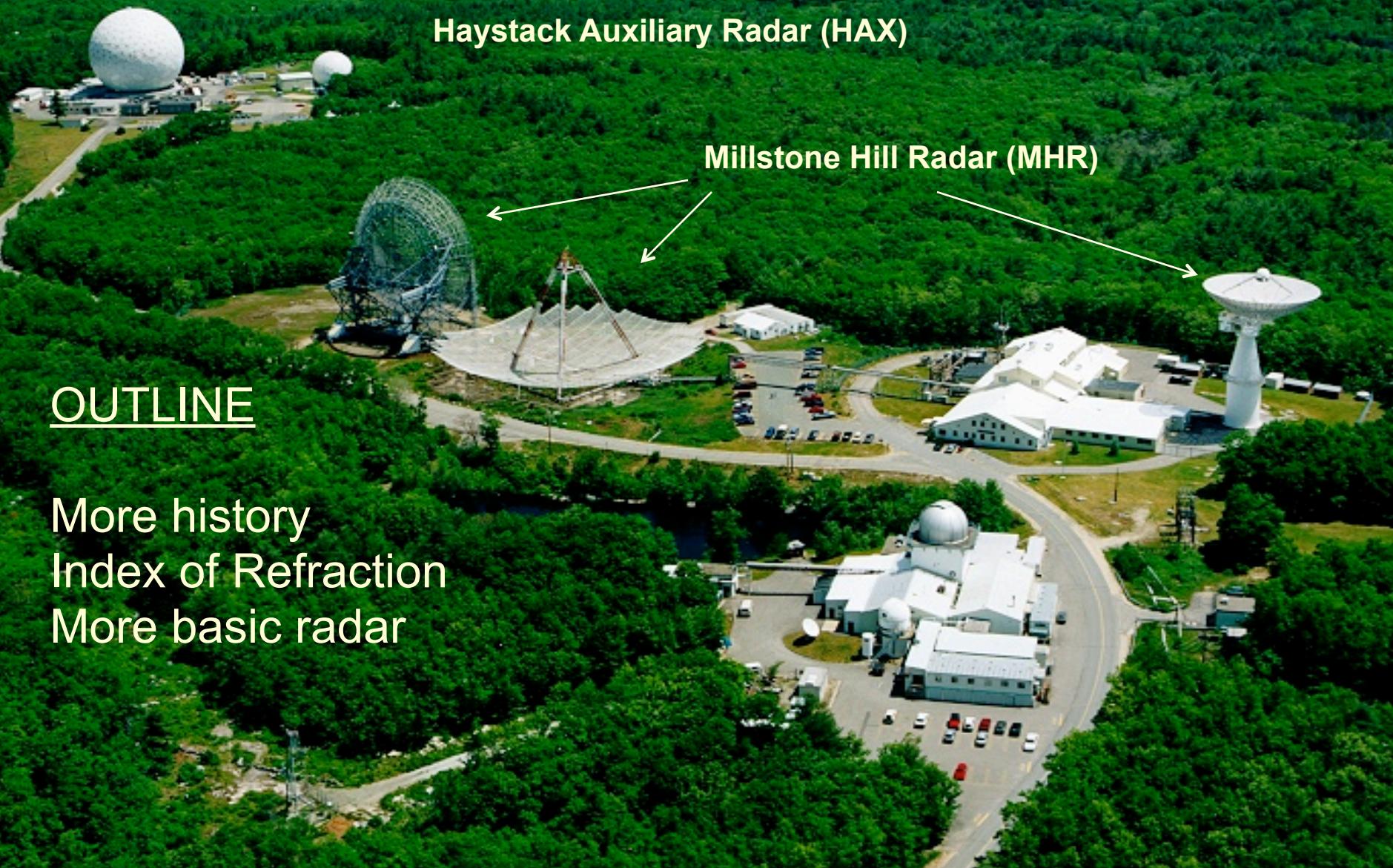
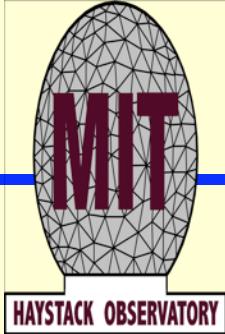


Haystack Radar (HY)

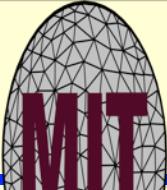


OUTLINE

More history
Index of Refraction
More basic radar



*It is frequently said that,
although the atomic bomb
ended World War II, it was
radar that won the war.*

