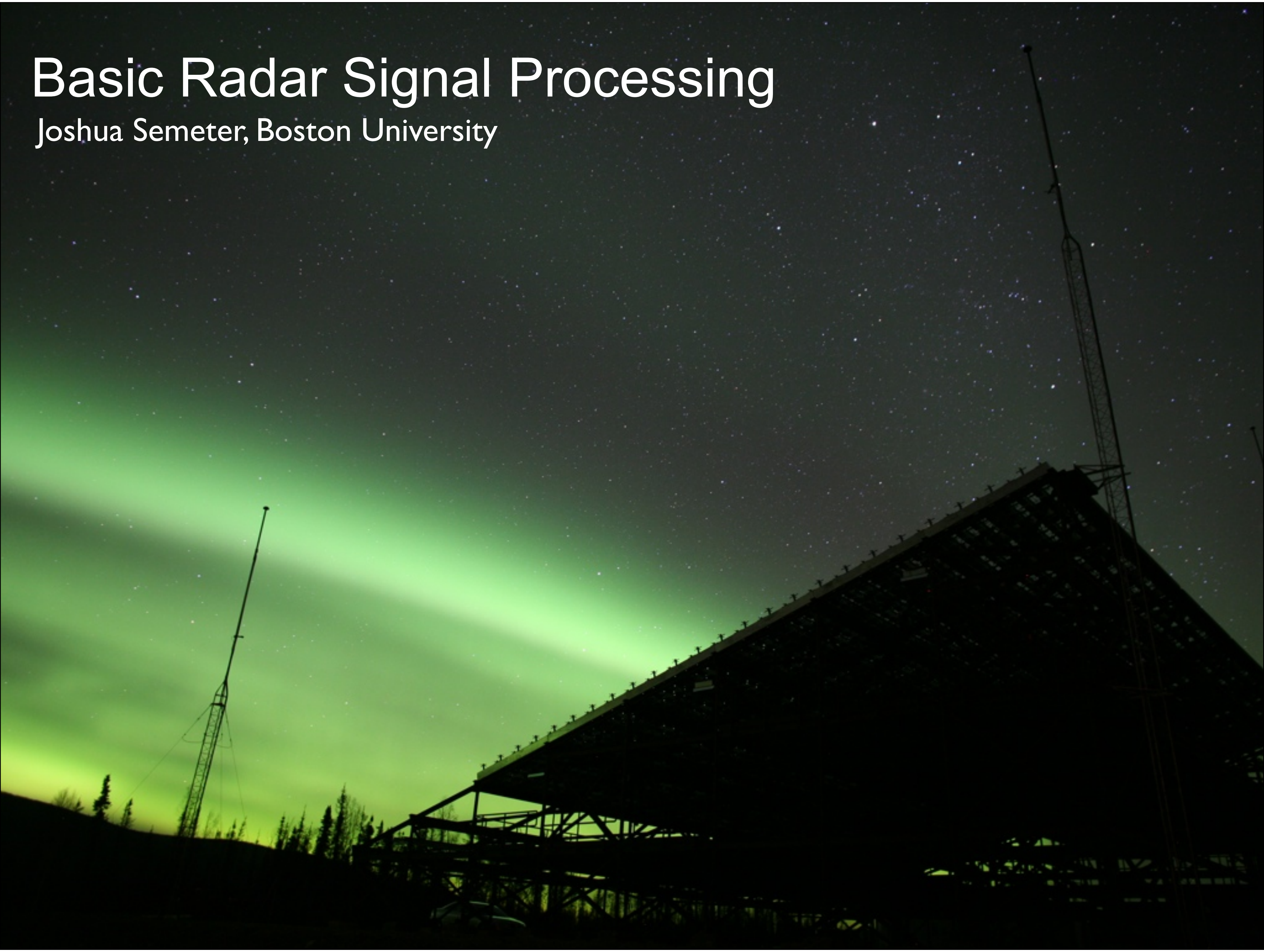
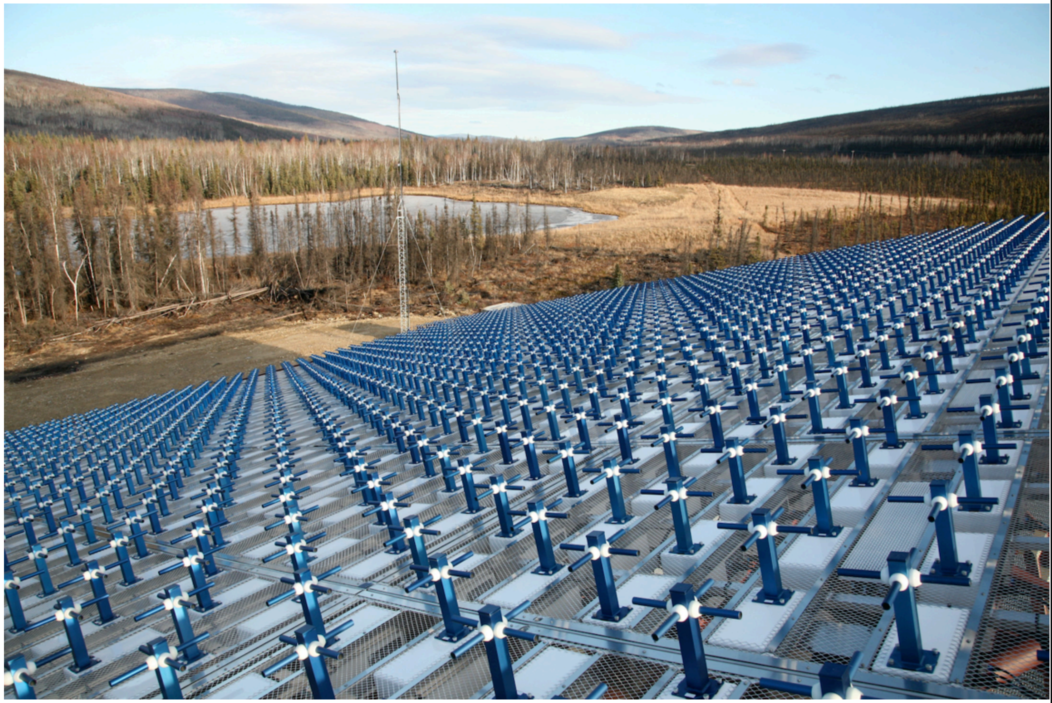
# Basic Radar Signal Processing

Joshua Semeter, Boston University



# Dishes versus Arrays



-FOV:  Entire sky
-Integration at each position before moving
-Power concentrated at Klystron
-Significant mechanical complexity

-FOV:  +/- 20 degrees from boresight
-Simultaneous integration over multple positions
-Power supply is distributed
-Solid state
-4096 crossed dipole antennas

# Why study ISR?

- You get to learn about many useful things, in substantial depth.
  - Plasma physics
  - Radar
  - Coding
  - Electronics (Power, RF, DSP)
  - Signal Processing

# Outline

- Principle of Pulsed Doppler Radar
- The Doppler spectrum of the ionospheric plasma
- Matched filtering and pulse compression
- Doppler Processing

# The deciBel (dB)

The relative value of two quantities expressed on a logarithmic scale

$$SNR = 10 \log_{10} \frac{P_1}{P_2} = 20 \log_{10} \frac{V_1}{V_2} \qquad (\text{Power} \propto \text{Voltage}^2)$$

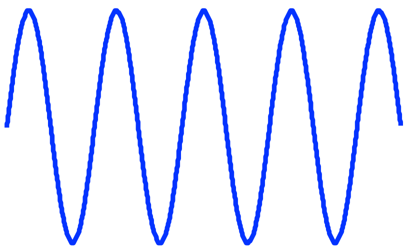
| Factor of: | Scientific Notation | dB |
|---|---|---|
| 0.1 | $10^{-1}$ | -10 |
| 0.5 | $10^{0.3}$ | -3 |
| 1 | $10^{0}$ | 0 |
| 2 | $10^{0.3}$ | 3 |
| 10 | $10^{1}$ | 10 |
| 100 | $10^{2}$ | 20 |
| 1000 | $10^{3}$ | 30 |
| 1,000,000 | $10^{6}$ | 60 |

Other forms used in radar:

| | |
|---|---|
| dBW | dB relative to 1 Watt |
| dBm | dB relative to 1 mW |
| dBsm | dB relative to 1 $m^2$ of radar cross section |
| dBi | dB relative to isotropic radiation |

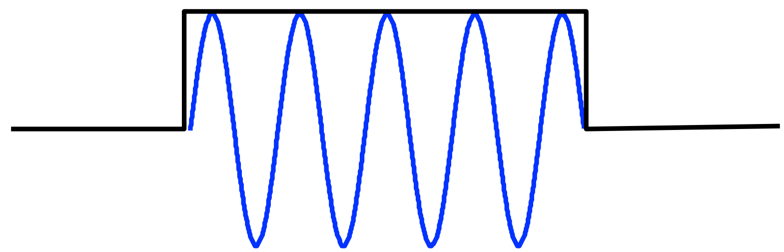
# Waves versus Pulses

**What do radars transmit?**

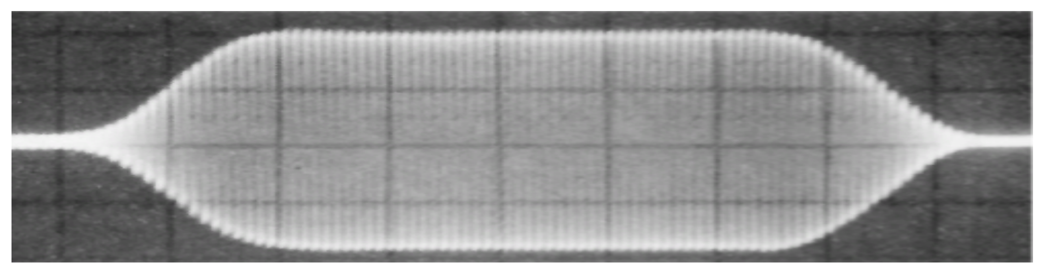


**Waves?**

**or Pulses?**



**Waves, modulated by “on-off” action of pulse envelope**



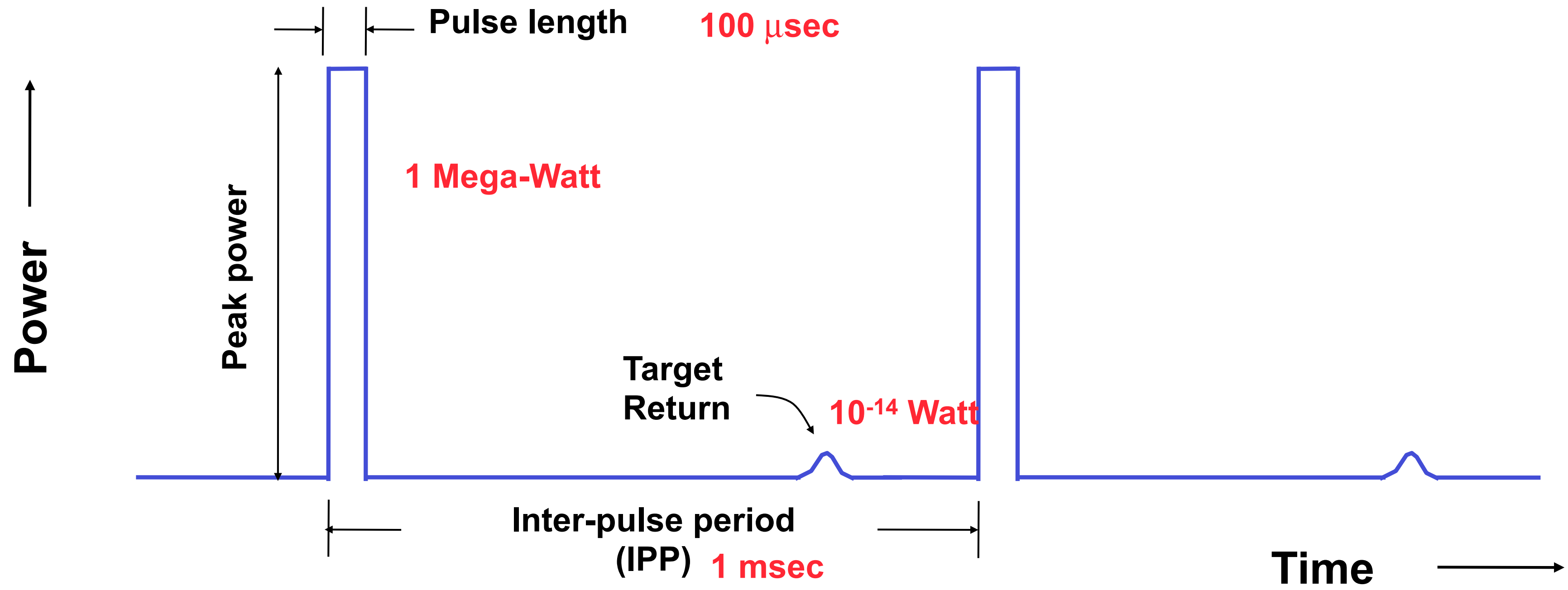
**How many cycles are in a typical pulse?**

**PFISR frequency: 449 MHz**

Typical long-pulse length: 480 μs

**215,520 cycles!**

# Pulsed Radar




Duty cycle = $\frac{\text{Pulse length}}{\text{Pulse repetition interval}}$ 10%

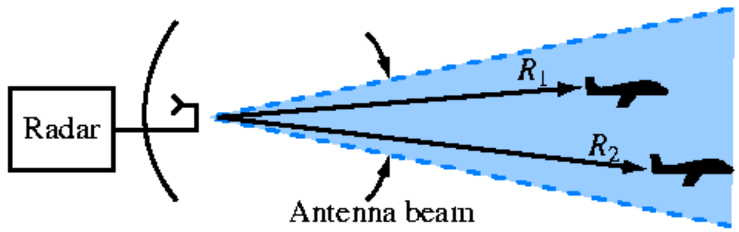
Average power = Peak power * Duty cycle 100 kWatt

Pulse repetition frequency (PRF) = 1/(IPP) 1 kHz

Continuous wave (CW) radar: Duty cycle = 100% (always on)

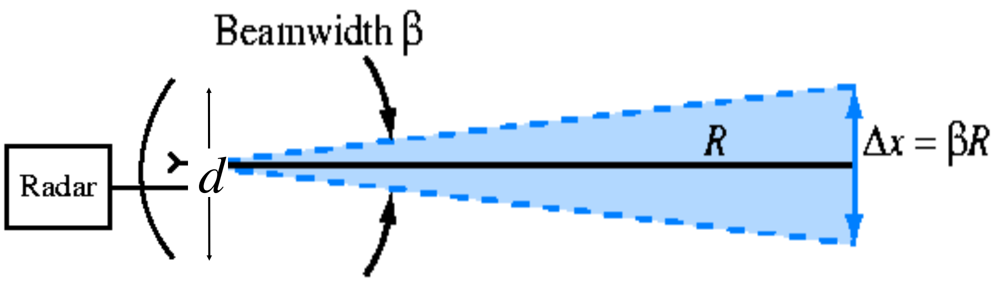
# Range

**Range resolution:** **Set by pulse length, given in units of time, $\tau_p$, or length, c $\tau_p$**



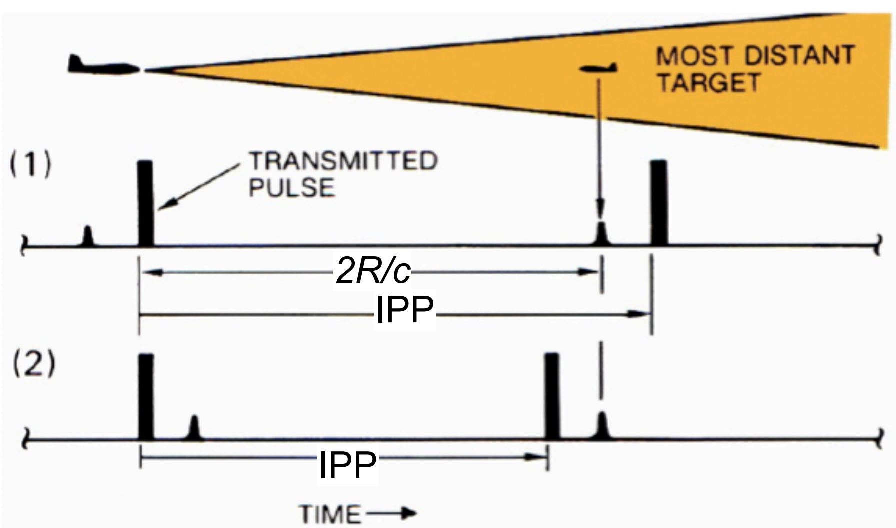

$$\Delta R = R_2 - R_1 = \frac{c\tau_p}{2}$$

**Cross-range resolution:** **Set by "beam width" (in degrees) and target range**




$$\beta \approx \frac{\lambda}{d} \quad \text{radians}$$

**Maximum unambiguous range:** **Set by Inter-pulse Period (IPP)**




IPP = Interpulse period (s)
PRF = pulse repetition frequence
= 1/IPP (Hz)

$$R_u = \frac{c\ \text{IPP}}{2}$$

# Doppler

Transmitted signal: $\cos(2\pi f_o t)$

After return from target: $\cos\left[2\pi f_o\left(t + \frac{2R}{c}\right)\right]$

To measure frequency, we need to observe signal for at least one cycle.
So we will need a model of how *R* changes with time. Assume constant velocity:

$$R = R_o + v_o t$$

Substituting:

$$\cos\left[2\pi\left(f_o + \underbrace{f_o\frac{2v_o}{c}}_{-f_D}\right)t + \underbrace{\frac{2\pi f_o R_o}{c}}_{\text{constant}}\right]$$

$$\boxed{f_D = \frac{-2f_o v_o}{c} = \frac{-2v_o}{\lambda_o}}$$

By convention, positive Doppler frequency shift $\Longleftrightarrow$ Target and radar closing

# Two key concepts

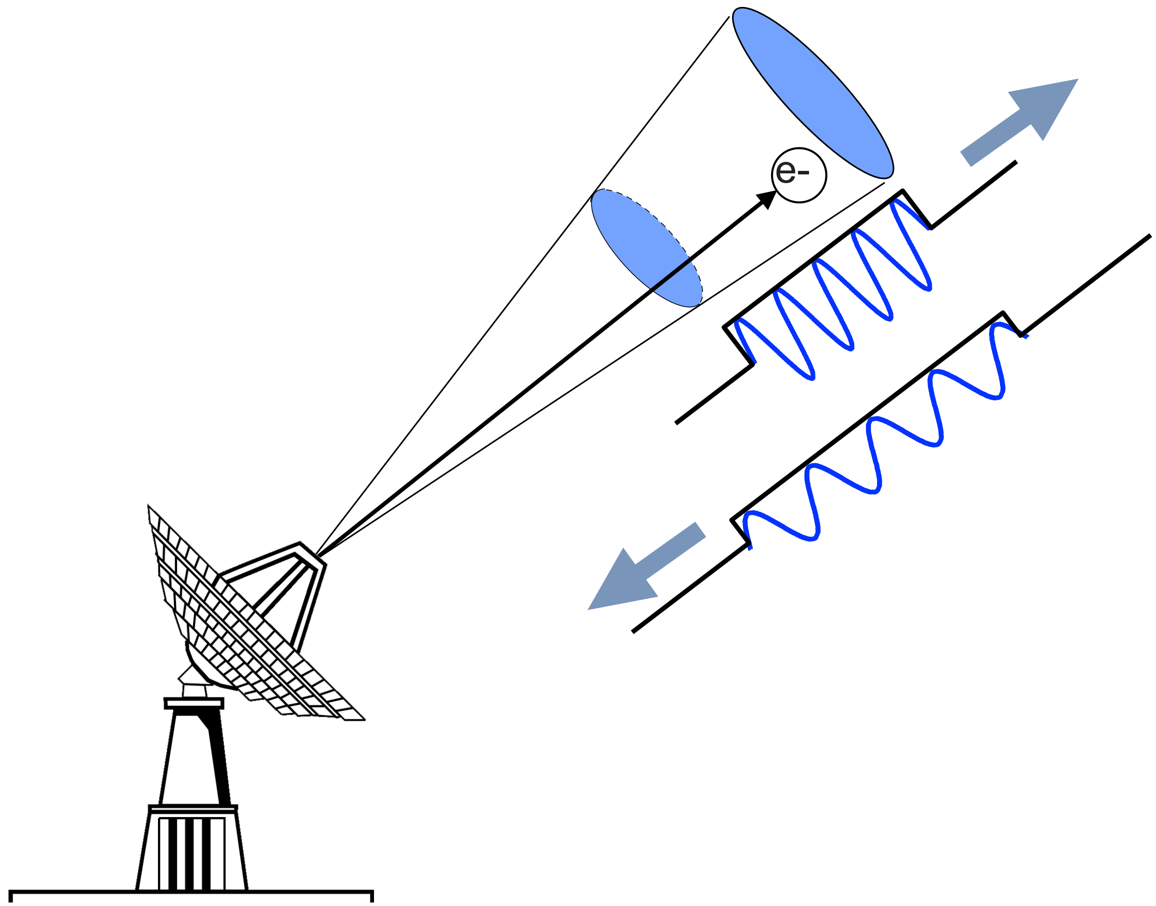
Two key concepts:

