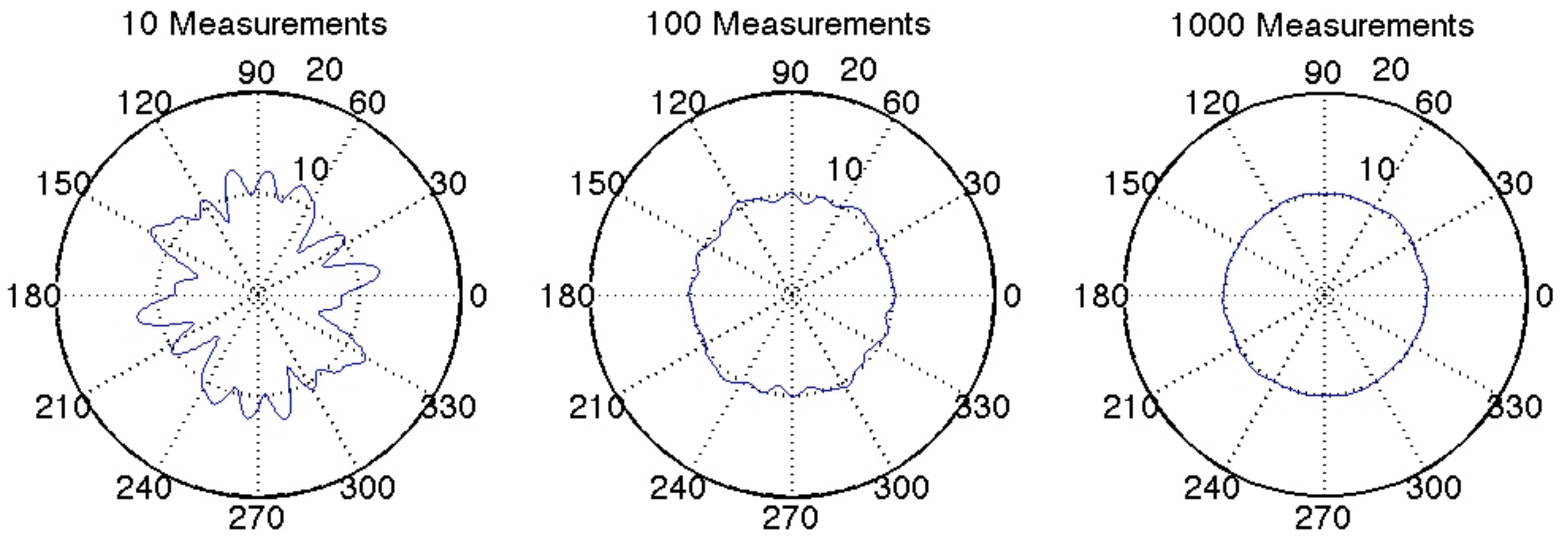


v3

‘Incoherent’ electron positions



Incoherent Integration



ISR Signal Strength

Differential received power

$$dP_r = \frac{P_T L \lambda^2 G_{TX}(\theta,\phi) G_{RX}(\theta',\phi') n_e(\theta,\phi,r)\sigma}{(4\pi)^3 r^4} dV$$

Assuming a narrow antenna beam and sufficiently short pulse

$$dV = \left(\frac{c\tau_P}{2}\right) r d\theta \cdot r \sin\theta \cdot d\phi$$

$$P_r(r) \approx \frac{P_T L \lambda^2 c\tau_P n_e(r)\sigma}{2(4\pi)^2 r^2} \frac{1}{4\pi} \iint G^2(\theta,\phi) \sin\theta \cdot d\theta \cdot d\phi$$

Defining the mean squared gain (backscatter gain) as

$$G_{BS} = \frac{1}{4\pi} \iint G^2(\theta,\phi) \sin\theta \cdot d\theta \cdot d\phi$$

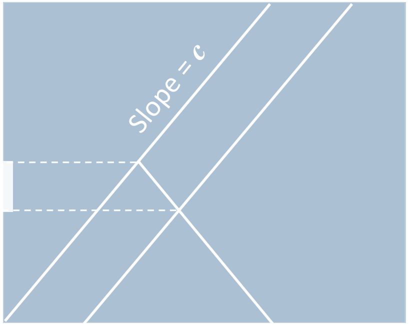
and from Hagen and Baumgartner (1996)

$$G_{BS} \approx C_{BS} \frac{4\pi A_{eff}}{\lambda^2}$$

$$P_r(r) \approx \frac{P_T L c\tau_P C_{BS} A_{eff} n_e(r)\sigma}{2(4\pi) r^2}$$

$$P_r(r) \approx \frac{P_T L c\tau_P C_{BS} A_{eff}}{8\pi r^2} \frac{n_e(r)\sigma_e}{\left(1 + k^2\lambda_D^2\right)\left(1 + k^2\lambda_D^2 + T_r\right)}$$

$$P_n = k_B T_{sys} BW$$



- P_T = transmitter peak power
- L = transmit feed line losses
- c = speed of light
- τ_P = transmit pulse duration
- C_{BS} = backscatter gain constant
- A_{eff} = antenna effective aperture
- n_e = electron number density
- σ_e = electron radar cross-section
- $k = 2\pi/\lambda$ = radar wave number
- λ_D = plasma debye length
- T_r = electron to ion temperature ratio
- k_B = Boltzmann constant
- T_{sys} = system noise temperature
- BW = receiver bandwidth

ISR Signal Strength

Signal-to-noise ratio

$$SNR = \frac{P_r}{P_n} = \frac{(P_T L)(C_{BS} A_{eff})\tau_P}{T_{sys} BW} \cdot \frac{c}{8\pi r^2 k_B} \frac{n_e(r)\sigma_e}{(1 + k^2\lambda_D^2)(1 + k^2\lambda_D^2 + T_r)}$$

$$std\left(\frac{\hat{P}_r}{P_r}\right) \propto \frac{1}{\sqrt{K_{meas}}}\left(\frac{P_r + P_n}{P_r}\right) = \frac{1}{\sqrt{K_{meas}}}\left(1 + \frac{1}{SNR}\right)$$