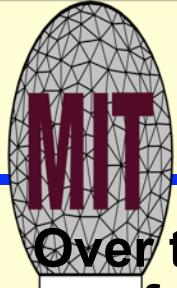


MIT Radiation Laboratory



E. G. Bowen (center), member of the British scientific mission that brought the first cavity magnetron to the United States in 1940, is shown an American-made copy by Radiation Laboratory director L. A. DuBridge (left) and associate director I. I. Rabi, a Nobel Prize winner (right). Photograph courtesy of the MIT Museum; from the Radiation Laboratory Negative Collection.

- The primary technical barrier to developing UHF systems was the lack of a usable source for generating high-power microwaves.
- In February 1940, John Randall and Harry Boot at Birmingham University in the UK built a resonant cavity magnetron.
- Bombing of London Sept 1940 – May 1941 (The Blitz)
- Britain was interested in developing practical applications for airborne microwave radar, but did not have the large-scale manufacturing ability to mass produce magnetrons.
- In 1940, Britain partnered with the US National Defense Research Committee (NDRC)
-



MIT Radiation Laboratory

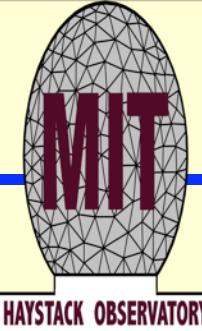
Over the course of five years, MIT researchers designed 50 percent of the radar used in World War II and invented over 100 different radar systems.

Including:

- Airborne bombing radars
- Shipboard search radars
- Harbor and coastal defense radars
- Interrogate-friend-or-foe beacon systems
- Long-range navigation (LORAN) system
- Critical contributions of the Radiation Laboratory were:
 - the microwave early-warning (MEW) radars, which effectively nullified the V-1 threat to London, and
 - air-to-surface vessel (ASV) radars, which turned the tide on the U-boat threat to Allied shipping.



Building 20 on the MIT campus, home of the Rad Lab from 1943 to 1945.

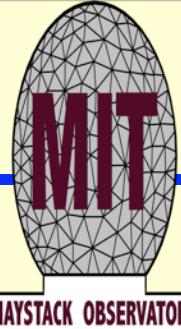


Millstone

The BMEWS Prototype



**Millstone Radar
1957**



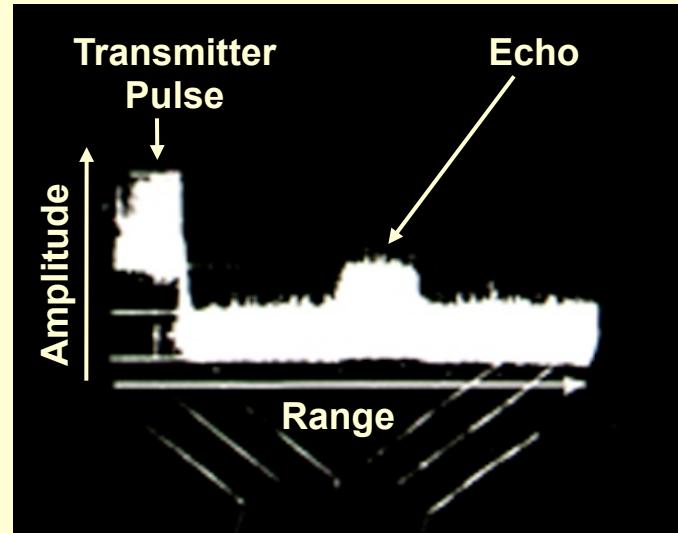
Millstone

The BMEWS Prototype

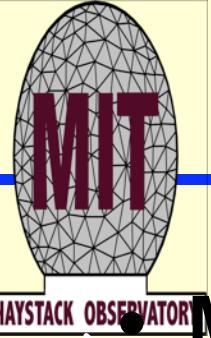


Millstone Radar
1957

First in Space Surveillance



Sputnik
A-Scope Trace



Outline

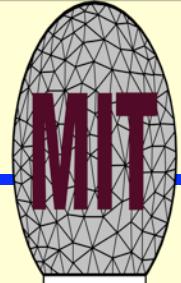
• More History

 Index of Refraction

- More Basic Radar

Appleton - Hartree

but also ...