Distant ⟺ Time

$$R = c\Delta t/2$$

Velocity ⟺ Frequency

$$v = -f_D \lambda_0 / 2$$




A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

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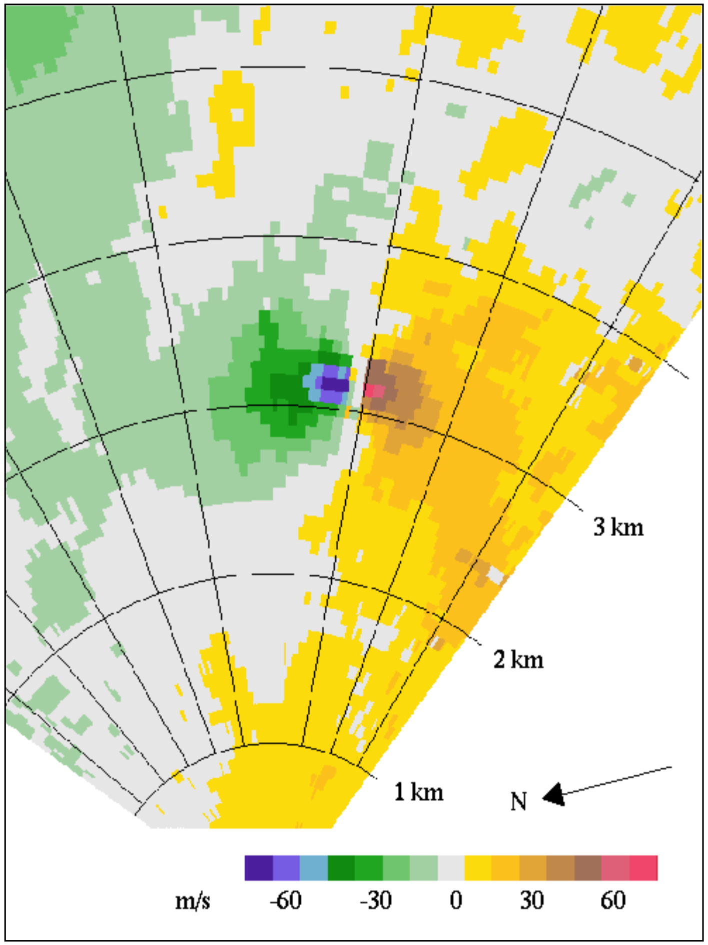


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# Concept of a "Doppler Spectrum"

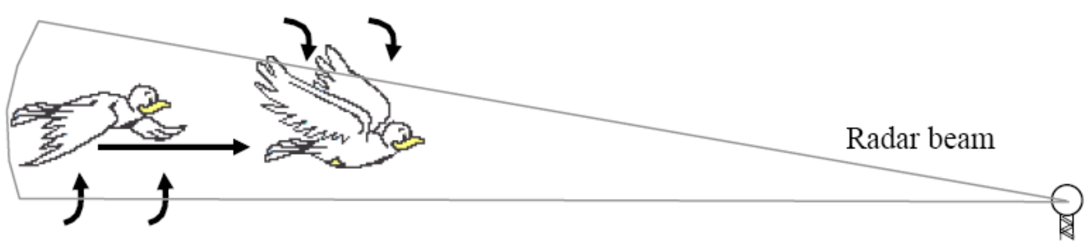
Two key concepts:

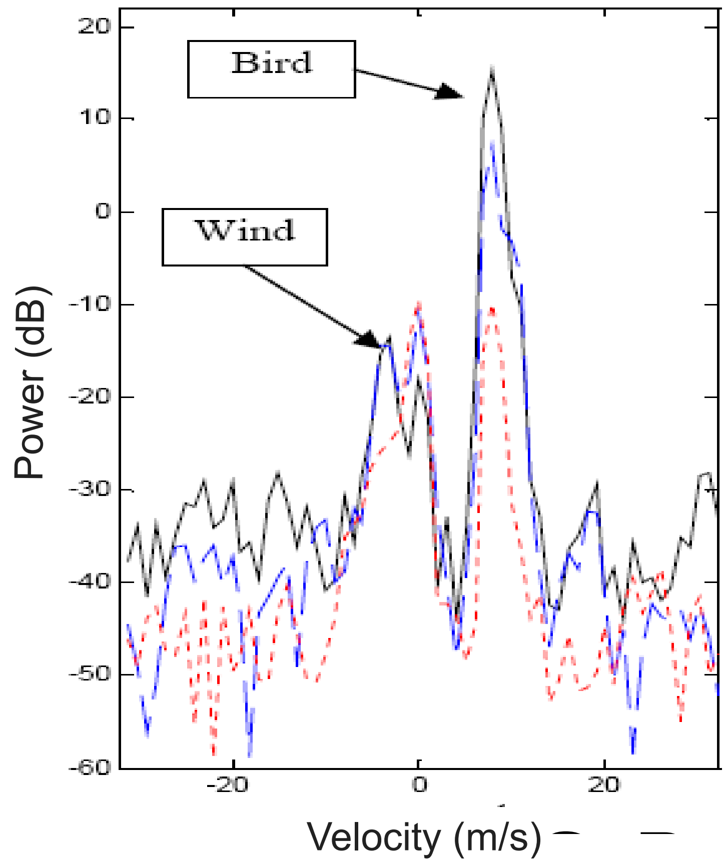
Distant ⟺ Time

$$R = c\Delta t/2$$

Velocity ⟺ Frequency

$$v = -f_D \lambda_0 /2$$




If there is a distribution of targets moving at different velocities (e.g., electrons in the ionosphere) then there is no single Doppler shift but, rather, a Doppler spectrum.

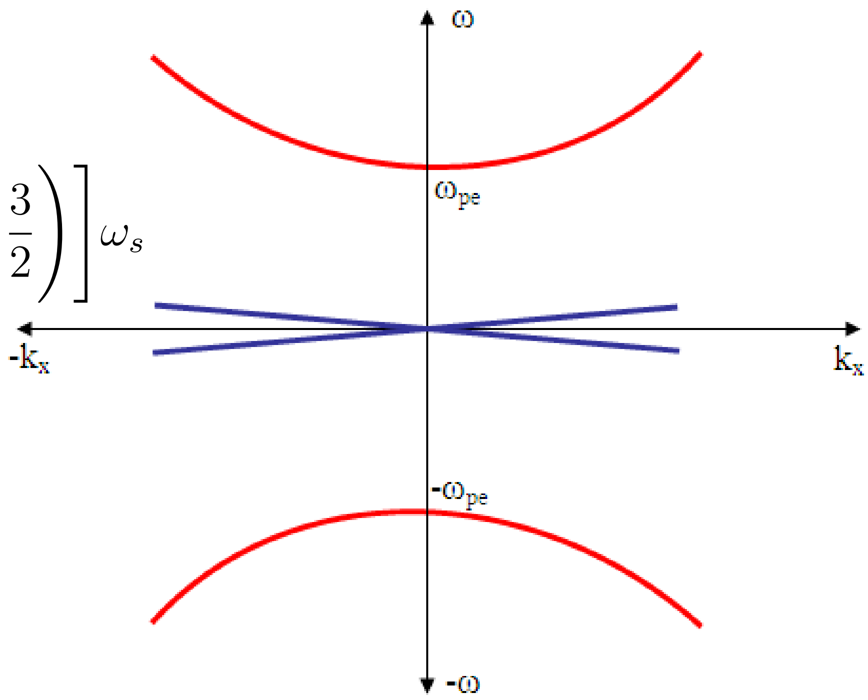
What is the Doppler spectrum of the ionosphere at UHF ($\lambda$ of 10 to 30 cm)?

# Longitudinal Modes in a Thermal Plasma

Ion-acoustic

$$\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}$$

$$\omega_{si} = -\sqrt{\frac{\pi}{8}}\left[\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} \exp\left(-\frac{T_e}{2T_i} - \frac{3}{2}\right)\right]\omega_s$$




Langmuir

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the} \lambda_{De} k^2$$

$$\omega_{Li} \approx -\sqrt{\frac{\pi}{8}}\frac{\omega_{pe}^3}{k^3}\frac{1}{v_{the}^3} exp\left(-\frac{\omega_{pe}^2}{2k^2 v_{the}^2} - \frac{3}{2}\right)\omega_L$$

# Simulated ISR Doppler Spectrum

Particle-in-cell (PIC):

$$\frac{d\,\mathbf{v}_i}{d\,t} = \frac{q_i}{m_i}(\mathbf{E}(\mathbf{x}_i) + \mathbf{v}_i \times \mathbf{B}(\mathbf{x}_i))$$

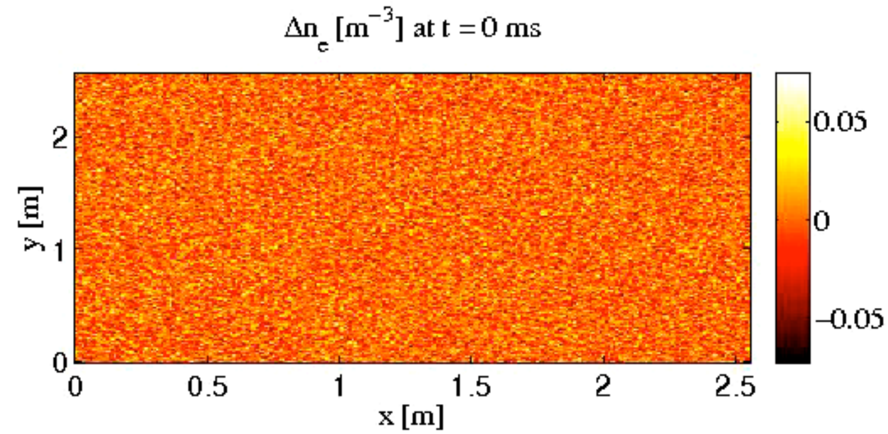
$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t}$$

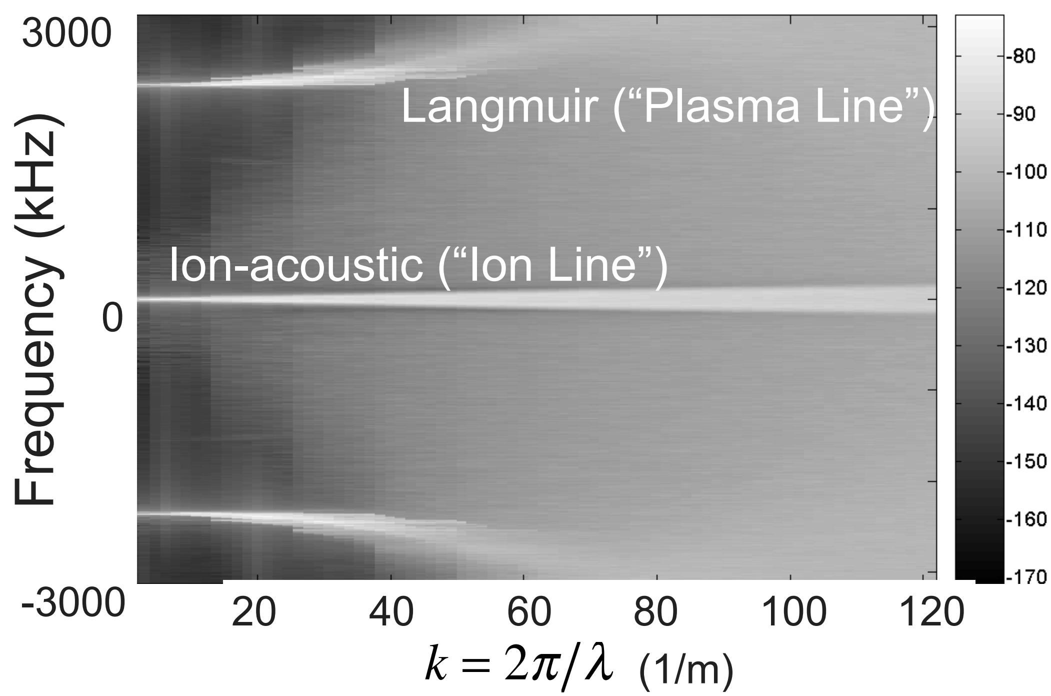
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

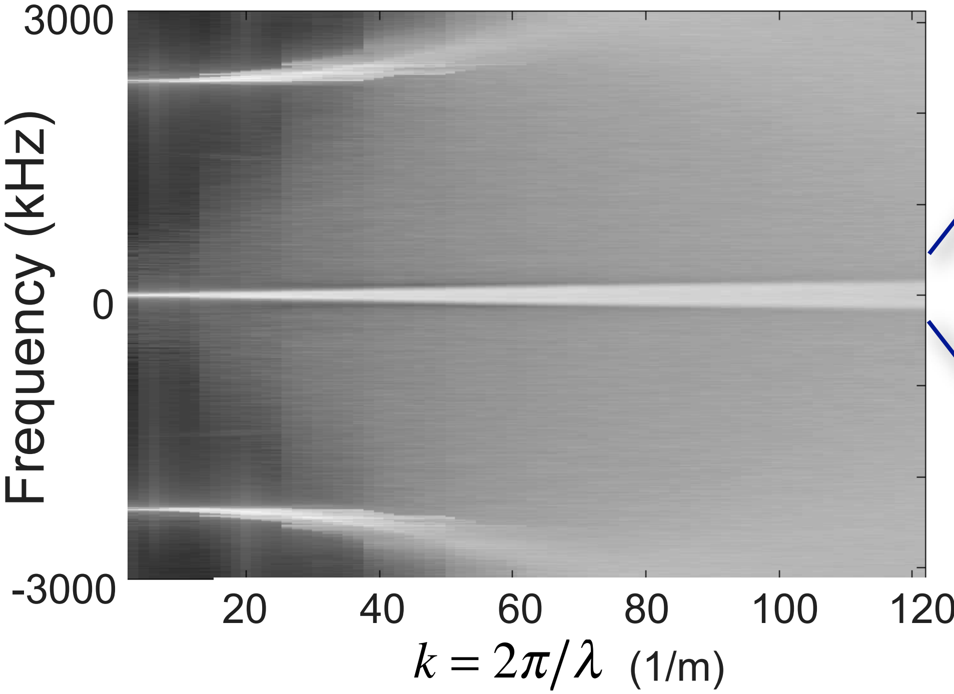
$$\nabla \cdot \mathbf{B} = 0$$

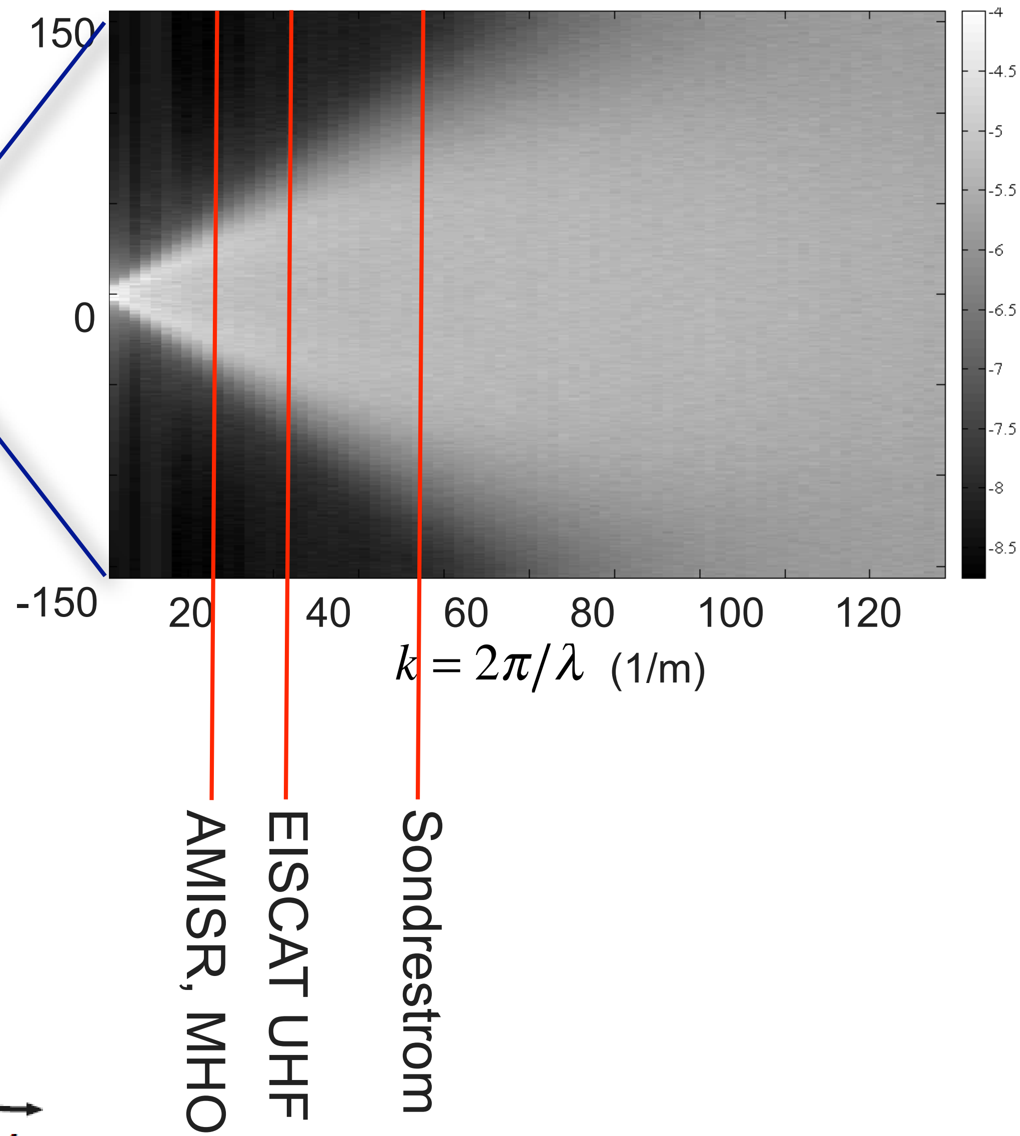
Simple rules yield complex behavior



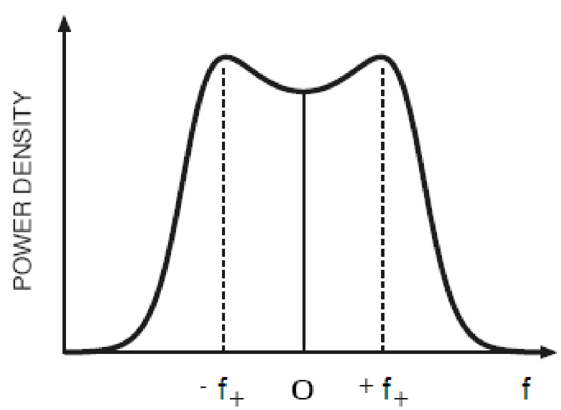



# ISR Measures a Cut Through This Surface




Ion-acoustic "lines" are broadened by Landau damping



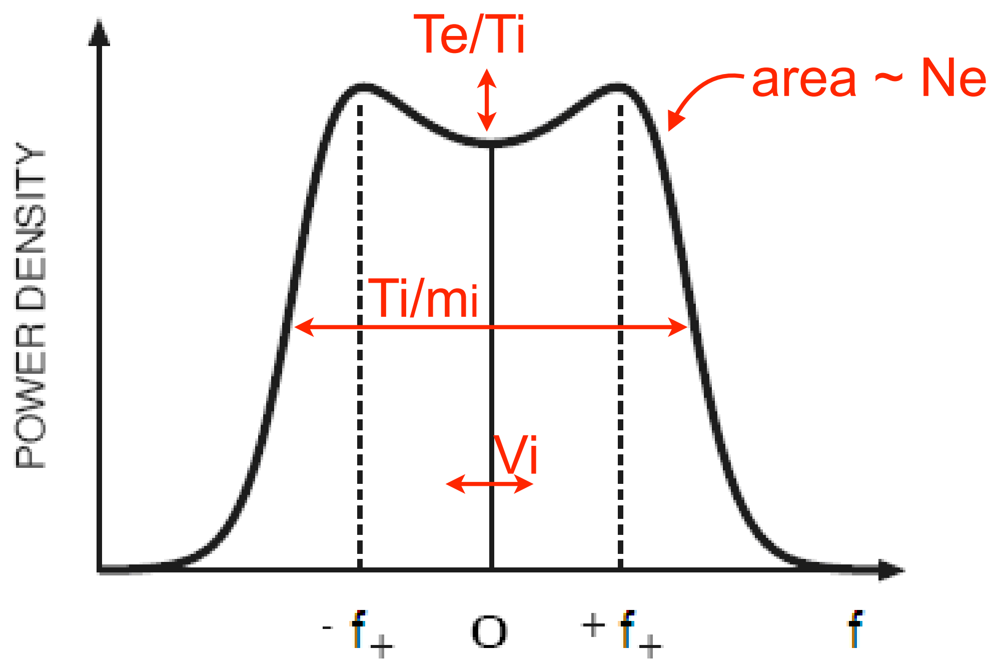
# The ISR model

$$\sigma(\omega) = \frac{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \sum_1 \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right|^2 \overline{\left|N_e^0(\omega)\right|^2} + \left(\frac{\lambda}{4\pi D_e}\right)^4 \left|F_e(\omega)\right|^2 \sum_i \overline{\left|N_i^0(\omega)\right|^2}}{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \left\{\left(\frac{1}{D_e}\right)^2 \cdot F_e(\omega) + \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right\}\right|^2}$$

where:

$$F_e(\omega) = 1 - \omega \int_o^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\cos(\omega\tau)\,d\tau$$