To obtain an $SNR = 1$ with the following parameters

$L = 1$ (no feed line losses) $C_{BS} = 0.4$

$\tau_P = 300\ \mu$sec (45 km range resolution) $n_e = 10^{11}$ m^{-3}

$T_{sys} = 100$ K $BW = 50$ kHz

$k^2\lambda_D^2 = 0$ (sufficiently high n_e) $T_r = 1$

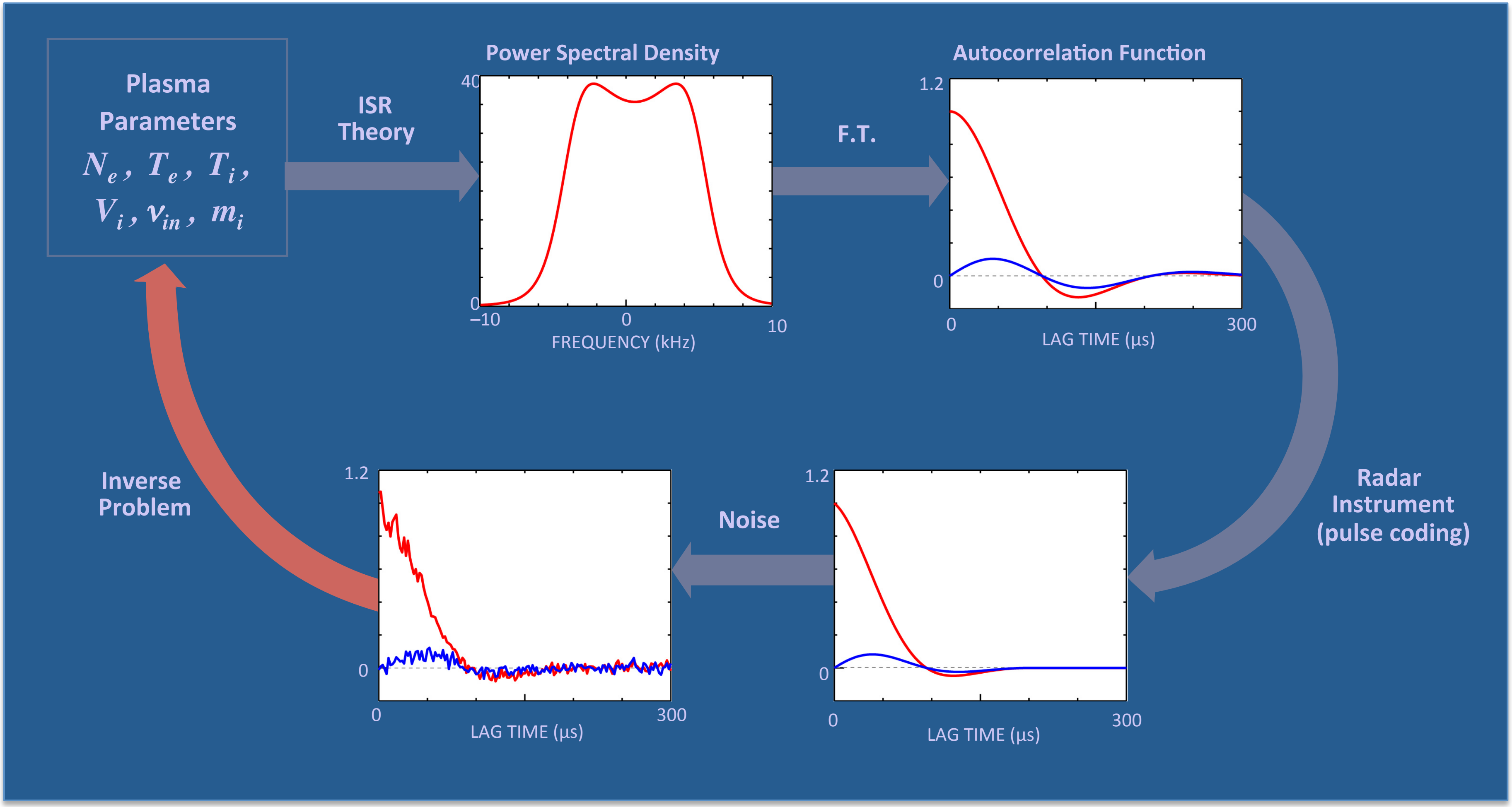
we need

$P_T A_{eff} = 8.7 \times 10^8$ Wm^2

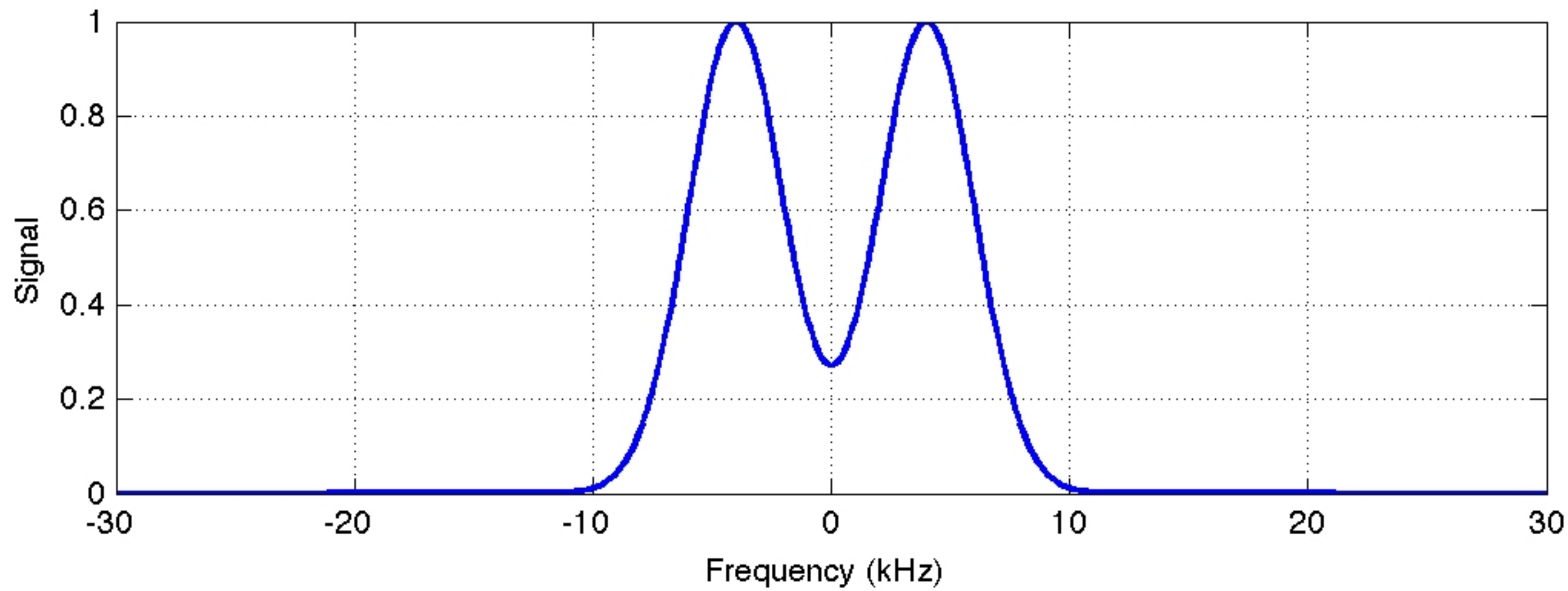
for $A_{eff} = 400$ m^2

$P_T = 2.2$ MW

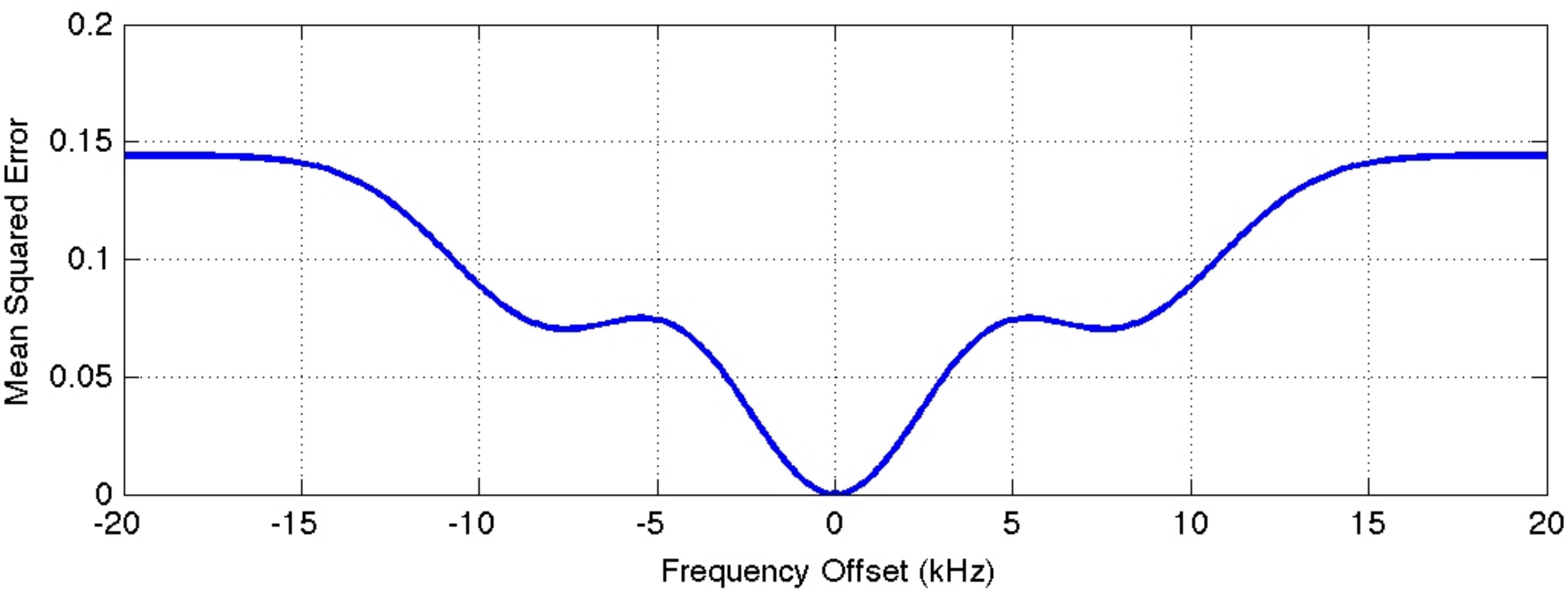
Incoherent Scatter Radar Data Fitting



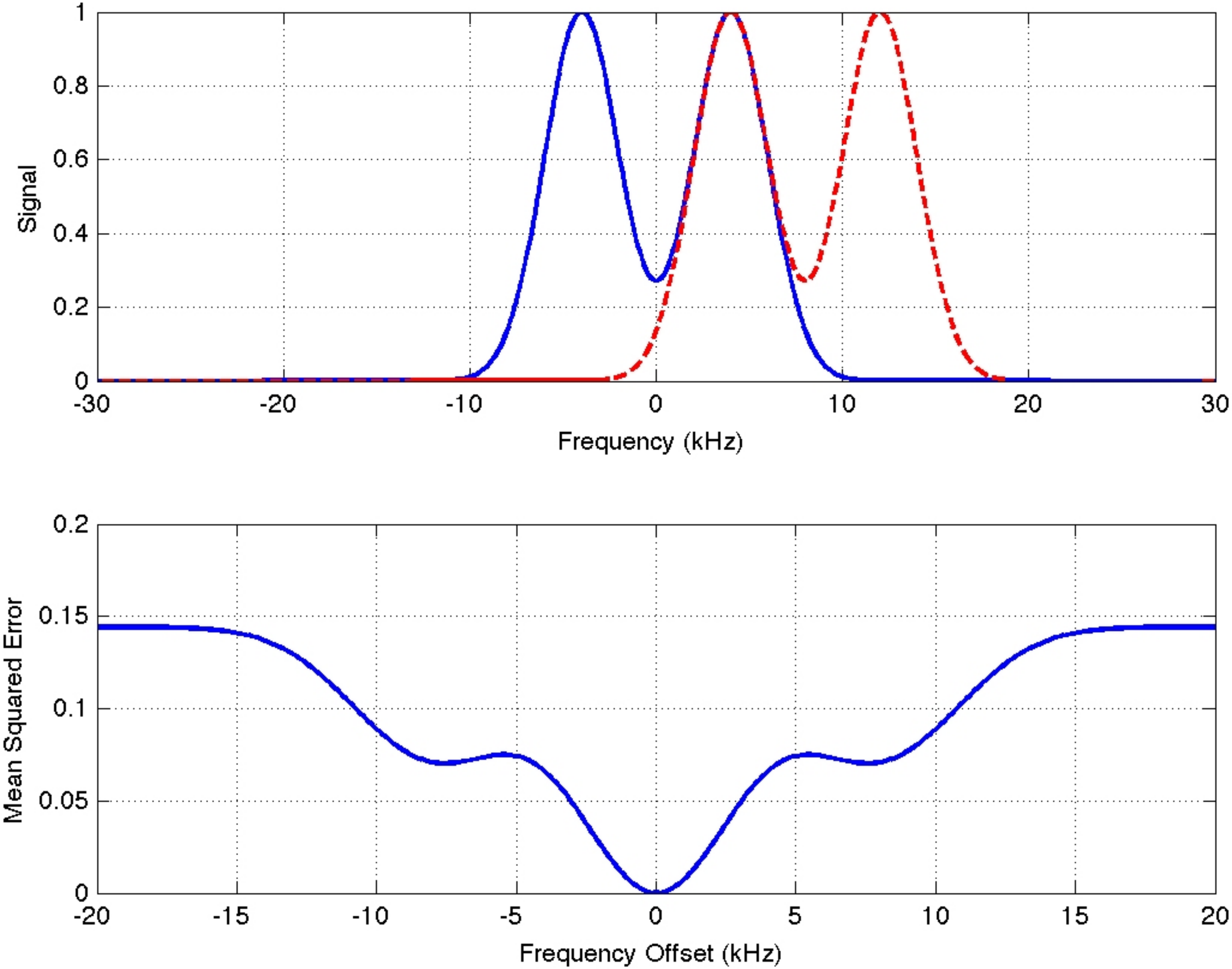
v3



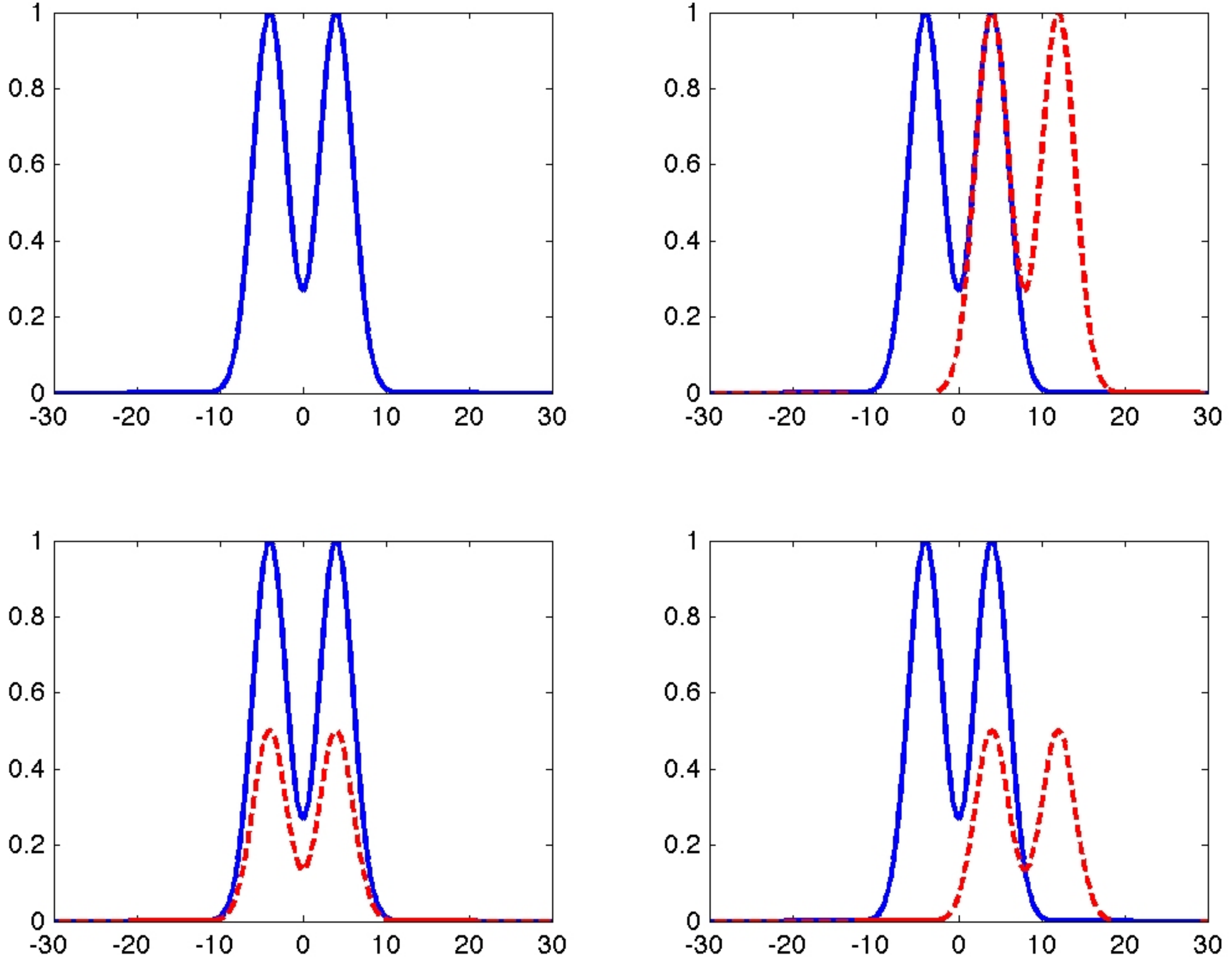
Signal
1
0.8
0.6
0.4
0.2
0
-30
-20
-10
0
10
20
30
Frequency (kHz)