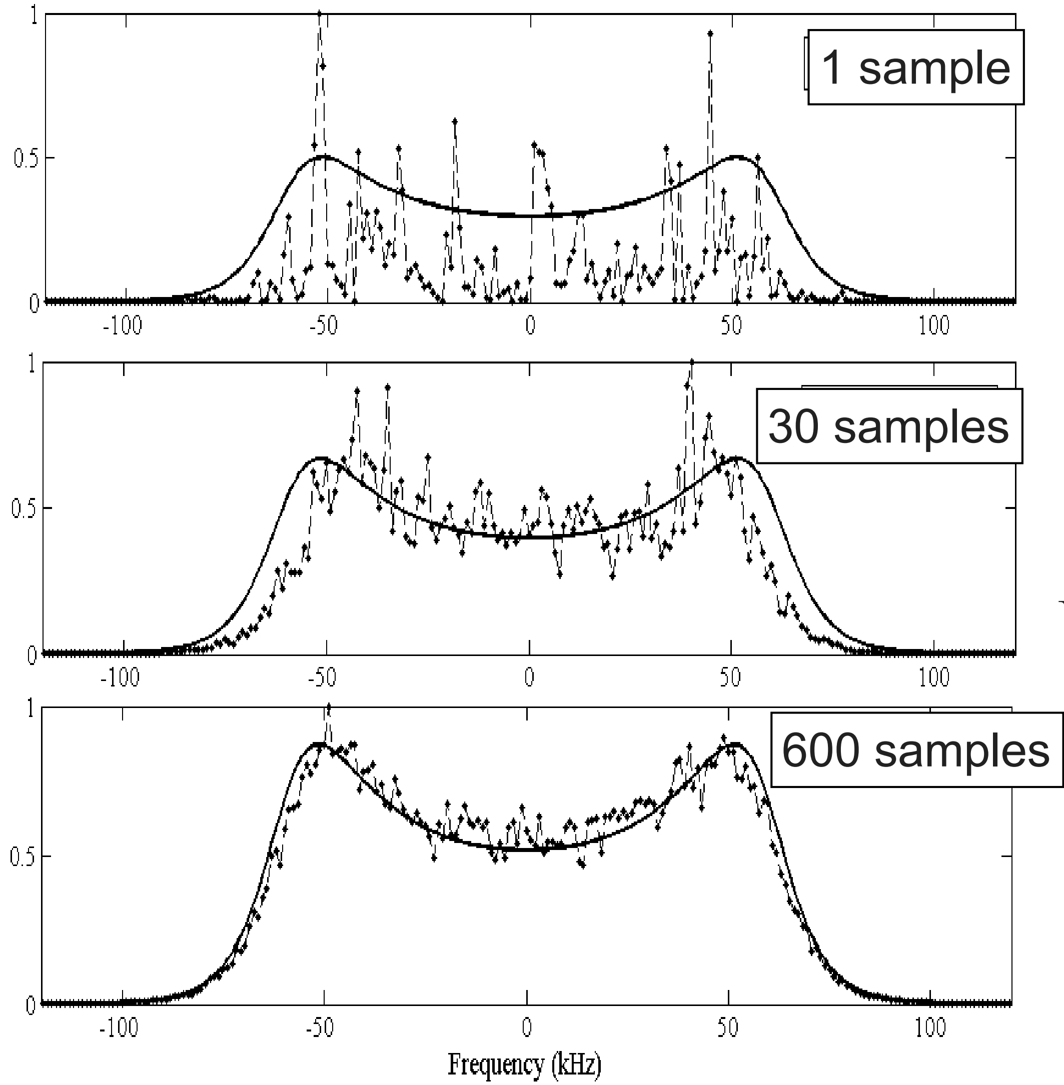
$$F_i(\omega) = 1 - \omega \int_o^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\cos(\omega\tau)\,d\tau$$




From Evans, IEEE Transactions, 1969

# Incoherent Averaging

Normalized ISR spectrum for different integration times at 1290 MHz




We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent "realizations" of the process.

$$\text{Uncertainties} \propto \frac{1}{\sqrt{\text{Number of Samples}}}$$

# ISR in a nutshell

**Here's what we measure:**

$$SNR = \frac{P_r}{P_n} = \left(\frac{P_t}{4\pi R^2}\right)\left(\frac{\sigma(\omega)}{4\pi R^2}\right)\left(\frac{GA}{KTBN_{sys}}\right)$$

The "radar equation"

$P_r$ = Received power

$P_n$ = Received noise power

$P_t$ = Transmitted power

$\sigma$ = Radar cross section

$G$ = Antenna gain

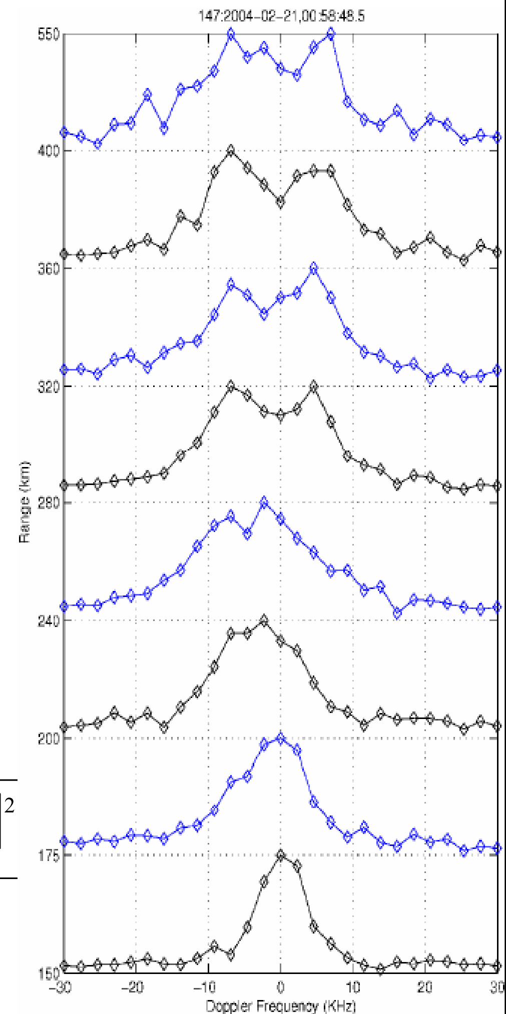
$A$ = Antenna area

$k_B$ = Boltzman's constant

$T$ = Temperature

$B$ = Bandwidth

$N_{sys}$ = System noise temperature



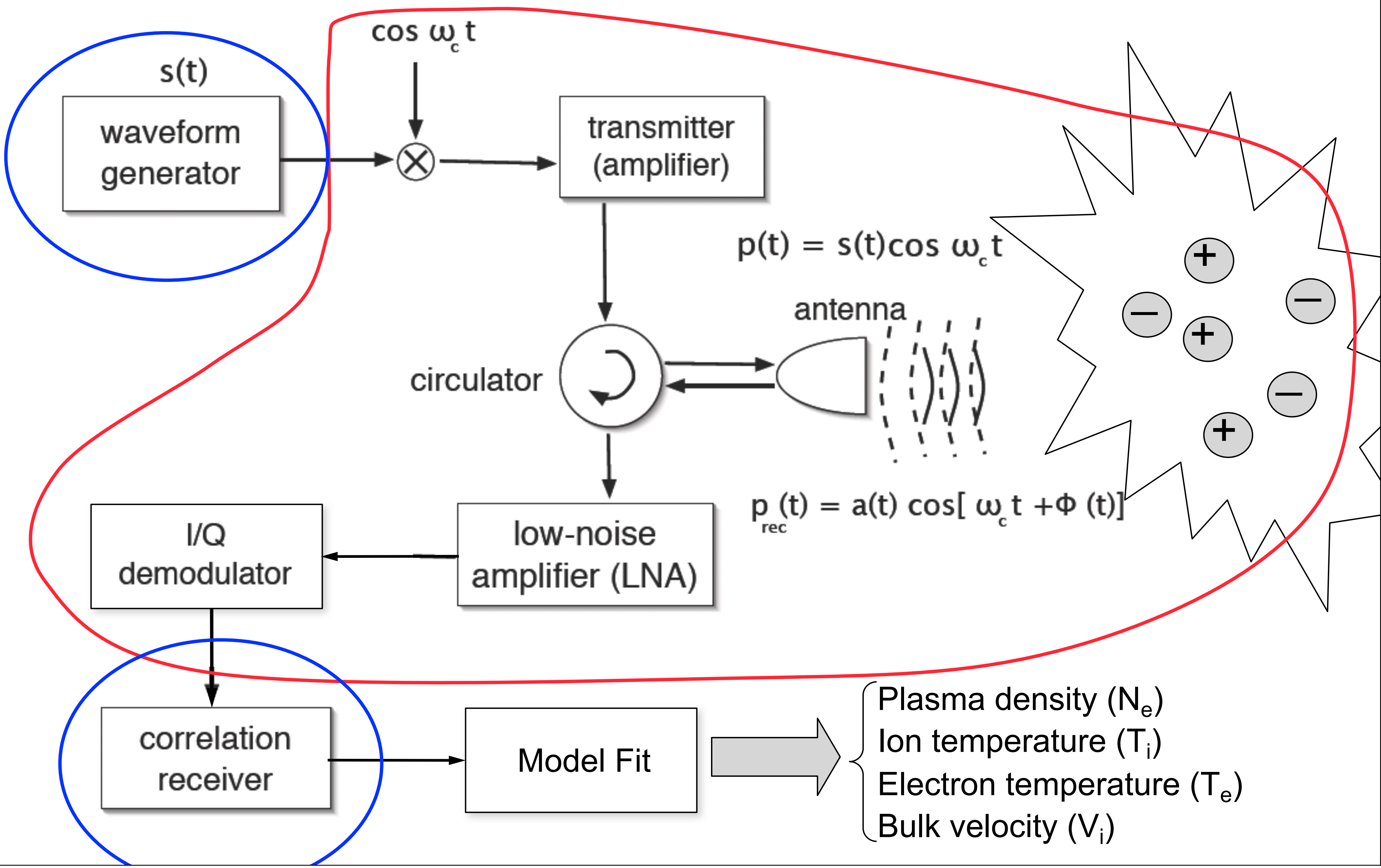

**Here's the theory:**

$$\sigma(\omega) = \frac{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \sum_1 \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right|^2 \overline{\left|N_e^0(\omega)\right|^2} + \left(\frac{\lambda}{4\pi D_e}\right)^4 \left|F_e(\omega)\right|^2 \sum_i \overline{\left|N_i^0(\omega)\right|^2}}{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \left\{\left(\frac{1}{D_e}\right)^2 \times F_e(\omega) + \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right\}\right|^2}$$

$$F_e(\omega) = 1 - \omega \int_o^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega \int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\cos(\omega\tau)\,d\tau$$

$$F_i(\omega) = 1 - \omega \int_o^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega \int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\cos(\omega\tau)\,d\tau$$

# Components of a Pulsed Doppler Radar



# Essential Mathematical Operations

**Euler:**

$$\begin{aligned} Ae^{j\phi t} &= A\cos(\phi) + jA\sin(\phi) \\ &= I + jQ \end{aligned}$$

**Fourier Transform:**

$$f(t) = \int_{-\infty}^{+\infty} F(f)e^{j2\pi ft}dt \quad \Longleftrightarrow \quad F(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-j2\pi ft}dt$$

**Convolution:**

$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau)g(\tau - t)d\tau \qquad f(t) * g(t) \Longleftrightarrow F(f)G(f)$$

**Correlation:**

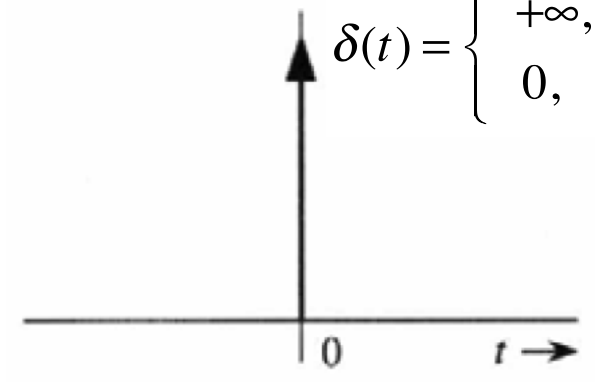
$$f(t) \circ g(t) = \int_{-\infty}^{+\infty} f^*(\tau)g(t + \tau)d\tau \qquad f(t) \circ g(t) \Longleftrightarrow F^*(f)G(f)$$

**Autocorrelation, Convolution, Power Spectrum (Wiener-Kintchine Theorem)**

$$u(t) \circ u(t) = u(t) * u^*(-t) \quad \Longleftrightarrow \quad |U(f)|^2$$

# Dirac Delta Function

$$\delta(t) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$




$\delta(t)$ is defined by the property that for all continuous functions

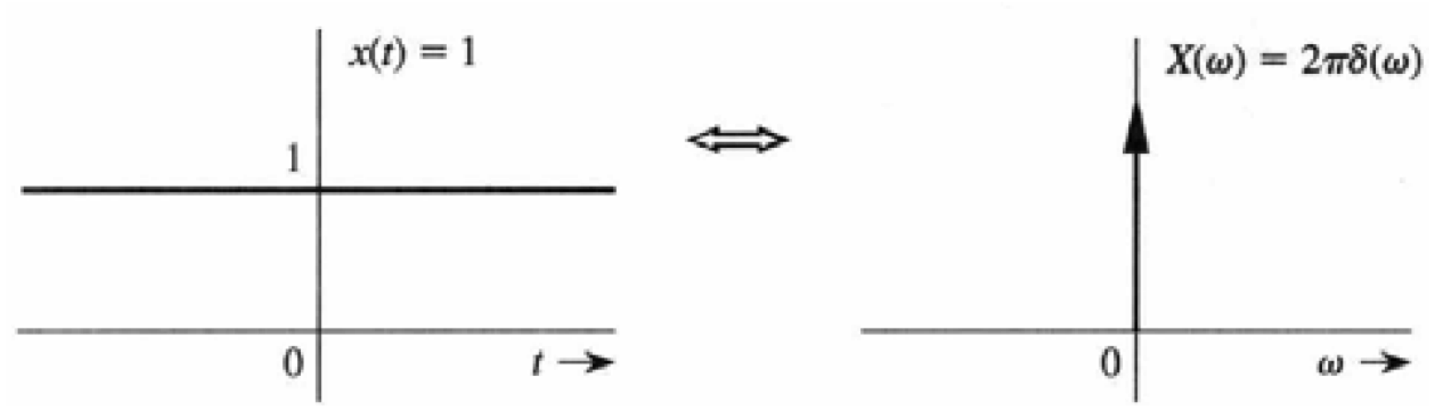
$$f(0) = \int_{-\infty}^{+\infty} \delta(t) f(t) dt$$

$$f(t-T) = f(t) * \delta(t-T)$$

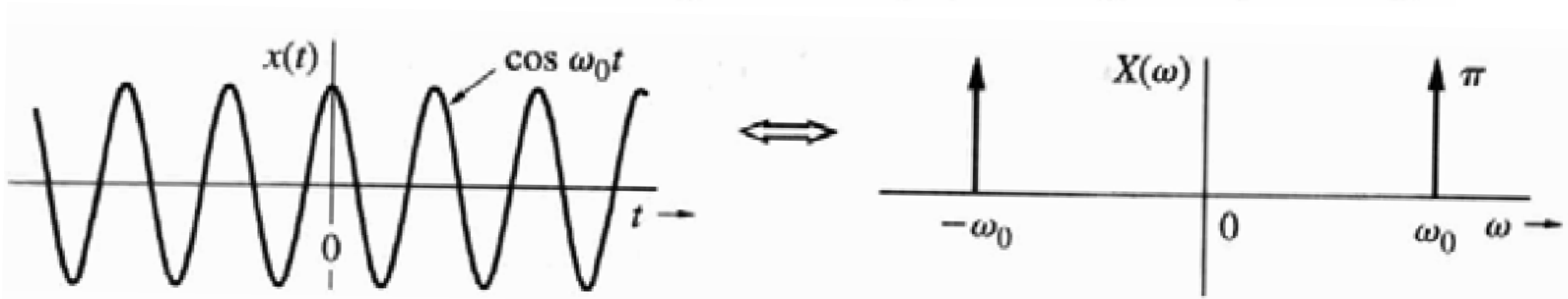
The Fourier Transform of a train of delta functions is a train of delta functions.

$$\sum_{n=-\infty}^{\infty} \delta(t - nT) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$$

# Harmonic Functions




$$\cos \omega_0 t \Longleftrightarrow \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$$



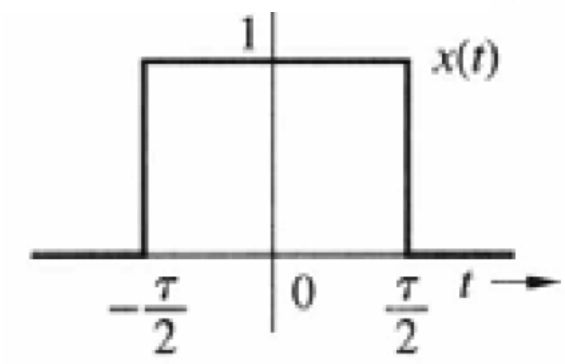

$$\sin \omega_0 t \Longleftrightarrow j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

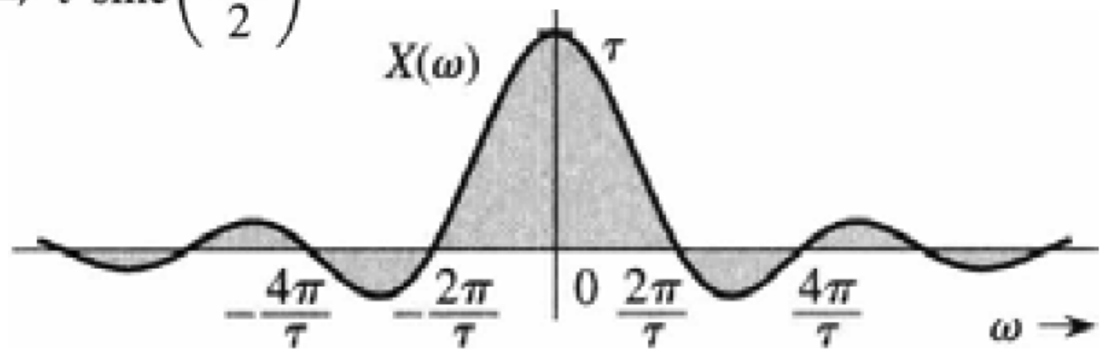
$$e^{j\omega_0 t} \Longleftrightarrow 2\pi\delta(\omega - \omega_0)$$

# Gate function

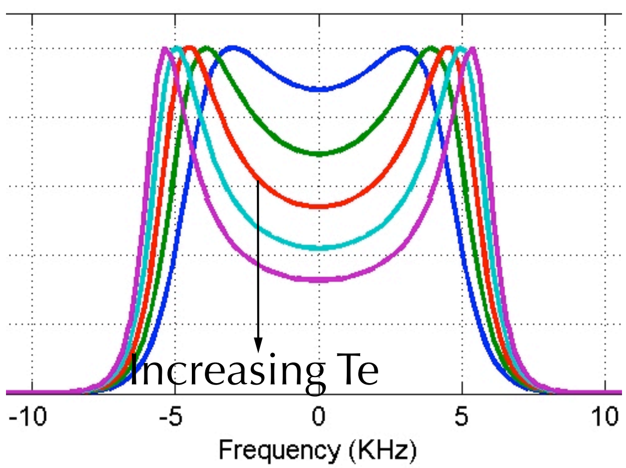
$$\mathrm{rect}(t/\tau) = \begin{cases} 1 & \text{for } -\tau/2 < t < \tau/2 \\ 0 & \text{otherwise} \end{cases}$$

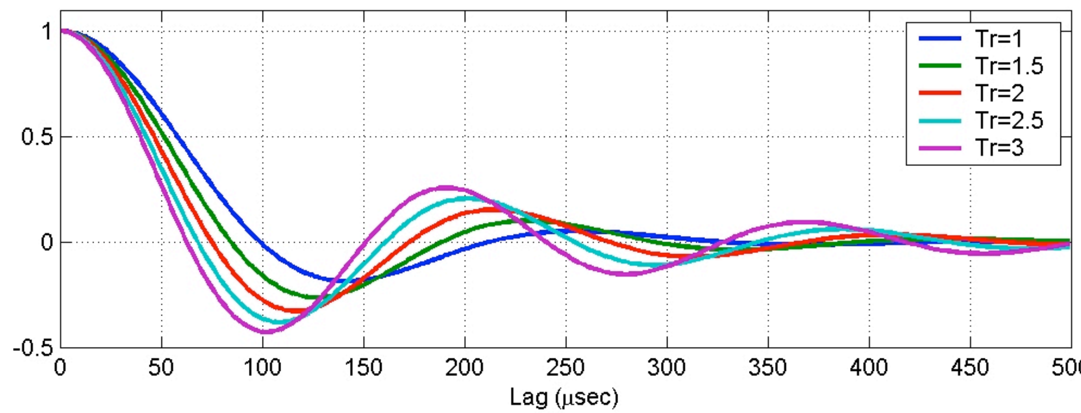
$$\mathrm{rect}\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$$




ISR spectrum $\Longleftrightarrow$ Autocorrelation function (ACF)

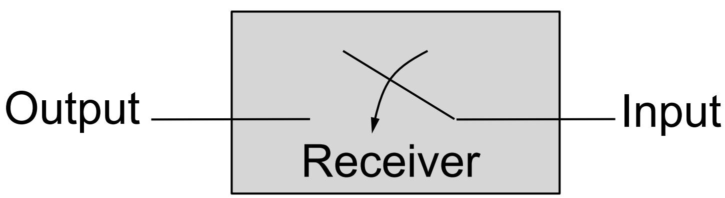


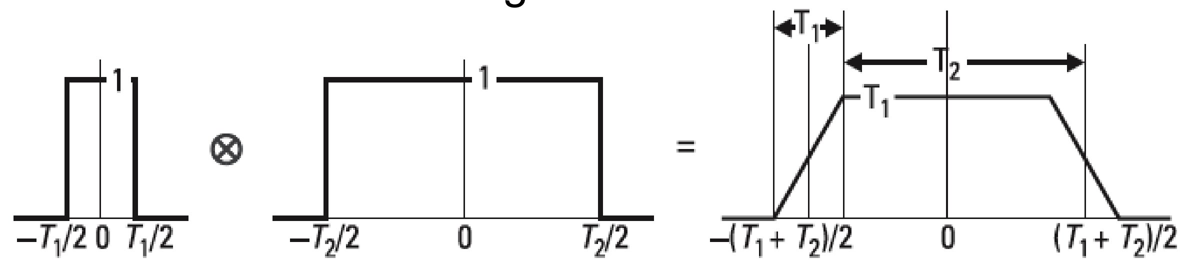

Not surprisingly, the ISR ACF looks like a sinc function…

# Strategies for radar reception

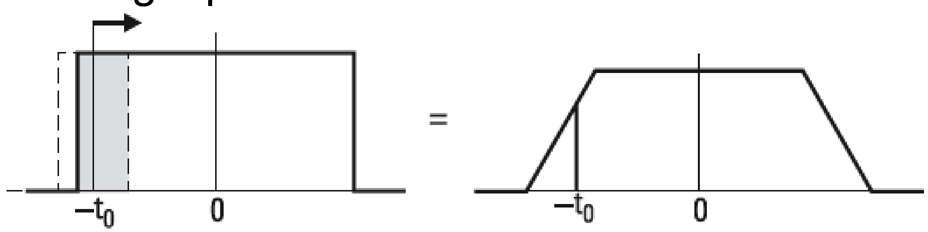
We send a pulse of duration $\tau$. How should we listen for the echo?




Convolution of two rectangle functions



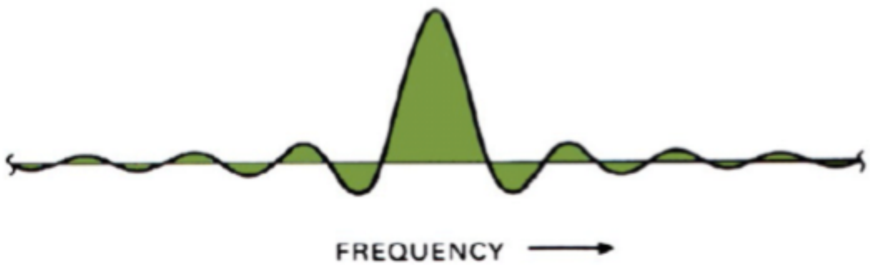

Value at a single point



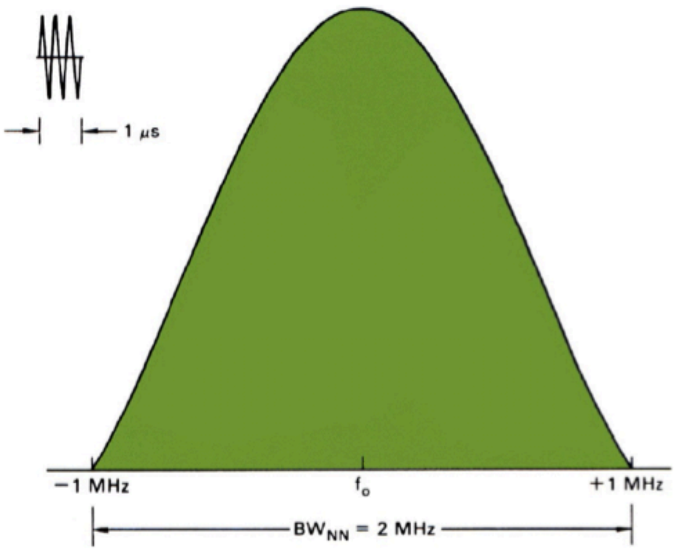

- To determine range, we only need to find the rising edge of the pulse we sent. So make $T_1 << T_2$.
- But that means large receiver bandwidth, lots of noise power, poor SNR.
- Could make $T_1 >> T_2$, then we're integrating noise in time domain.
- So how long should we close the switch?

# Bandwidth of a pulsed signal

**Spectrum of receiver output has sinc shape, with sidelobes half the width of the central lobe and continuously diminishing in amplitude above and below main lobe**




**A 1 microsecond pulse has a null-to-null bandwidth of the central lobe = 2 MHz**



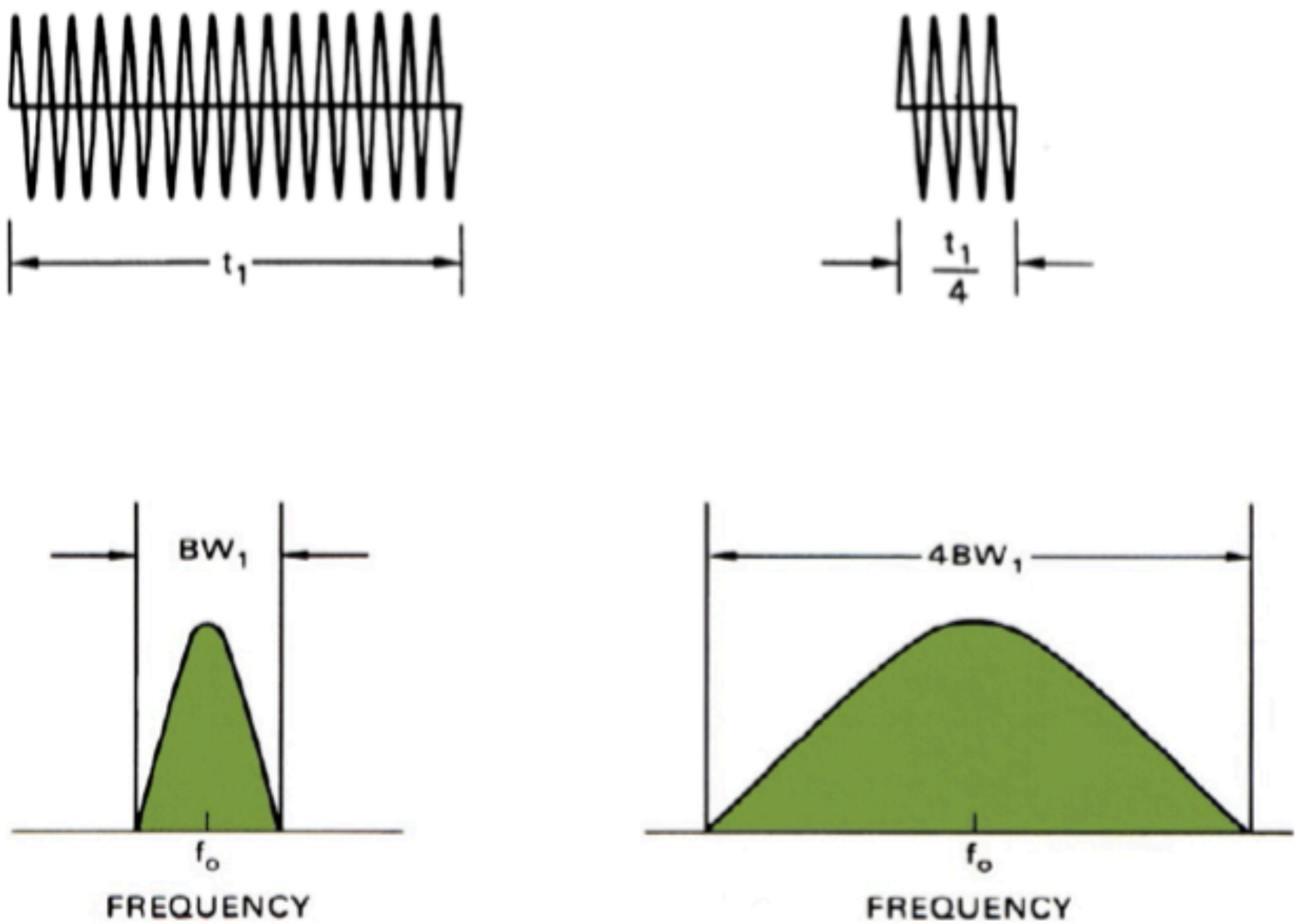

**Two possible bandwidth measures:**

**"null to null" bandwidth** $$B_{nn} = \frac{2}{\tau}$$

**"3dB" bandwidth** $$B_{3dB} = \frac{1}{\tau}$$

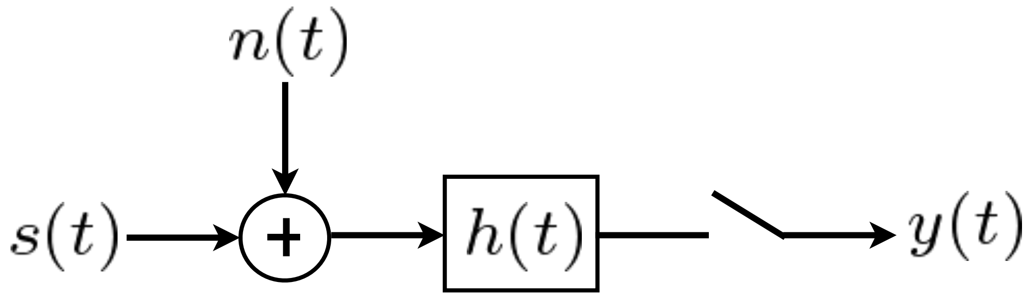
**Unless otherwise specified, assume bandwidth refers to 3 dB bandwidth**

# Pulse-Bandwidth Connection




**Shorter pulse ⟺ Larger bandwidth**

# Matched Filter




$$y(t) = \int [s(\tau) + n(\tau)]\, h(t-\tau) d\tau$$
$$= \int H(f) S(f) e^{j2\pi f T} df + \int H(f) N(f) e^{j2\pi f T} df$$

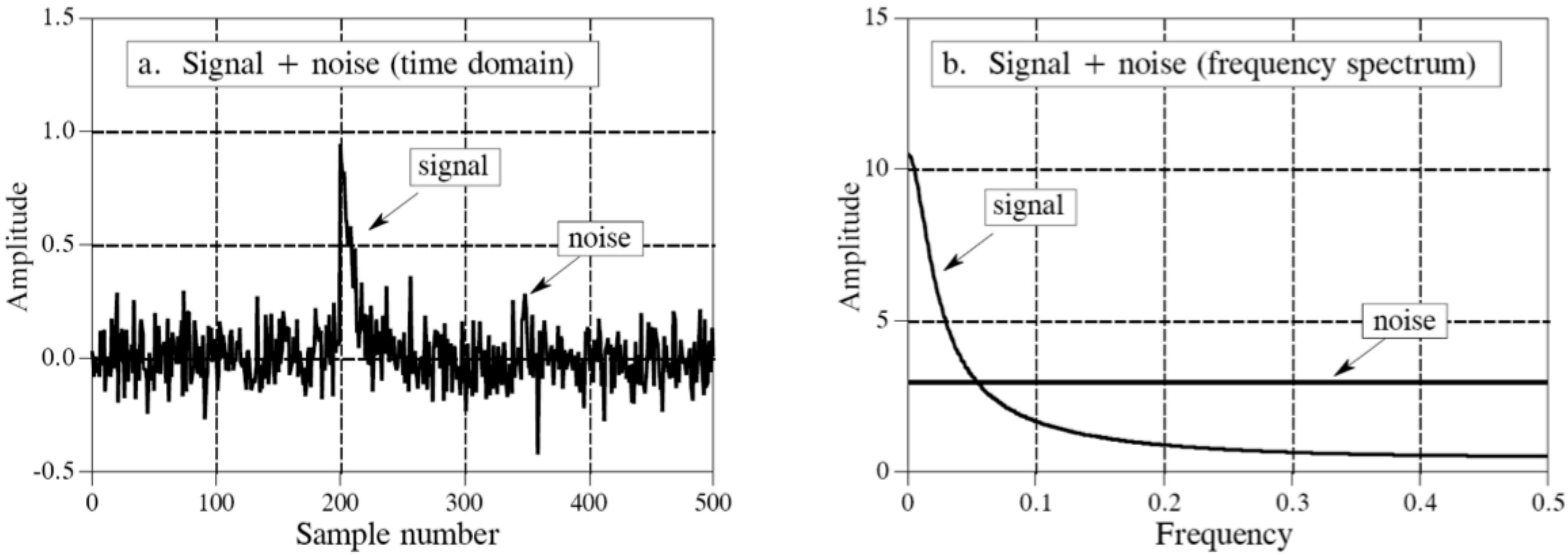
How should we choose *h(t)*⇔*H(f)* such that the output SNR is maximal?

$$SNR = \frac{\left| \int H(f) S(f) e^{j2\pi f T} df \right|^2}{E\left\{ \left| \int H(f) N(f) df \right|^2 \right\}}$$

Assuming white Gaussian noise, it can be shown that max SNR is when

$$\boxed{H(f) = S^*(f) \iff h(t) = s^*(-t)}$$

# Detection of a signal embedded in noise




Exponential pulse buried in random noise.  Sine the signal and noise overlap in both time and frequency domains, the best way to separate them is not obvious.

# Most important consideration: Match the bandwidth of the signal you are looking for

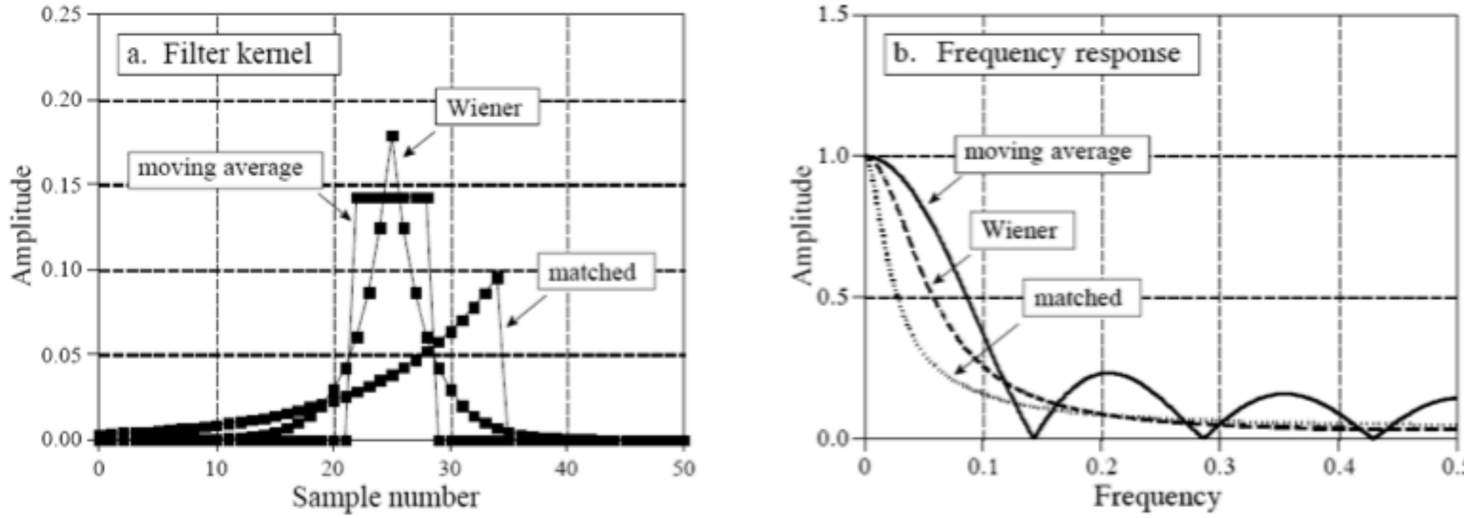
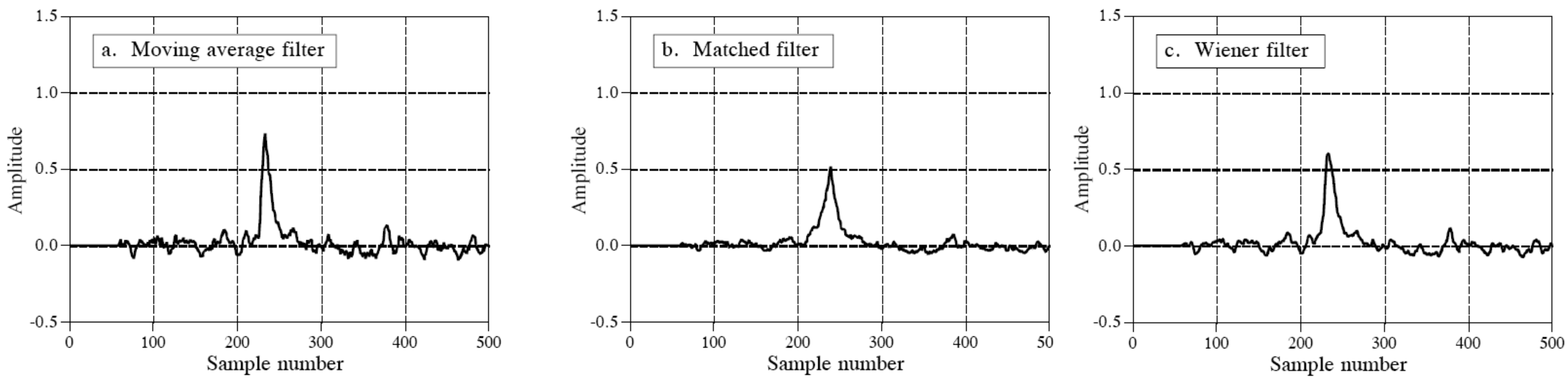
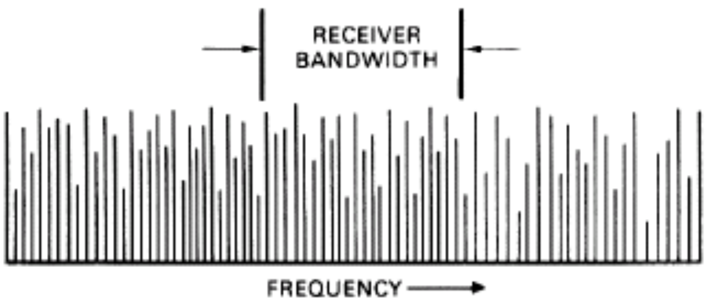



FIGURE 17-8
Example of optimal filters. In (a), three filter kernels are shown, each of which is optimal in some sense. The corresponding frequency responses are shown in (b). The moving average filter is designed to have a rectangular pulse for a filter kernel. In comparison, the filter kernel of the matched filter looks like the signal being detected. The Wiener filter is designed in the frequency domain, based on the relative amounts of signal and noise present at each frequency.