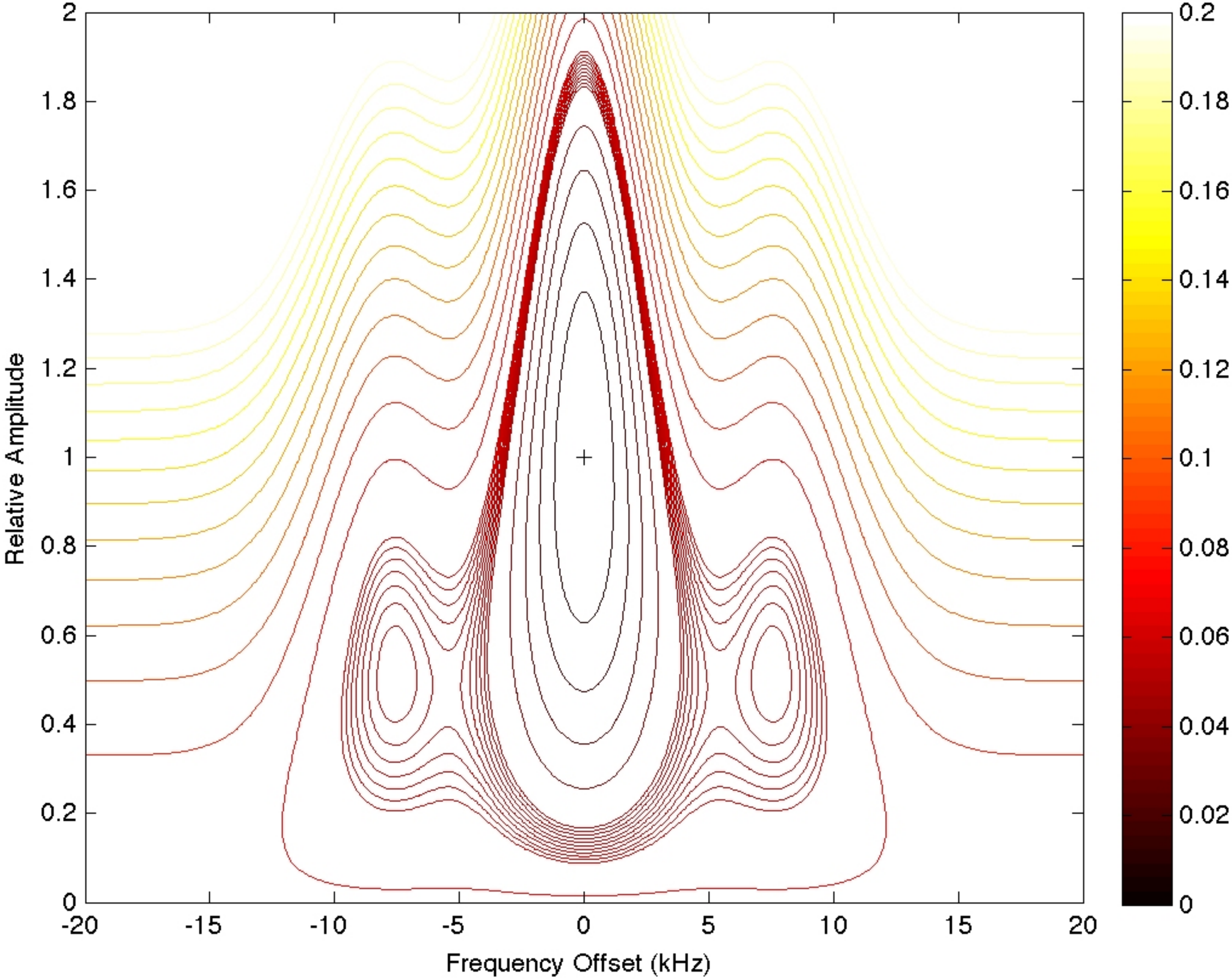
Mean Squared Error
0.2
0.15
0.1
0.05
0
-20
-15
-10
-5
0
5
10
15
20
Frequency Offset (kHz)



Signal
Frequency (kHz)
Mean Squared Error
Frequency Offset (kHz)



1
0.8
0.6
0.4
0.2
0
-30
-20
-10
0
10
20
30


Relative Amplitude
Frequency Offset (kHz)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
-20
-15
-10
-5
0
5
10
15
20
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