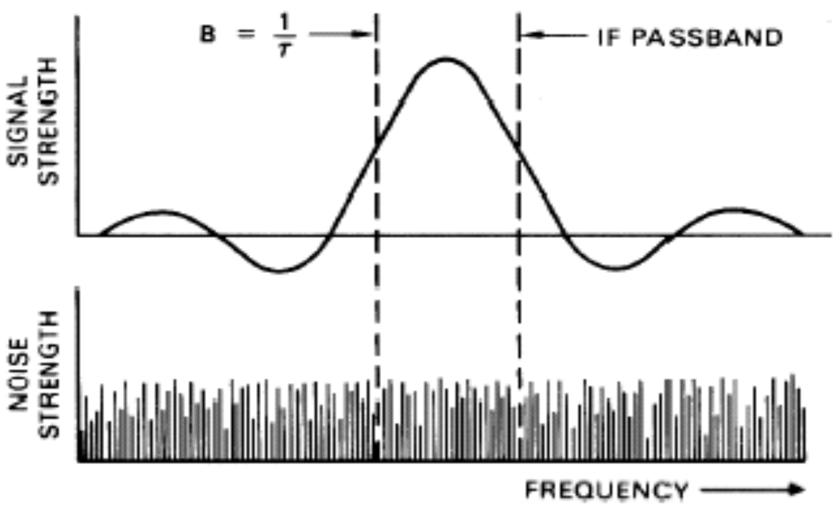


# The bandwidth-noise connection




6. Noise in receiver output is proportional to bandwidth of receiver.

The matched filter is a filter whose impulse response, or transfer function, is determined by a given signal, in a way that will result in the maximum attainable signal-to-noise ratio at the filter output when both the signal and white noise are passed through it.

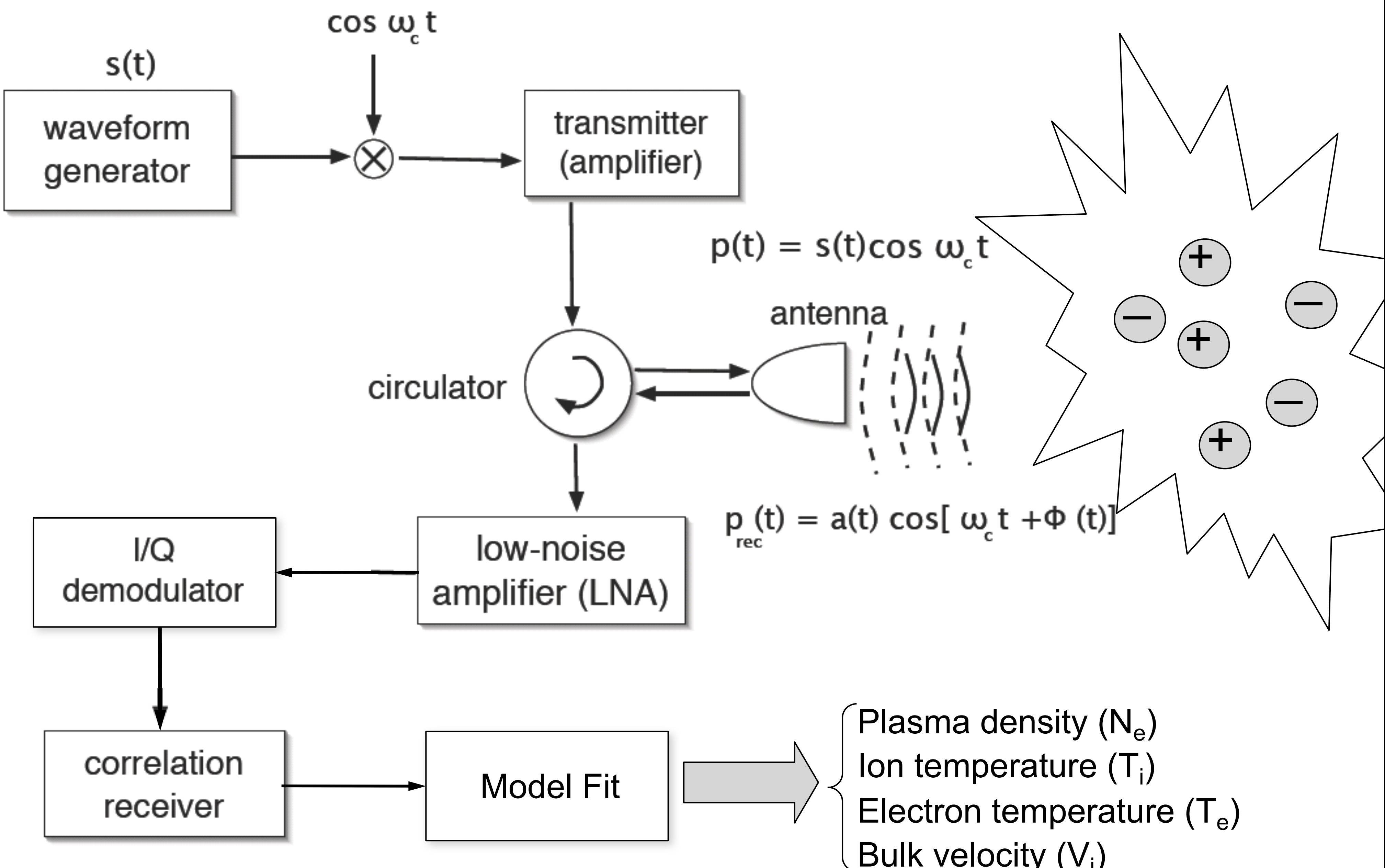



1. Signal-to-noise ratio may be maximized by narrowing the passband of the IF amplifier to the point where only the bulk of the signal energy is passed.

The optimum bandwidth of the filter, B, turns out to be very nearly equal to the inverse of the transmitted pulse width.

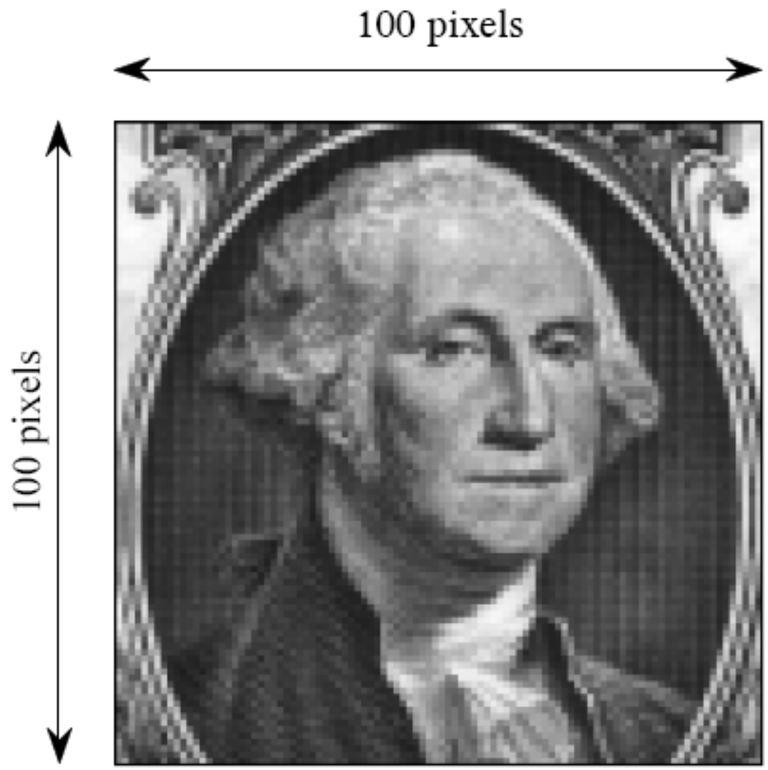
To improve range resolution, we can reduce $\tau$ (pulse width), but that means increasing the bandwidth of transmitted signal = More noise…

# Components of a Pulsed Doppler Radar

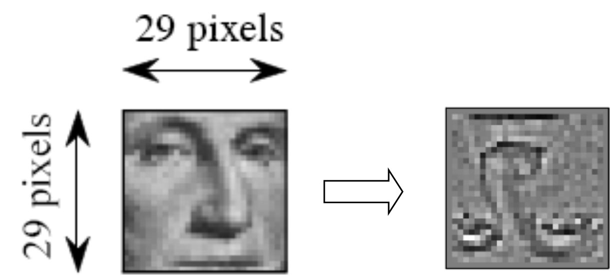


# Pulse compression and matched filtering

"If you know what you're looking for, it's easier to find."

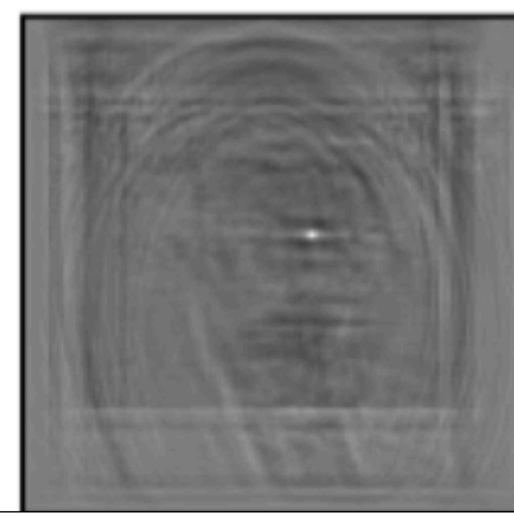



a. Image to be searched




b. Target

c. Kernel



Problem: Find the precise location of the target in the image.
Solution: Correlation

# Range detection: revisited

$$\Delta R = \frac{c\tau}{2} = \frac{c}{2B}$$

$\tau = \text{Pulse length}$

$B = \text{Bandwidth}$

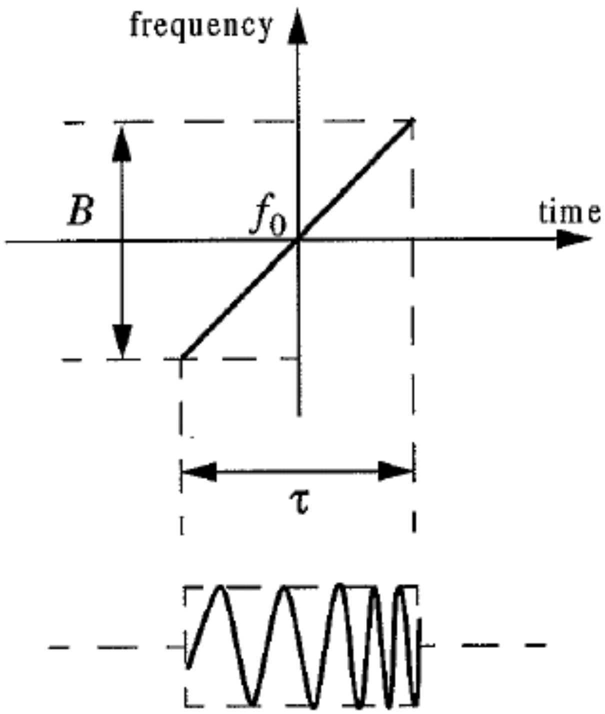
- **For high range resolution we want short pulse ⇔ large bandwidth**

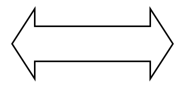
- **For high SNR we want long pulse ⇔ small bandwidth**

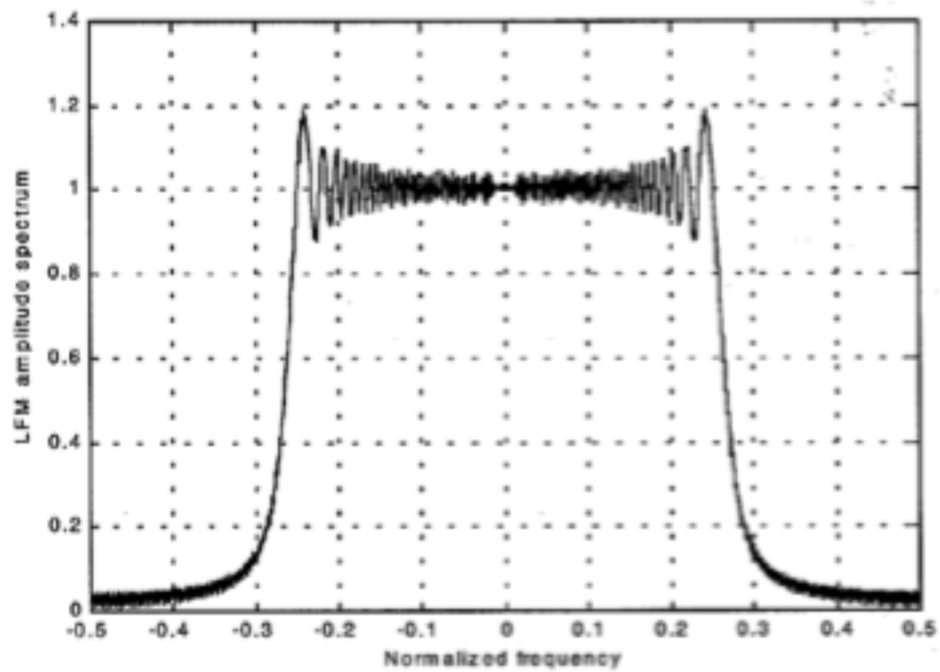
- **Long pulse also uses a lot of the duty cycle, can't listen as long, affects maximum range**

- **The Goal of pulse compression is to increase the bandwidth (equivalent to increasing the range resolution) while retaining large pulse energy.**

# Linear Frequency Modulation (LFM or "Chirp")



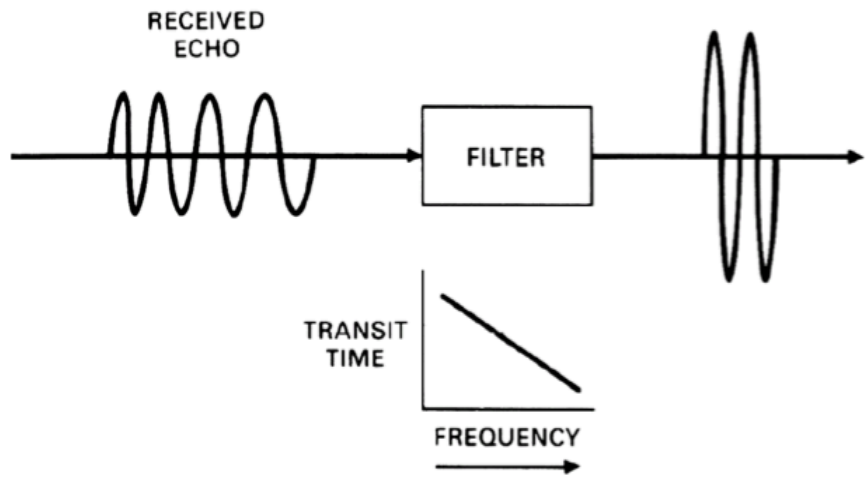



$$s_1(t) = e^{j2\pi f_0 t} s(t)$$

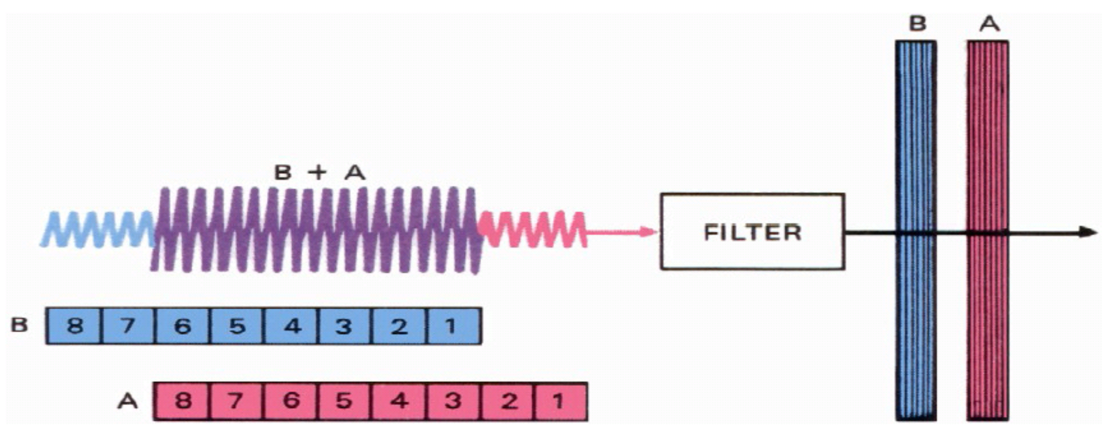
where

$$s(t) = Rect\left(\frac{t}{\tau}\right) e^{j\pi\mu t^2}$$

# Matched filter detection of a chirp

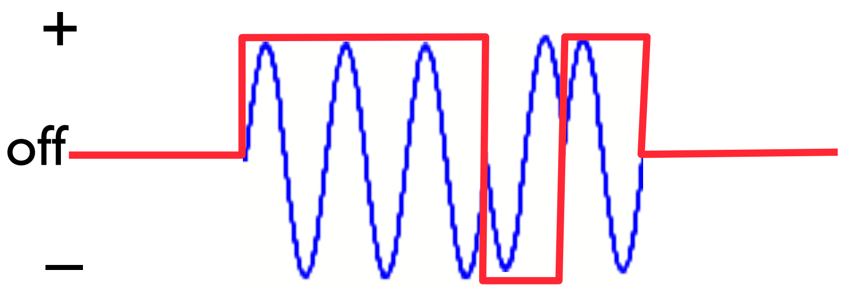



Since trailing portions of echo take less time to pass through filter, successive portions tend to bunch up: Amplitude of pulse is increased and width is decreased.




Echoes from closely spaced targets, A and B, are merged but, because of coding, separate in output of filter.

# Barker codes




| | | | | | | | | | correlator output |
|---|---|---|---|---|---|---|---|---|---|
| | | | | + | + | + | - | + | |
| + | + | + | - | + | | | | | 1 |
| | + | + | + | - | + | | | | -1+1=0 |
| | | + | + | + | - | + | | | 1-1+1=1 |
| | | | + | + | + | - | + | | 1+1-1-1=0 |
| | | | | + | + | + | - | + | 1+1+1+1+1=5 |

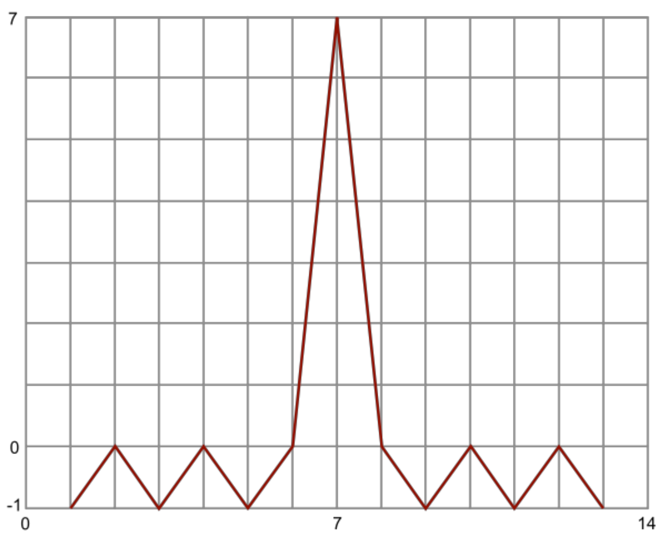



**TABLE 6.2 All Known Binary Barker Codes**

| Code Length | Code |
|---|---|
| 2 | 11 or 10 |
| 3 | 110 |
| 4 | 1110 or 1101 |
| 5 | 11101 |
| 7 | 1110010 |
| 11 | 11100010010 |
| 13 | 1111100110101 |

# Doppler Revisited

Transmitted signal: $\cos(2\pi f_o t)$

After return from target: $$\cos\left[2\pi f_o\left(t+\frac{2R}{c}\right)\right]$$

To measure frequency, we need to observe signal for at least one cycle.
So we will need a model of how *R* changes with time. Assume constant velocity:

$$R = R_o + v_o t$$

Substituting:

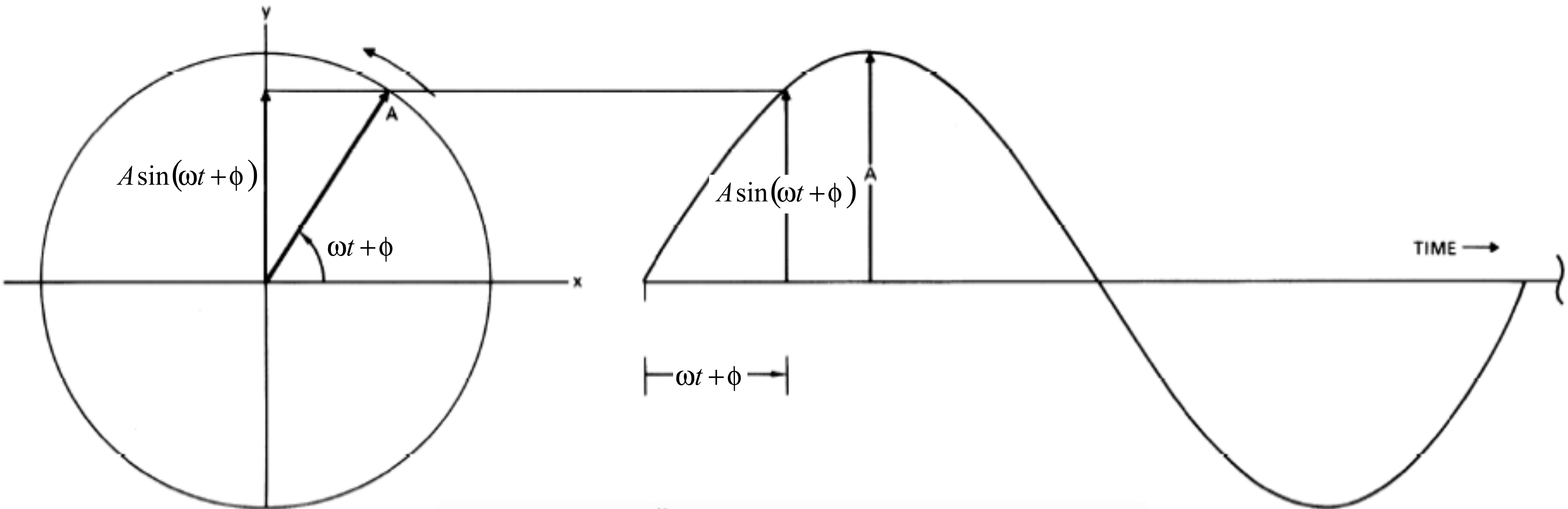
$$\cos\left[2\pi\left(f_o + \underbrace{f_o\frac{2v_o}{c}}_{-f_D}\right)t + \underbrace{\frac{2\pi f_o R_o}{c}}_{\text{constant}}\right]$$

$$f_D = \frac{-2 f_o v_o}{c} = \frac{-2v_o}{\lambda_o} = \frac{d\phi}{dt}$$

By convention, positive Doppler frequency shift $\Longleftrightarrow$ Target and radar closing

# Doppler Detection: Intuitive Approach

**Phasor diagram is a graphical representation of a sine wave**




**<u>I & Q components*</u>**

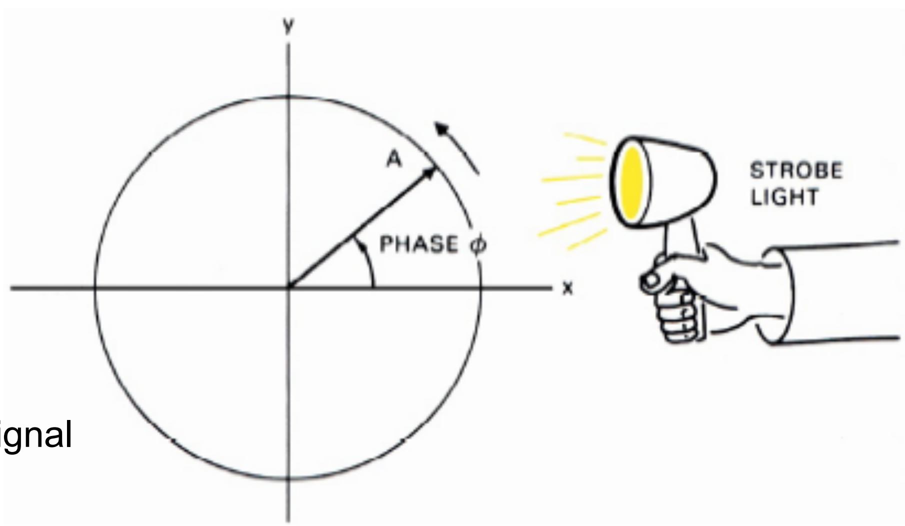
**I => in-phase component**

**Acos(ϕ)**

**Q => in-quadrature component**

**Asin(ϕ)**

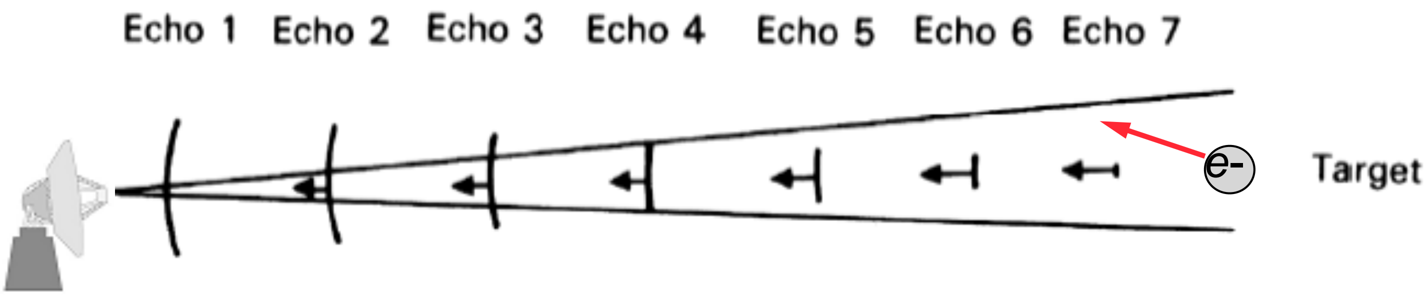
*relative to reference signal

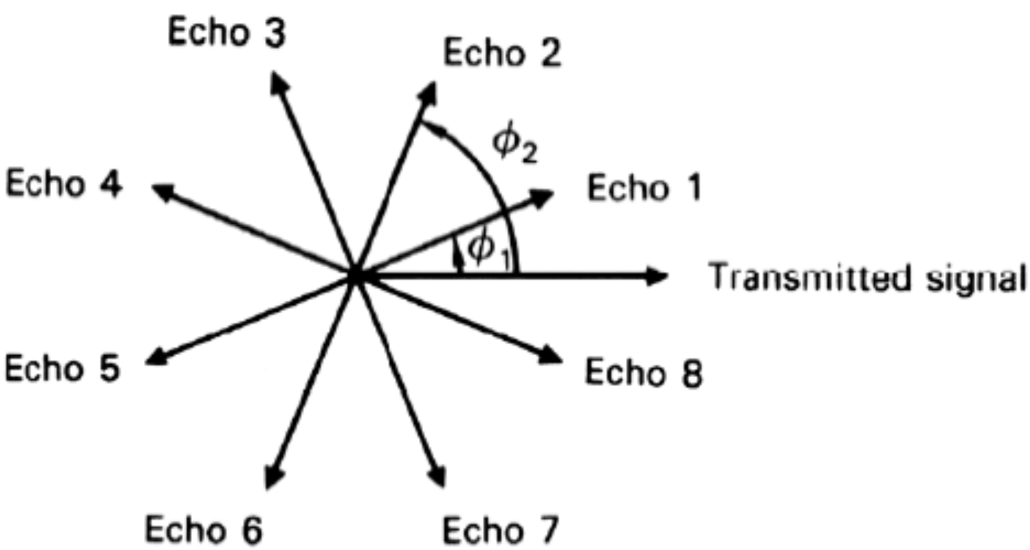


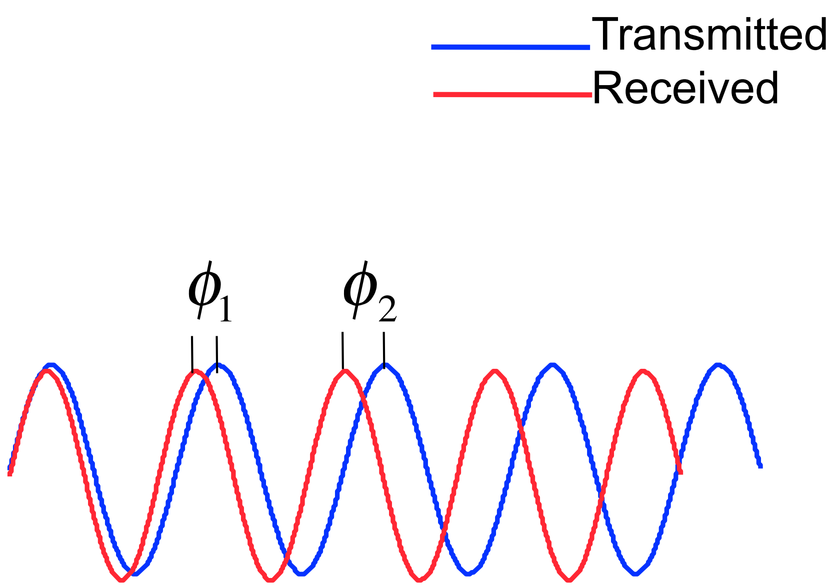

**Consider strobe light as cosine reference wave at same frequency but with initial phase = 0**

# Doppler Detection: Intuitive Approach

Closing on target – positive Doppler shift

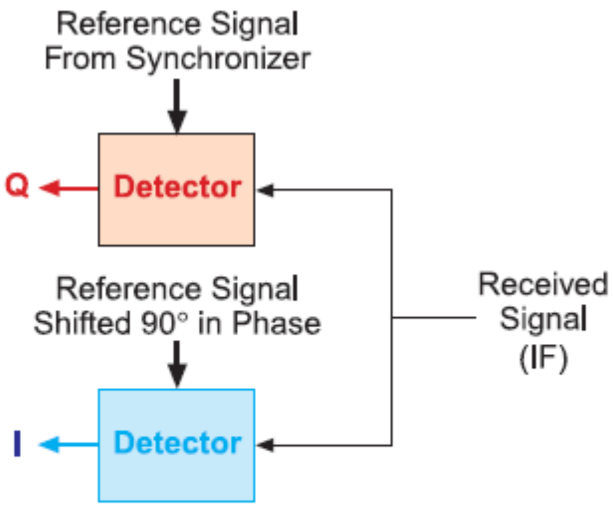


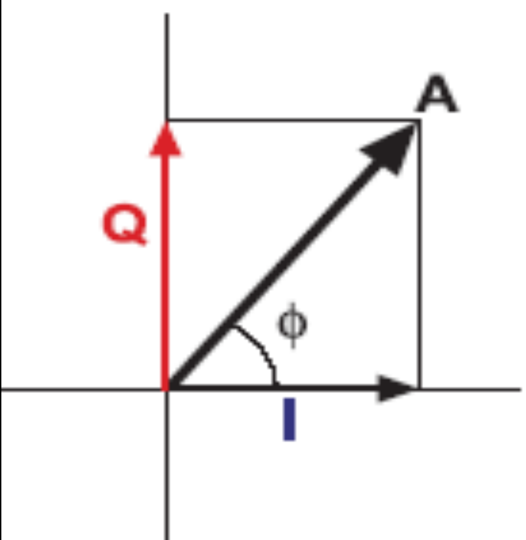




Target's Doppler frequency shows up as a pulse-to-pulse shift in phase.

# I and Q Demodulation




in-phase (I) channel:

$$p_{rec}(t)\cos(\omega_c t) = a(t)\cos(\phi(t)+\omega_c t)\cos(\omega_c t)$$

$$= a(t)\frac{1}{2}\left(\underbrace{\cos(\phi(t)+2\omega_c t)}_{\text{filter out}} + \cos\phi(t)\right)$$

quadrature (Q) channel (90° out of phase):

$$p_{rec}(t)\sin(\omega_c t) = a(t)\cos(\phi(t)+\omega_c t)\sin(\omega_c t)$$

$$= a(t)\frac{1}{2}\left(\underbrace{-\sin(\phi(t)+2\omega_c t)}_{\text{filter out}} + \sin\phi(t)\right)$$

I and Q channels together give the *analytic signal*

$$\boxed{s_{rec}(t) = a(t)e^{i\phi(t)}}$$

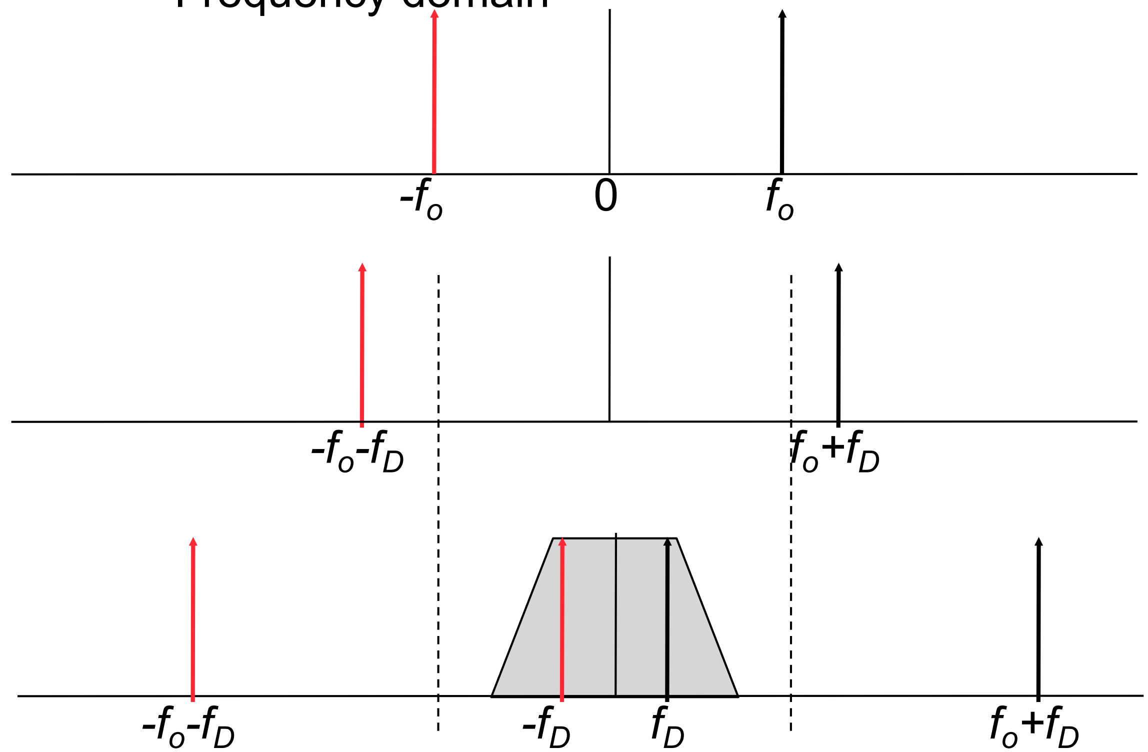
The fundamental output of a pulsed Doppler radar is a time series of complex numbers.

# I and Q Demodulation in Frequency Domain

Transmitted signal

$$\cos(2\pi f_o t)$$

Frequency domain




Doppler shifted

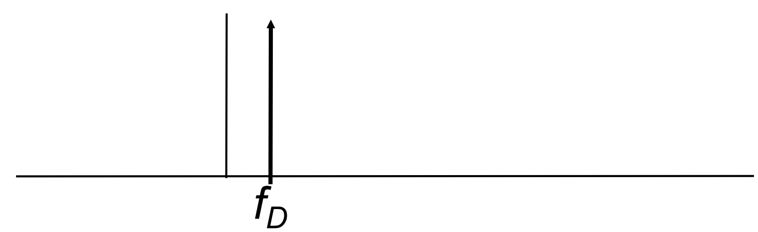
$$\cos(2\pi(f_o + f_D)t)$$

$$\cos(2\pi f_o t)$$

$$\cos(2\pi f_D t)$$

Cosine is even function, so sign of $f_D$ (and, hence, velocity) is lost.
What we need instead is:

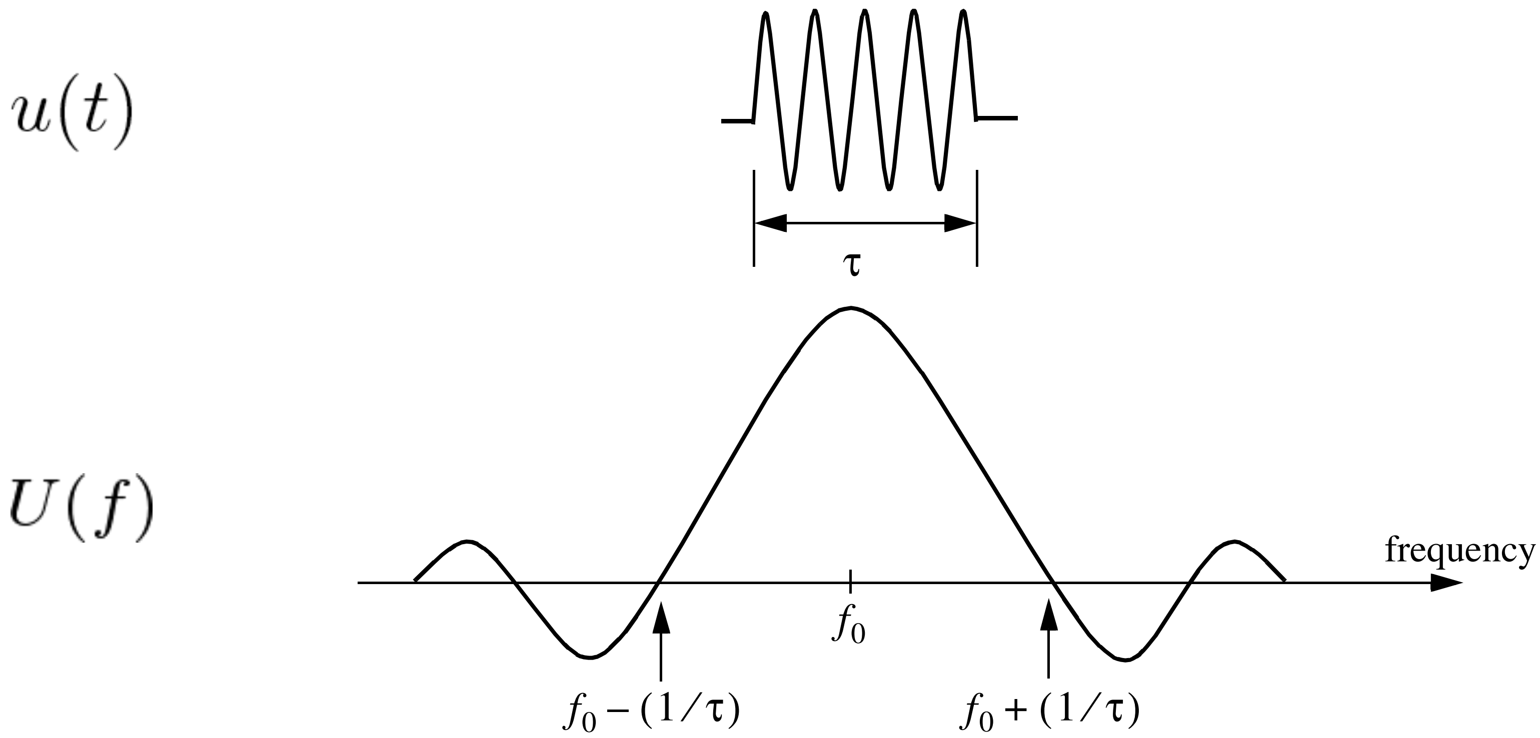
$$\exp(j2\pi f_D t) = \cos(2\pi f_D t) + j\sin(2\pi f_D t)$$




The analytic signal $\exp(j2\pi f_D t)$ cannot be measured directly, but the cos and sin components via mixing with two oscillators with same frequency but orthogonal phases. The components are called “in phase” (or *I*) and “in quadrature” (or *Q*):