ISR-Measurable Parameters

BASIC PARAMETERS

Ne, Te, Ti, Vi, ν_{in}, ion composition

ELECTRODYNAMIC PARAMETERS

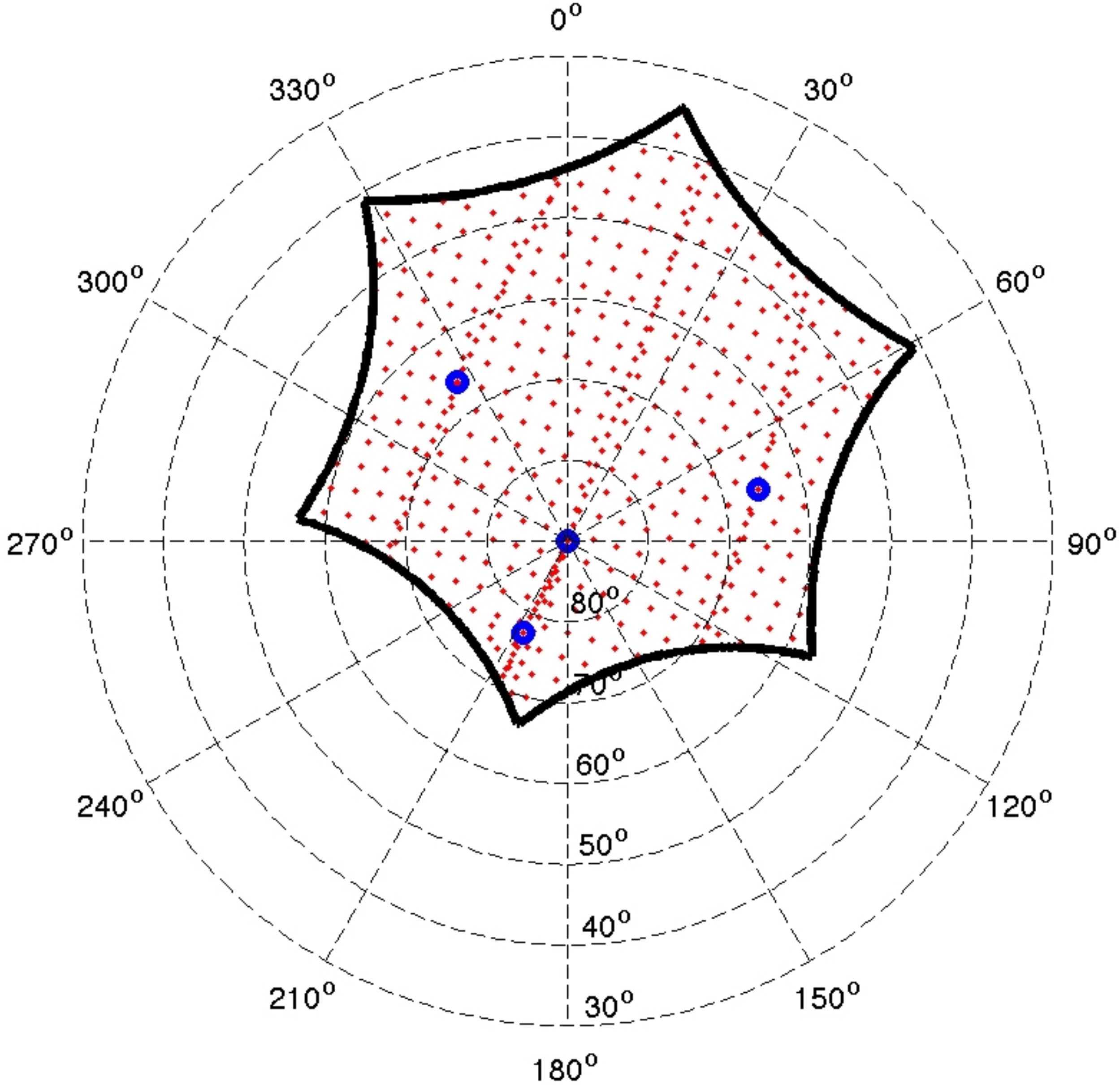
E, σ_H and Σ_H, σ_P and Σ_P, $J_\perp$ and $J_{||}$

NEUTRAL PARAMETERS

U_{merid}, U, T_{inf}

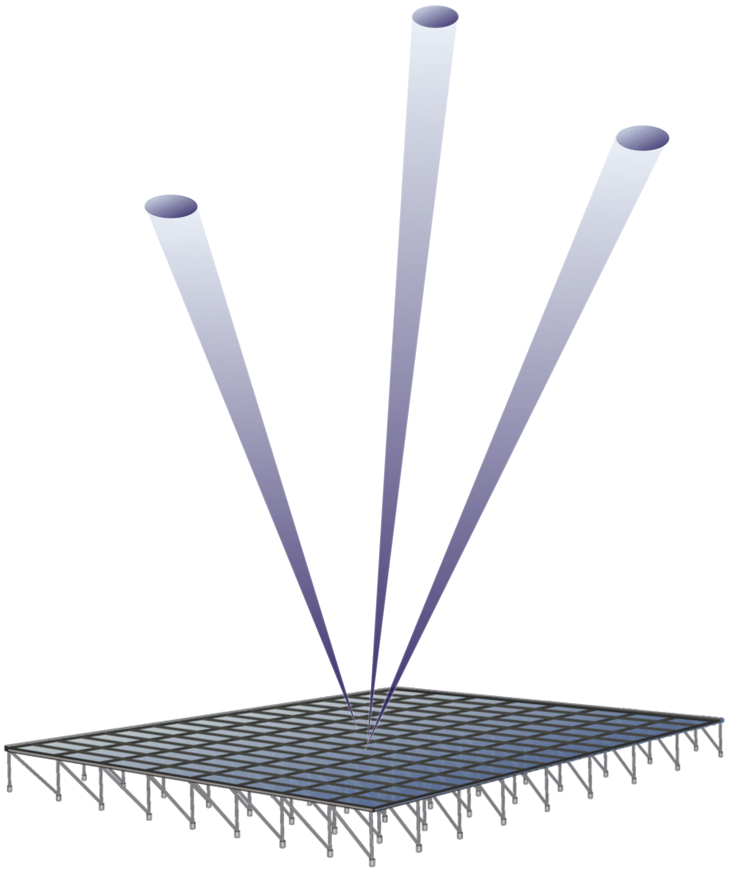
ENERGY DEPOSITION

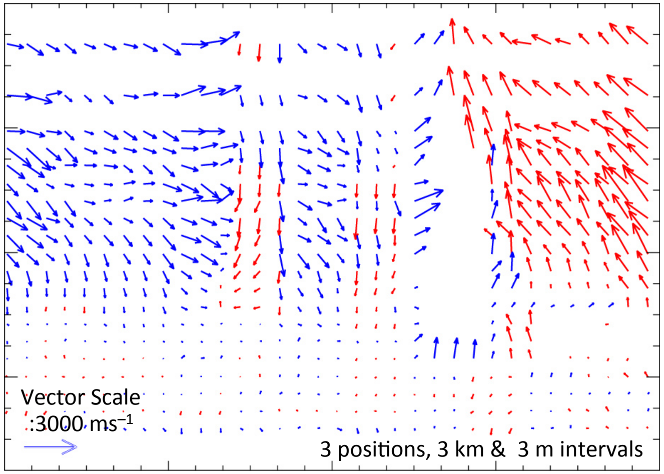
f(E)

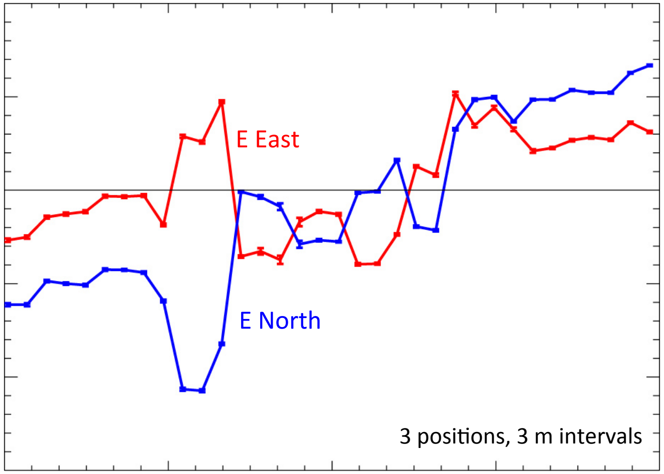


0°
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°
80°
70°
60°
50°
40°
30°

AMISR Ion Velocity Estimation







v12

E-region Electrodynamics

Ion momentum equation

$$n_i m_i \frac{D\vec{V}_i}{Dt} = -\vec{\nabla}P_i + n_i m_i \vec{g} + n_i m_i \Omega_i \left(\frac{\vec{E}}{B} + \frac{\vec{V}_i \times \vec{B}}{B} \right) - n_i m_i \nu_{in} \left(\vec{V}_i - \vec{U}_n \right)$$

Steady state

$$0 = \Omega_i \left(\frac{\vec{E}}{B} + \frac{\vec{V}_i \times \vec{B}}{B} \right) - \nu_{in} \left(\vec{V}_i - \vec{U}_n \right)$$

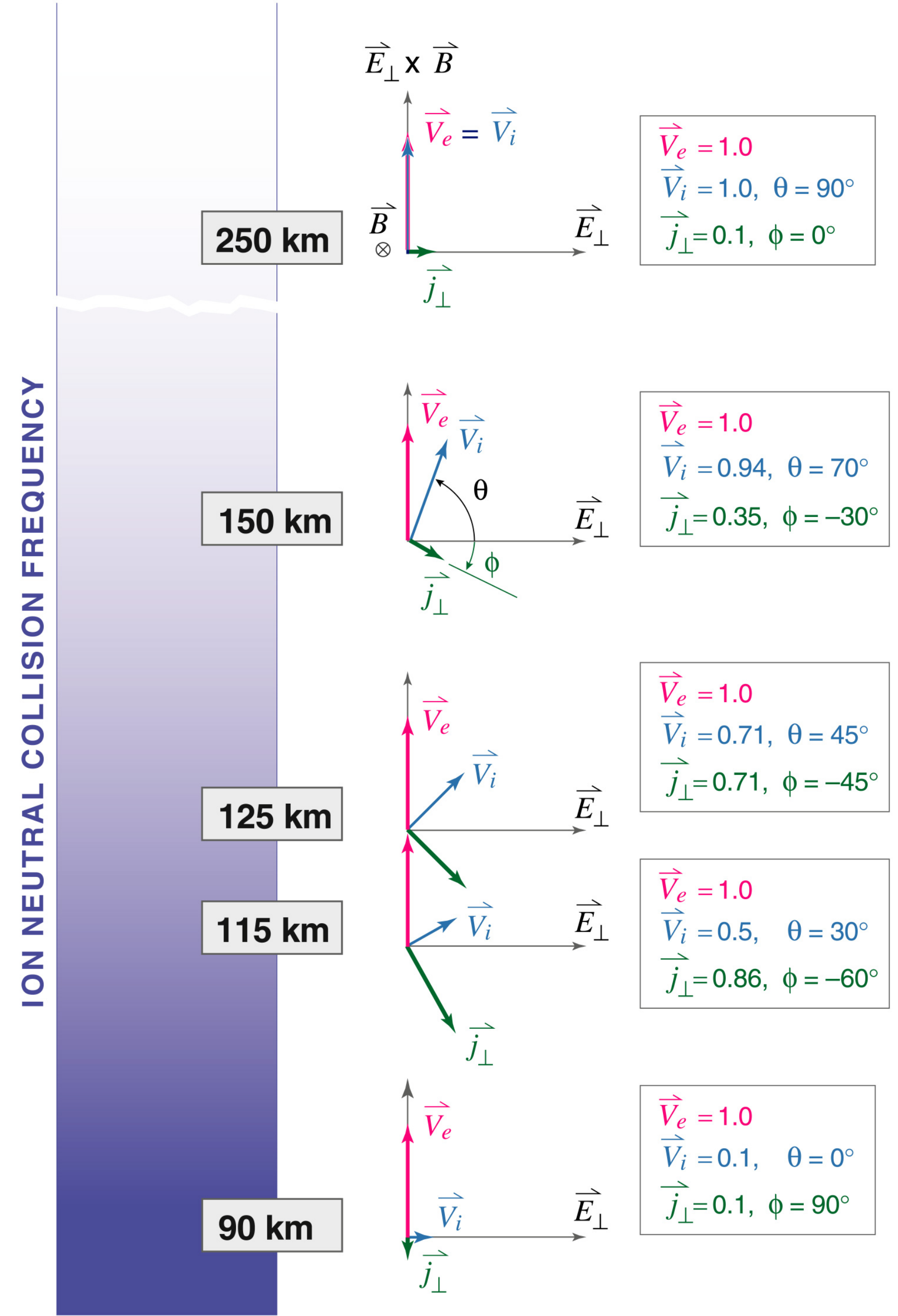
Ion motion with no neutral wind

$$\theta = \arctan\left(\frac{\Omega_i}{\upsilon_{in}} \right)$$

$$\left| \vec{V}_i(z) \right| = \sin\theta \, \frac{E}{B}$$

Ion motion with neutral wind