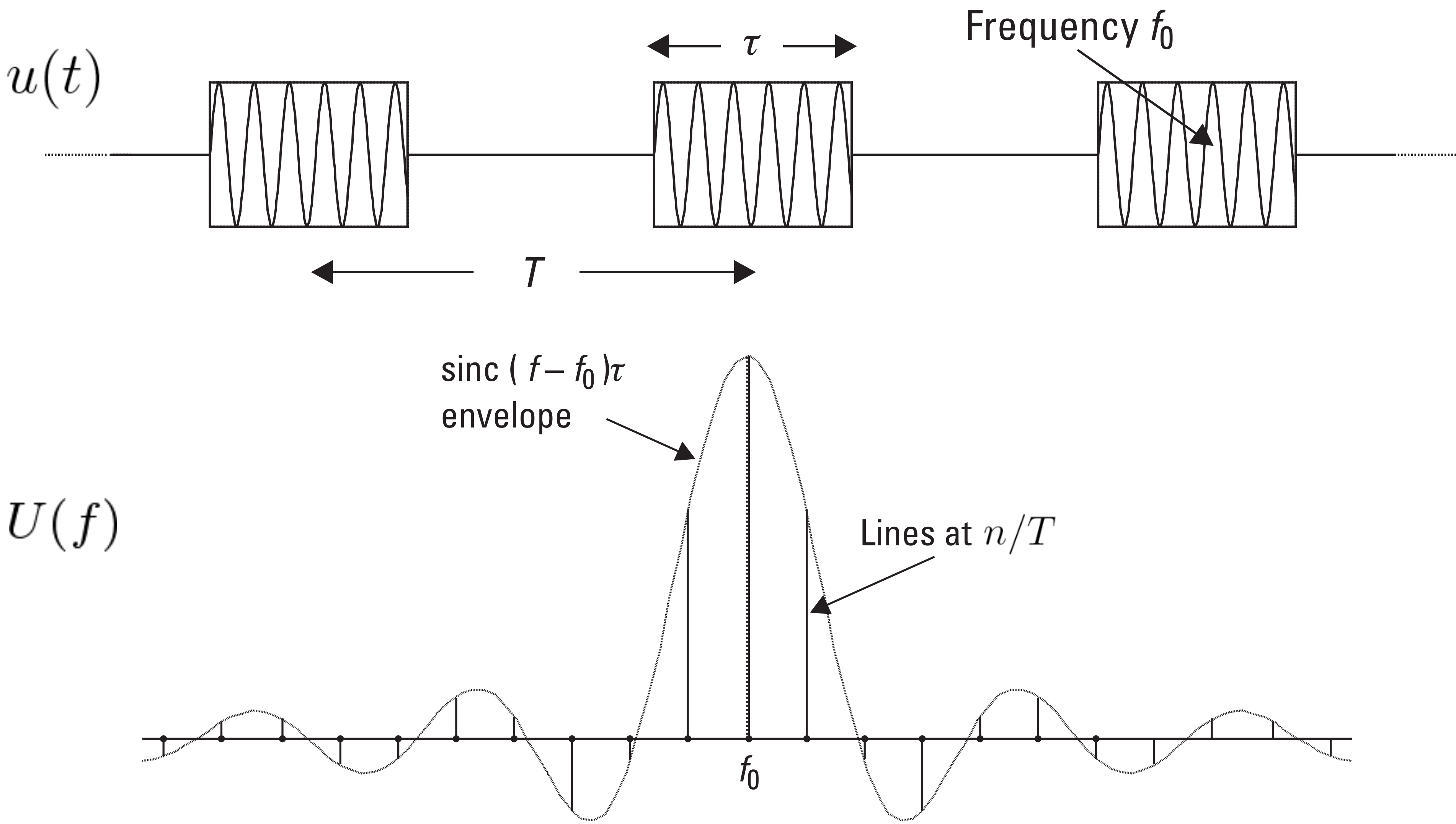
$$A\exp(j2\pi f_D t) = I + jQ$$

# Fourier Transform of an RF Pulse



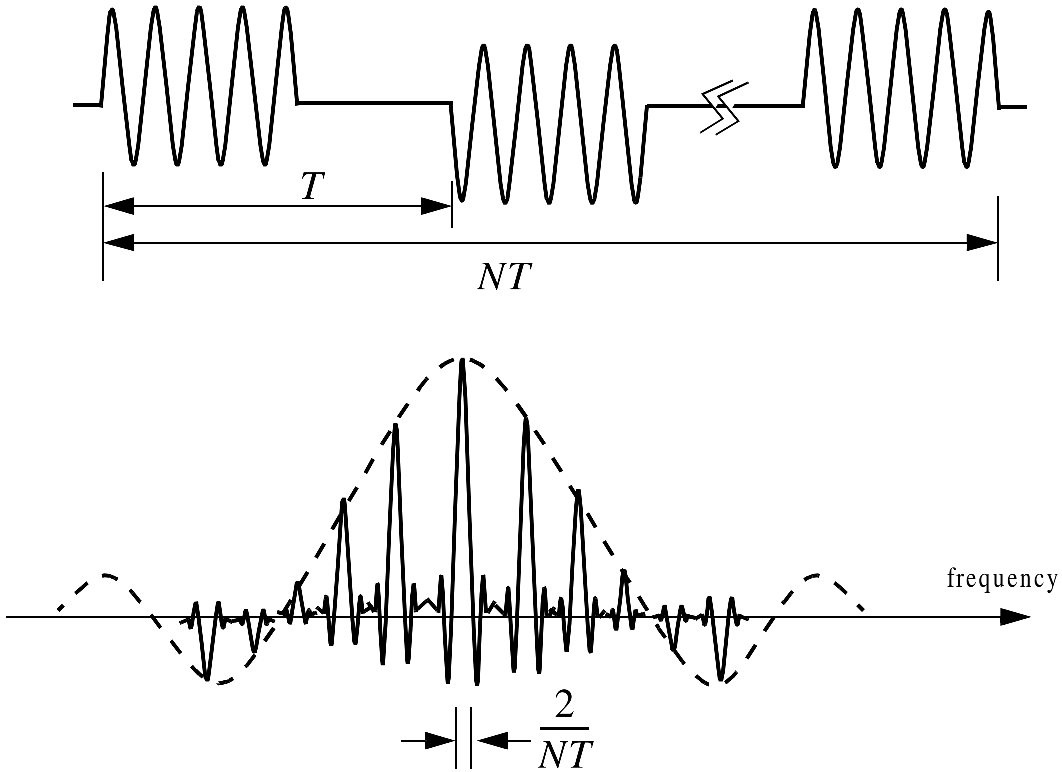

The F.T. of a simple RF pulse is a sync function shifted to the carrier frequency (by convolution property and definition of delta function).

# Fourier Transform of a Long Pulse Train




The F.T. of an infinite train of RF pulses is a line spectrum with lines separated by the pulse repetition frequency.

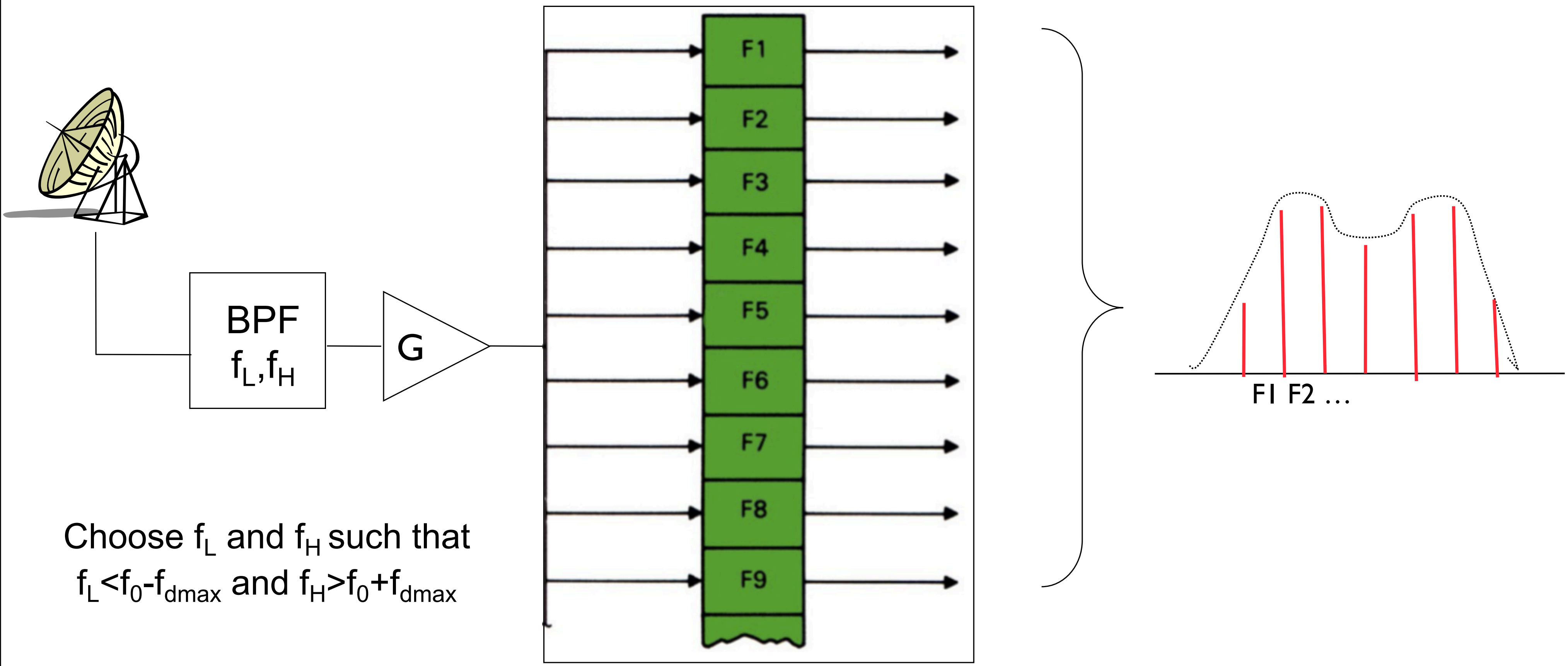
# Fourier Transform of a Finite Pulse Train




The F.T. of a finite train of RF pulses is a decaying train of sync functions with width inversely proportional to the total number of pulses, and separation inversely proportional to the Interpulse Period (IPP).

# ISR Receiver: Doppler filter bank approach

Doppler Filter Bank     Doppler Range

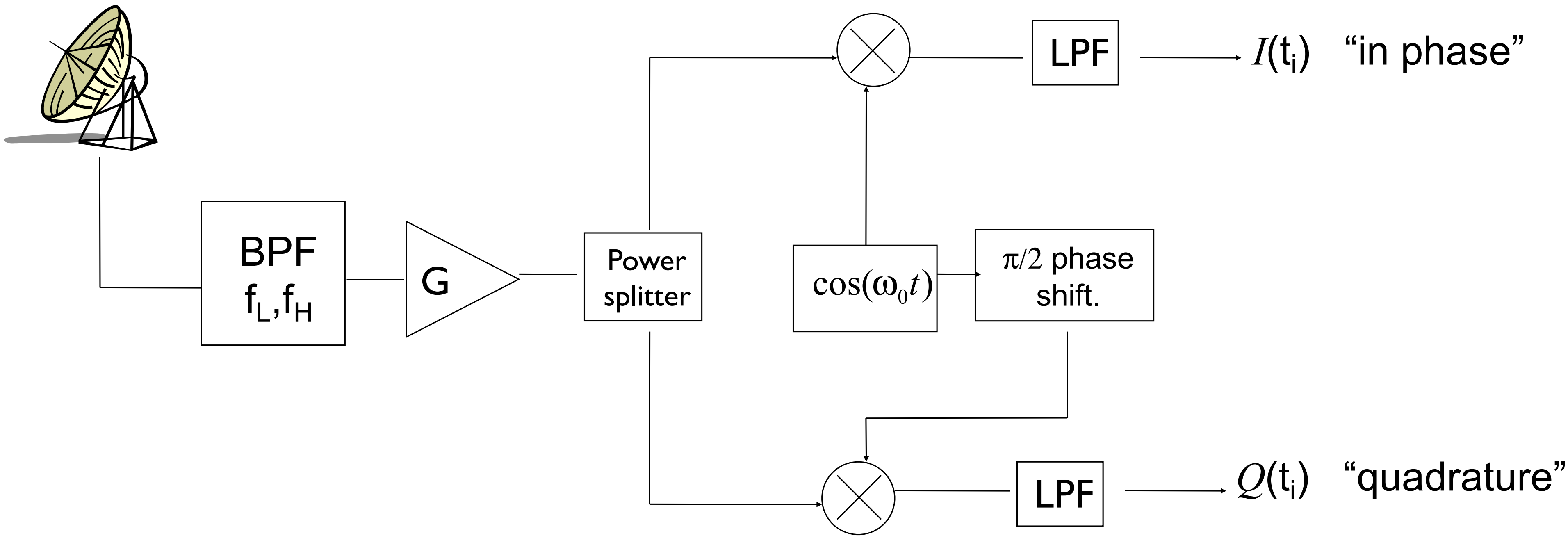



Choose $f_L$ and $f_H$ such that
$f_L < f_0 - f_{dmax}$ and $f_H > f_0 + f_{dmax}$

Practical Problem: It is hard to make narrow band (High Q) RF filters:

$$Q = \frac{f_0}{f_H - f_L}$$

# ISR Receiver: I and Q plus correlation




**We have time series of *V(t) =I(t) + jQ(t)*, how do I compute the Doppler spectrum?**

Estimate the autocorrelation function (ACF) by computing products of complex voltages ("lag products")

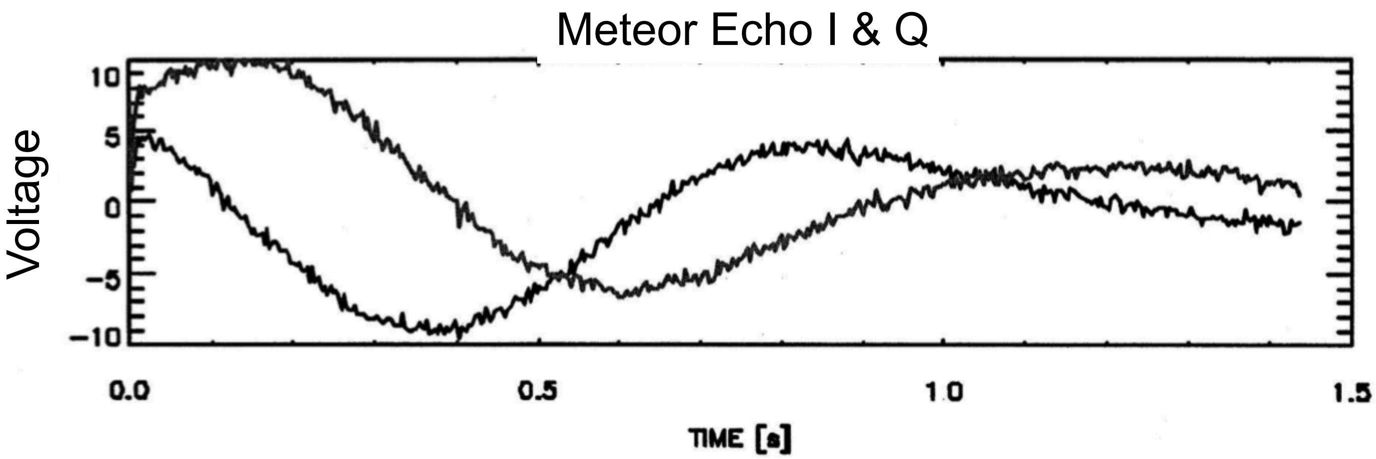
$$\rho(\tau) = \frac{\left\langle V(t)V^*(t+\tau)\right\rangle}{S}$$

FFT

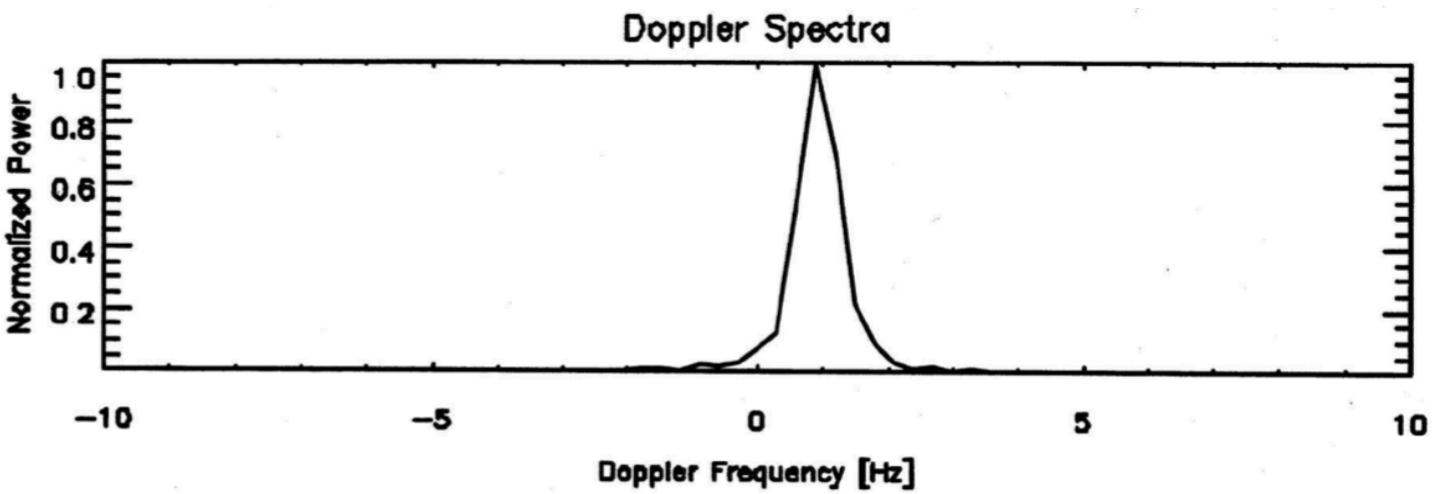
Power spectrum is Fourier Transform of the ACF

# Example: Doppler Shift of a Meteor Trail

- **Collect N samples of $I(t_k)$ and $Q(t_k)$ from a target**
- **Compute the complex FFT of $I(t_k)+jQ(t_k)$, and find the maximum in the frequency domain**
- **Or compute "phase slope" in time domain.**



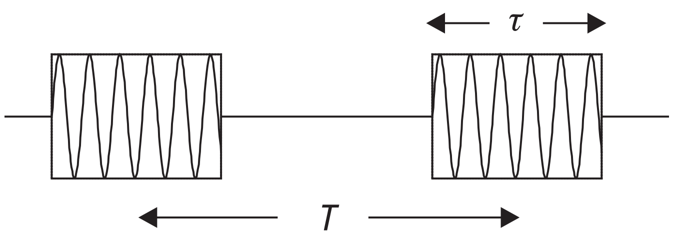


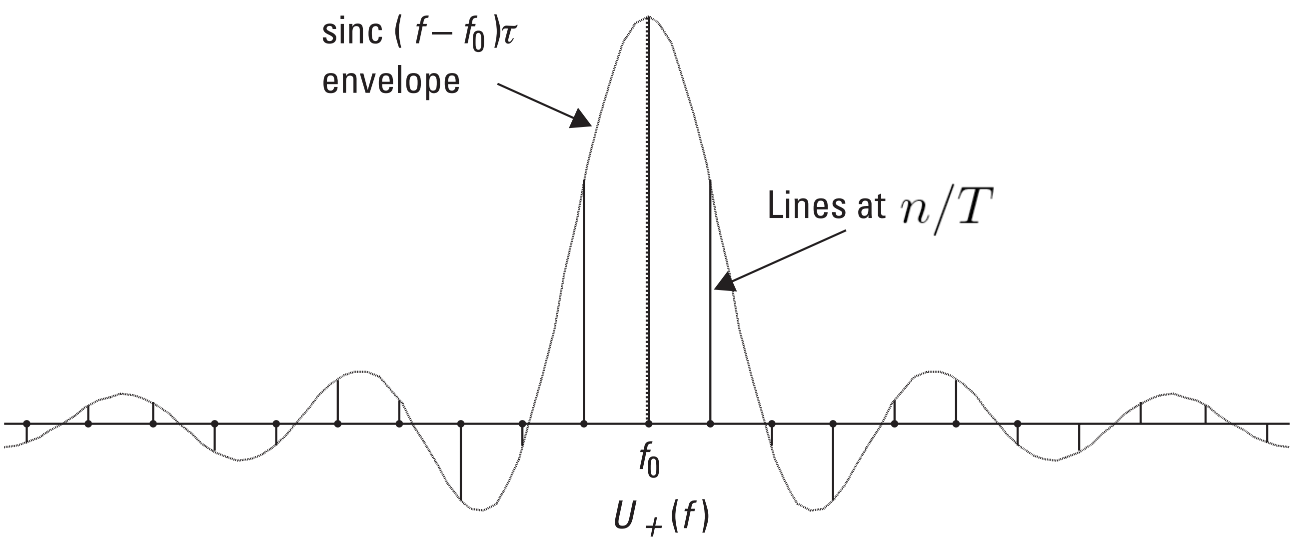
# Does this strategy work for ISR?

Typical ion-acoustic velocity: 3 km/s
Doppler shift at 450 MHz: 10kHz
Correlation time: 1/10kHz = 0.1 ms
Required PRF to probe ionosphere (500km range): 300 Hz

Plasma has completely decorrelated by the time we send the next pulse.

Alternately, the Doppler shift is well beyond the max unambiguous Doppler defined by the Inter-Pulse Period *T*.