$$\vec{V}_i(z) = U_n(z) + \frac{\Omega_i}{\upsilon_{in}} \left[\frac{\vec{E}}{B} + \frac{\vec{V}_i(z) \times \vec{B}}{B} \right]$$



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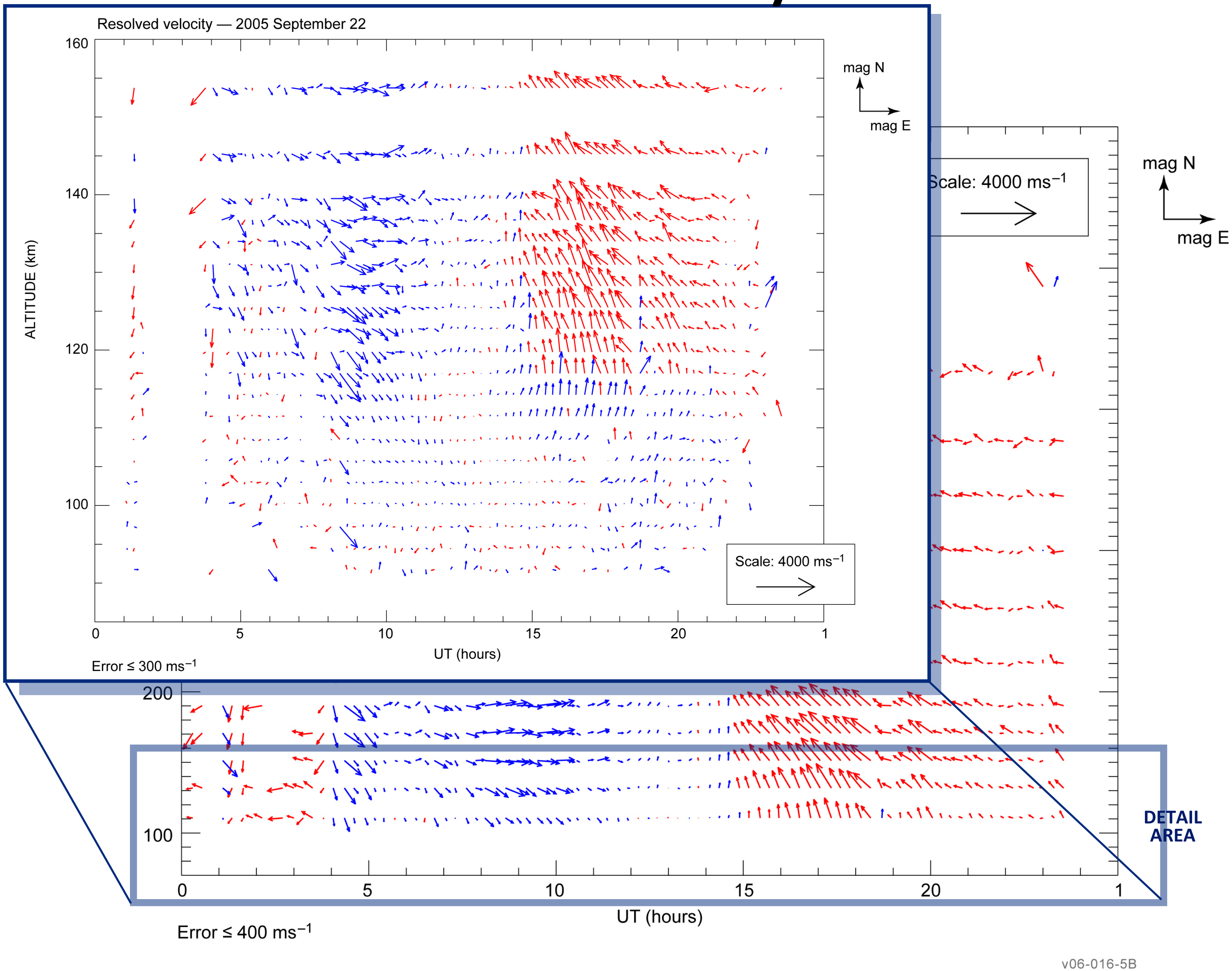
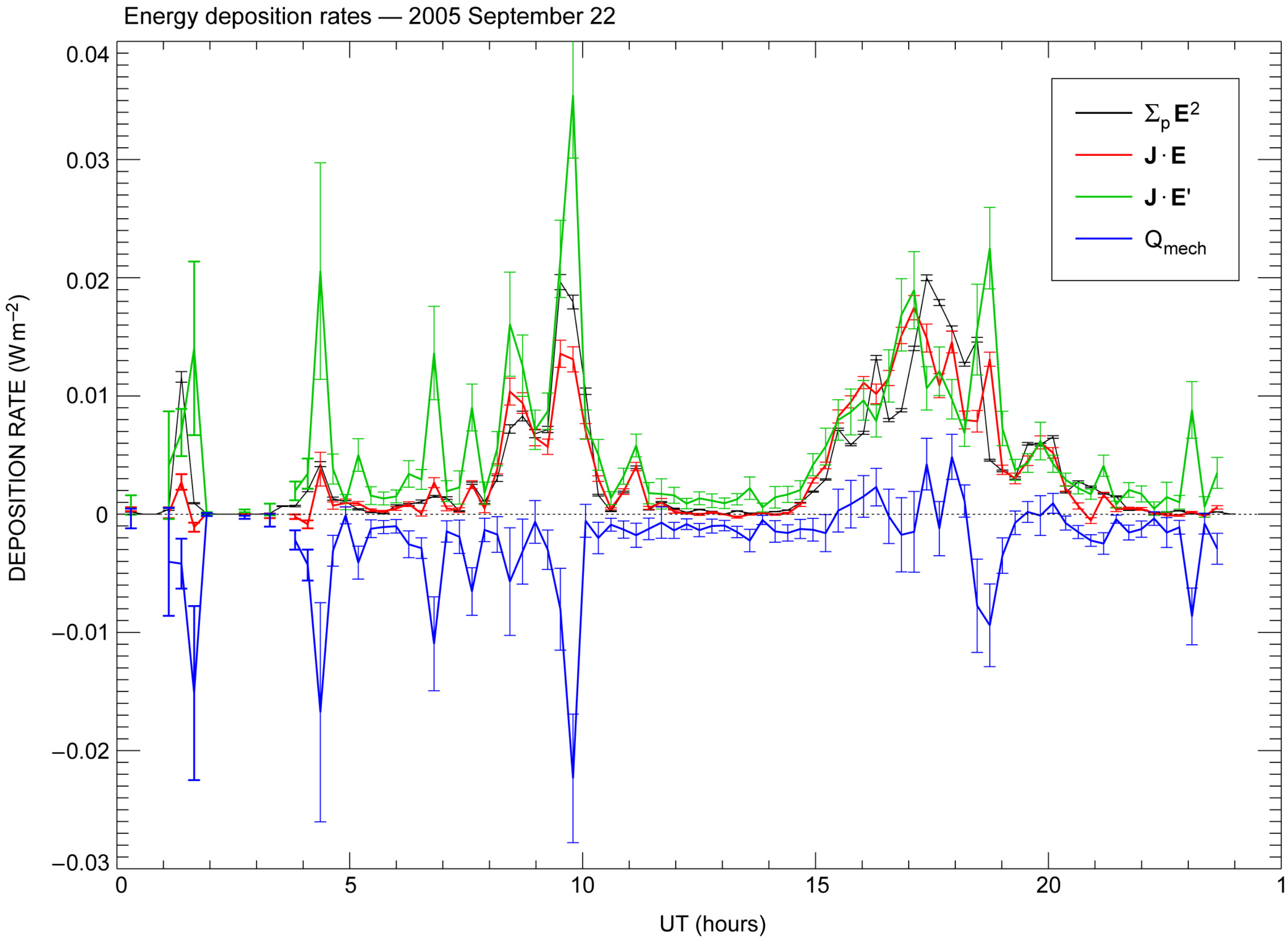
Local Electrodynamics



Local Energy Deposition



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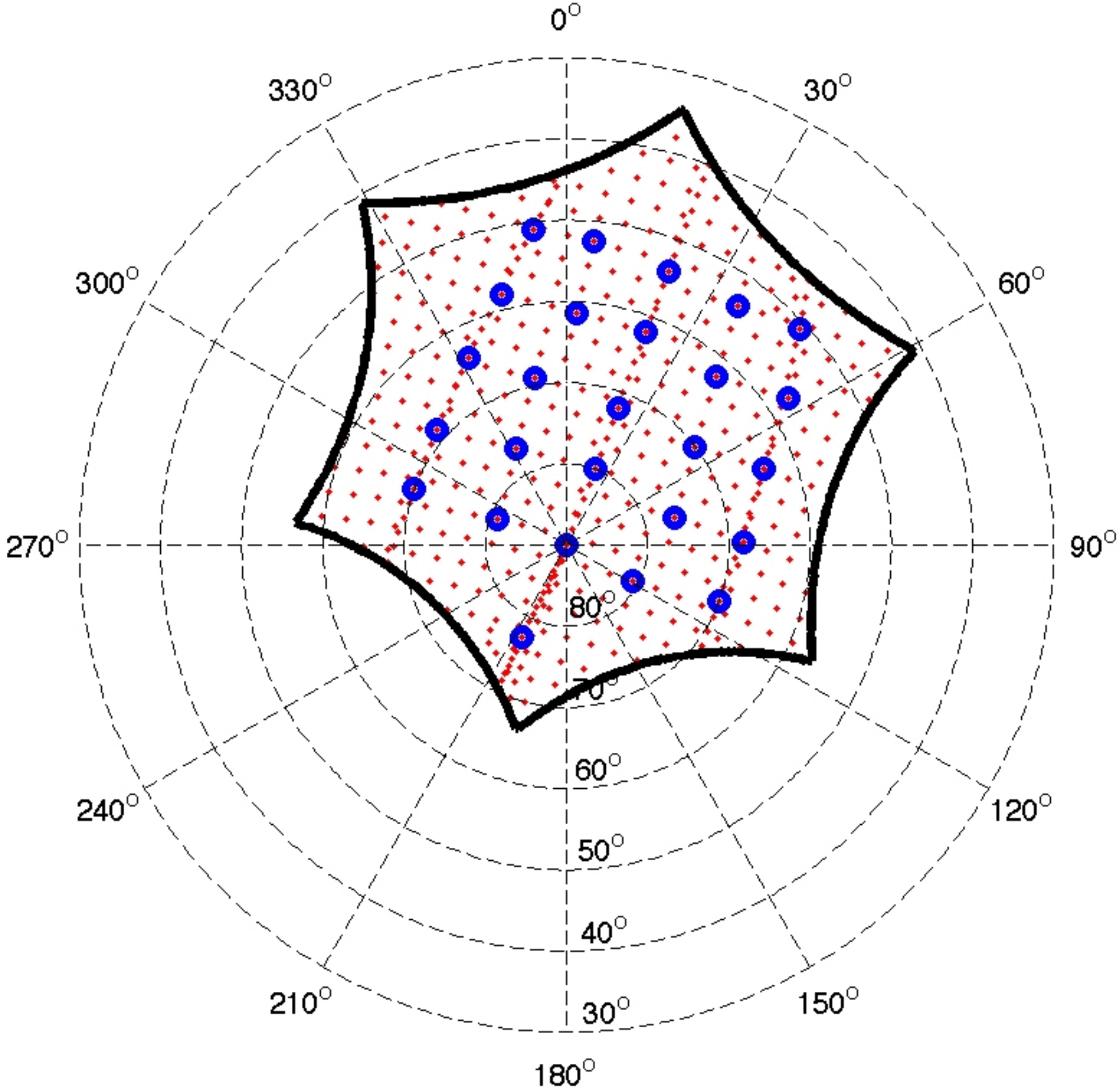
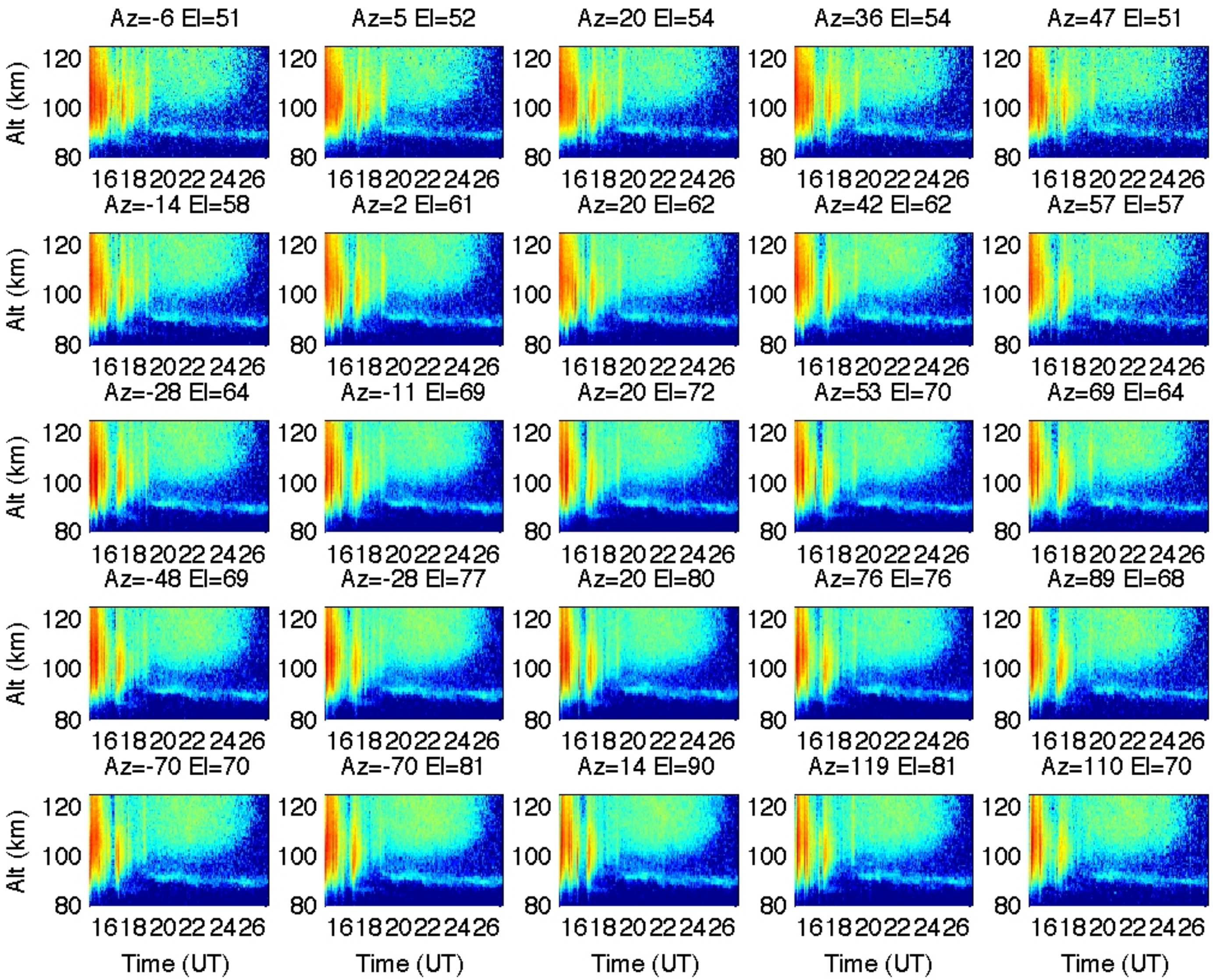
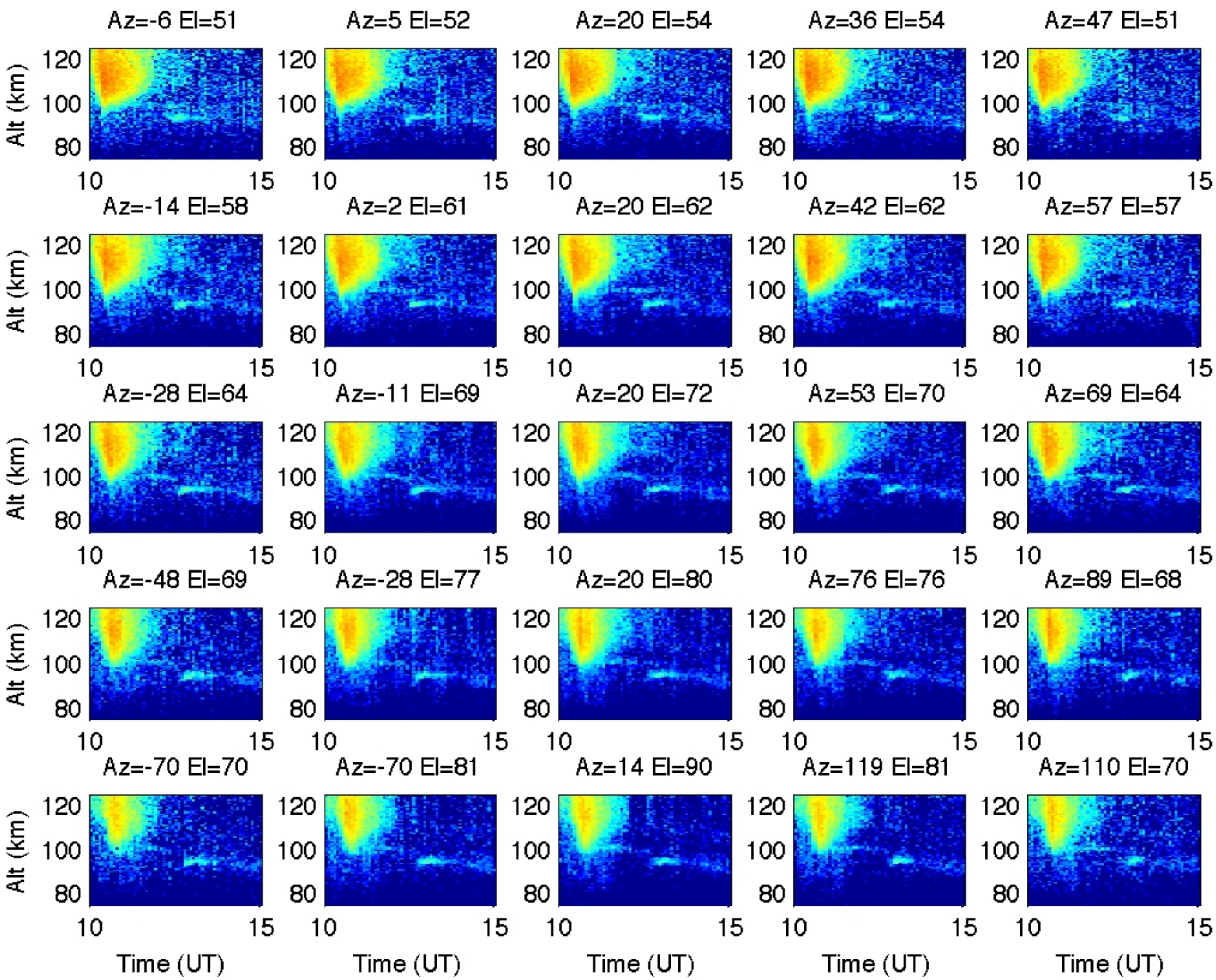


0°
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°
80°
70°
60°
50°
40°
30°

PFISR 2007-10-16



PFISR 2007-11-01



PFISR 2007-11-01