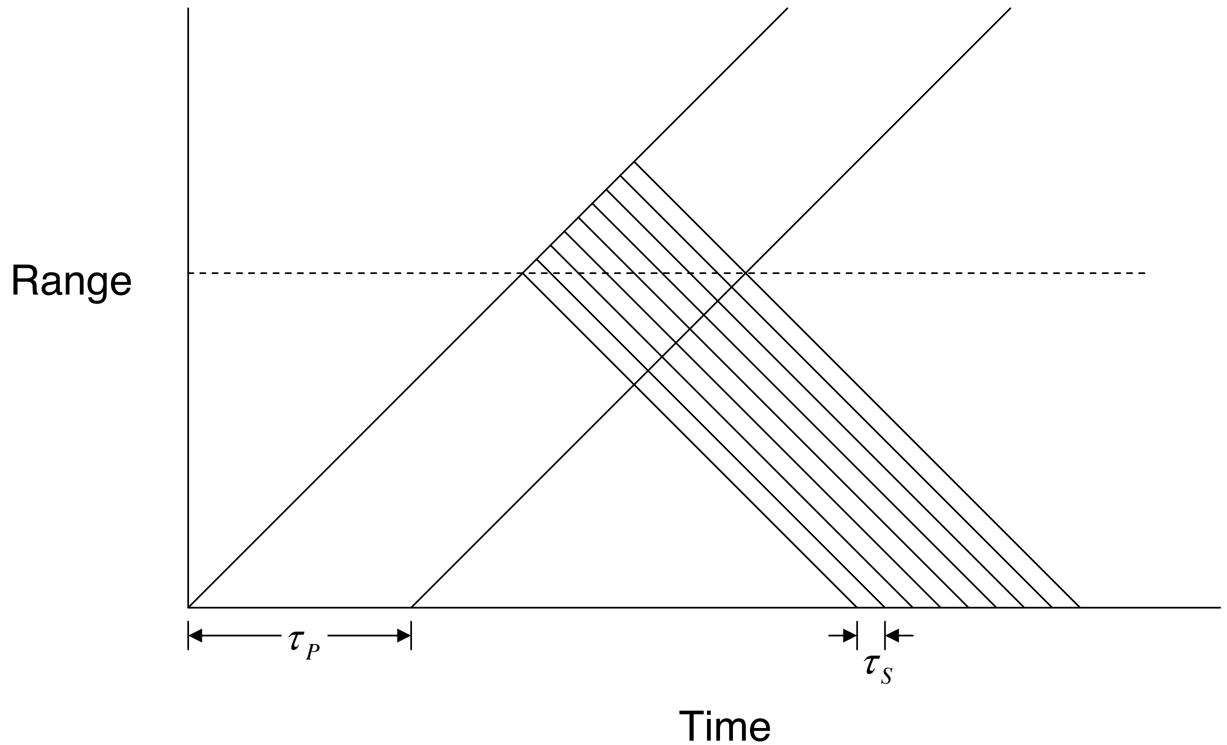


# The ISR target is “overspread”

$f_d >> 1/\tau$ (Doppler changes significantly during one pulse)

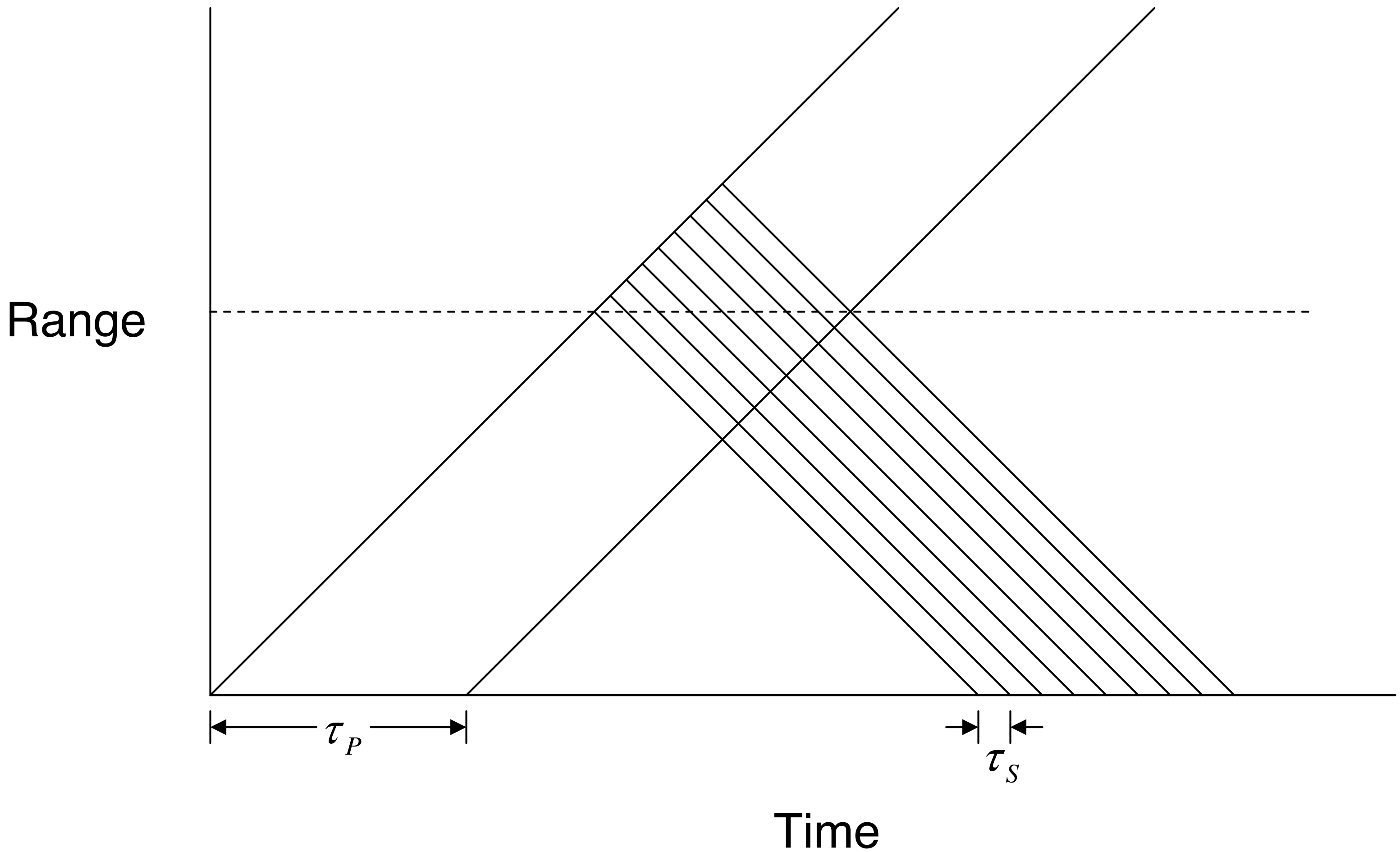
- Must sample multiple times per pulse
- Result: Doppler can be determined from single pulse.




$\tau_P$ = Length of RF pulse

$\tau_s$ = Sample Period (typically ~ 1/10 pulse length)

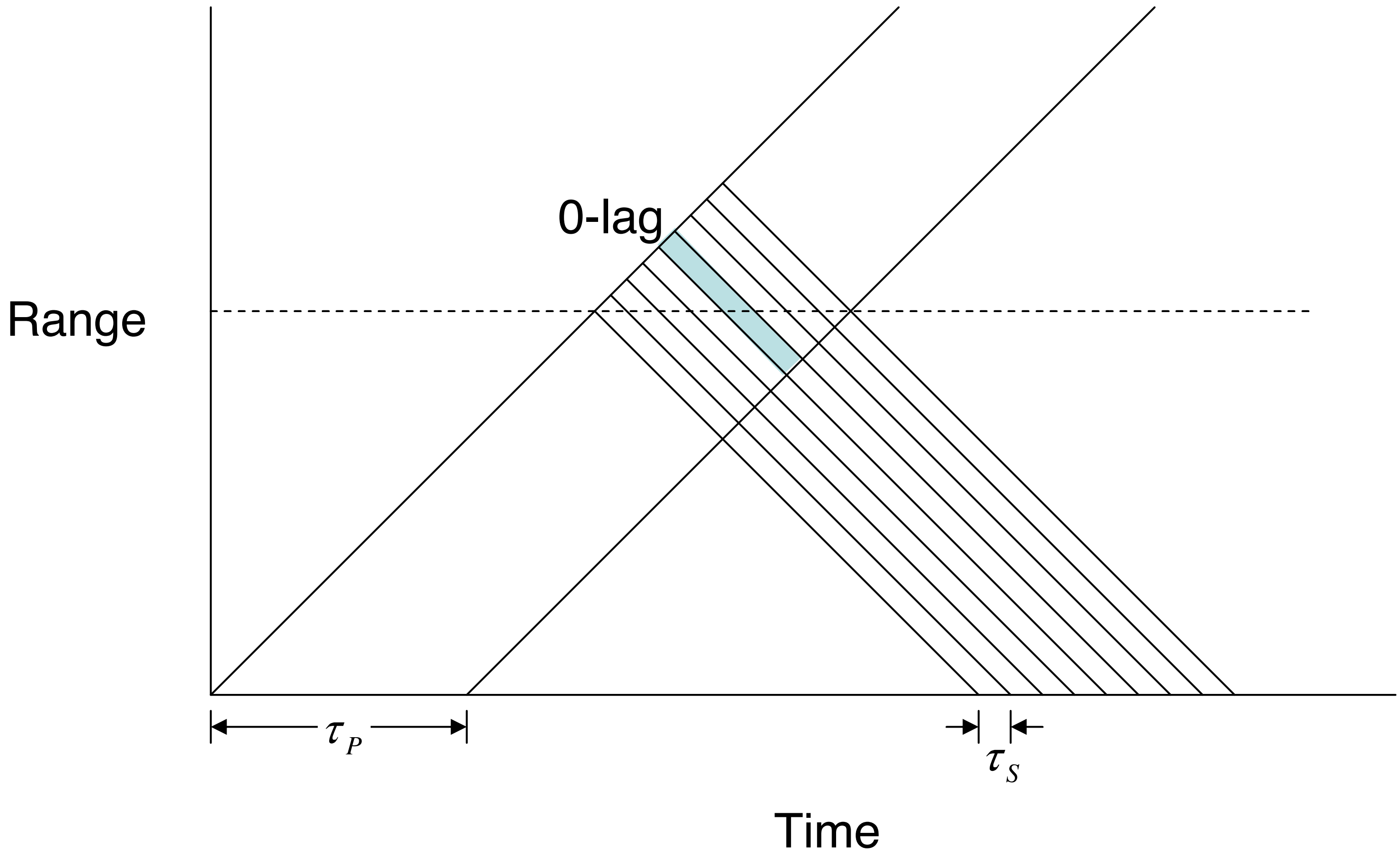
# Computing the ACF



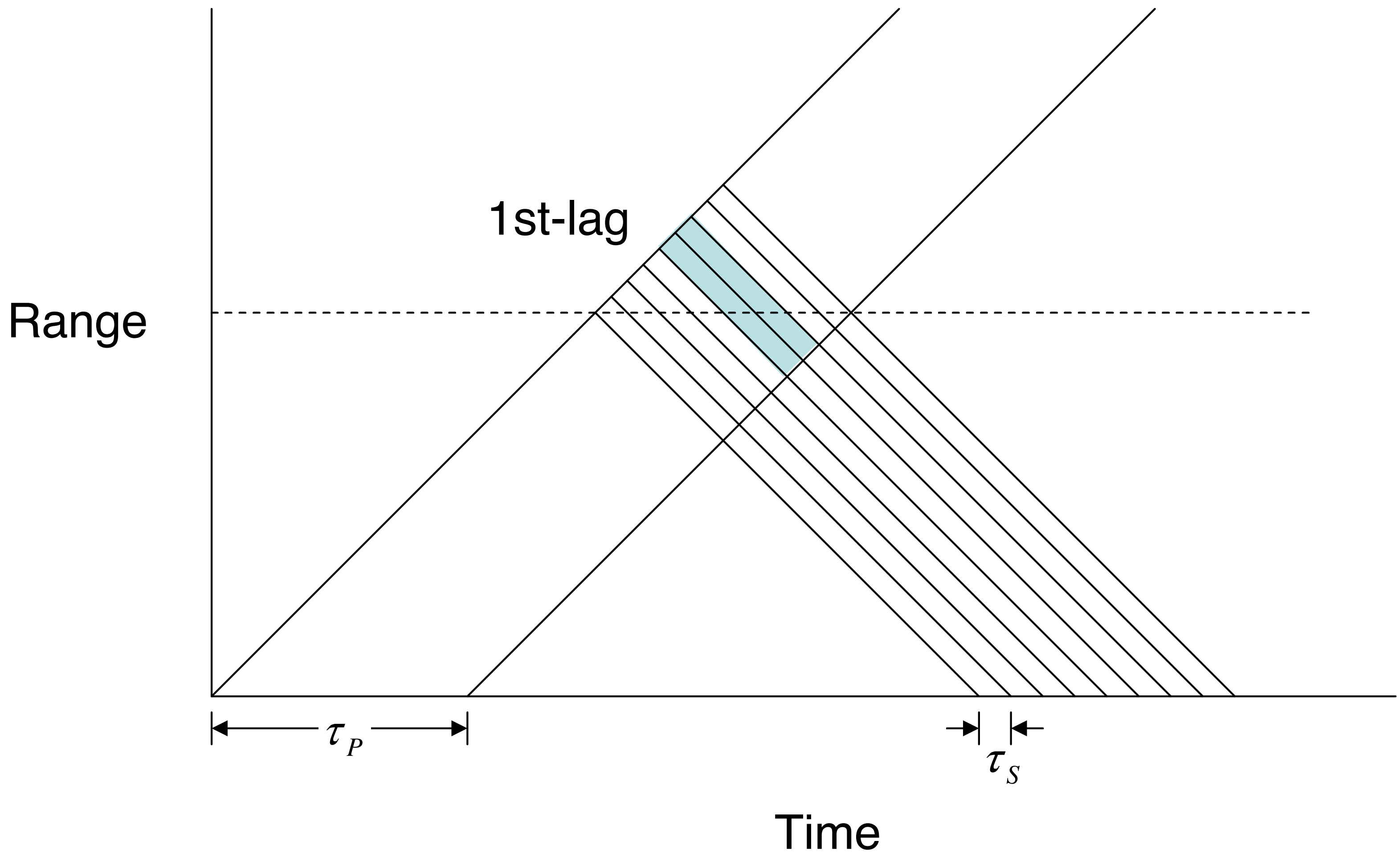

$\tau_P$ = Length of RF pulse

$\tau_s$ = Sample Period (typically ~ 1/10 pulse length)

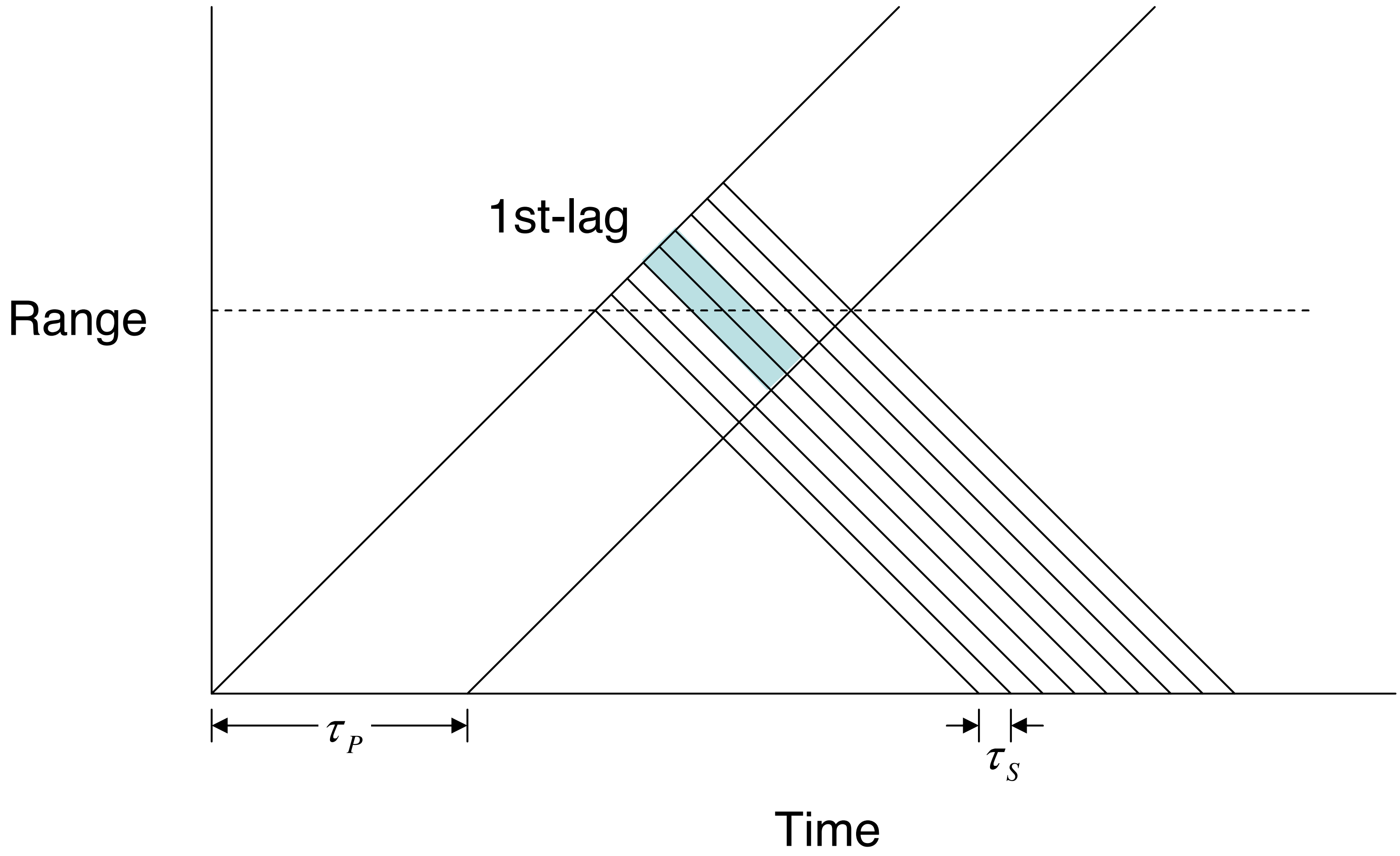
# Computing the ACF



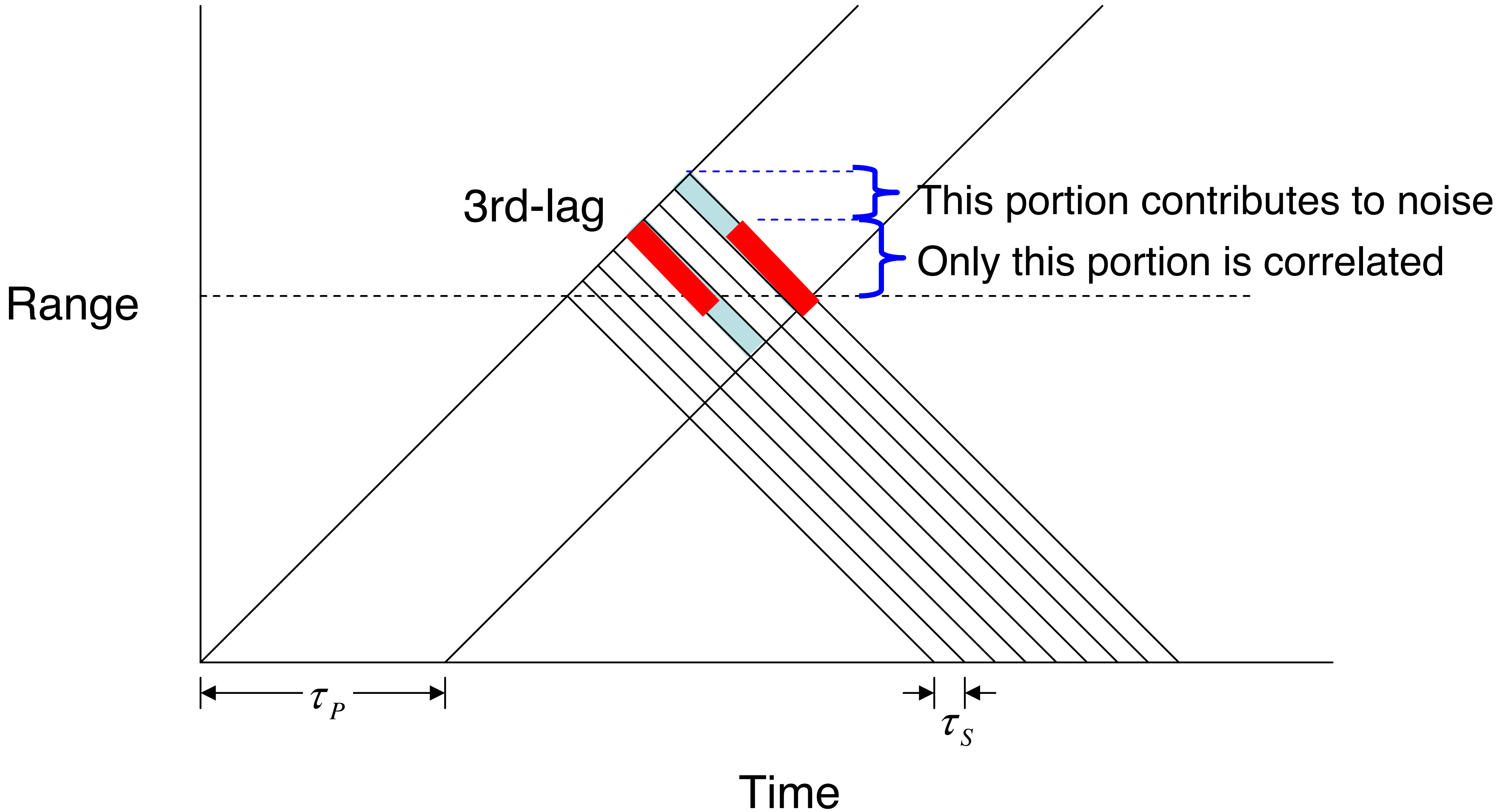
# Computing the ACF



# Computing the ACF



# Computing the ACF



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*For fun:*

- http://mathforum.org/johnandbetty/