Outline

• More History

→ Index of Refraction

• More Basic Radar



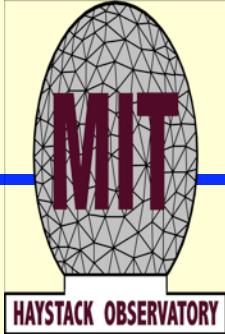
Appleton

- Hartree

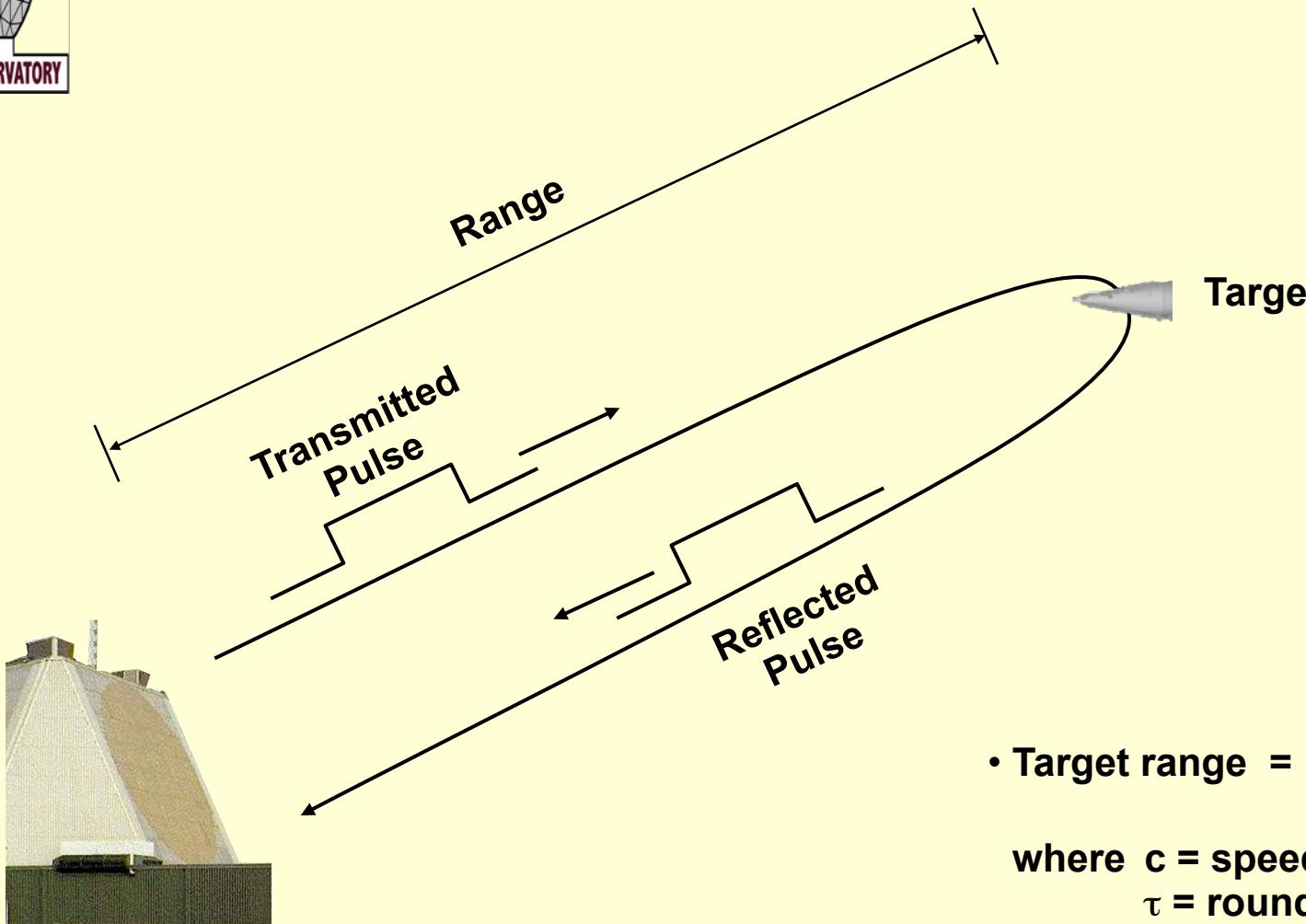
but also ...



Until 1976, it was almost unknown that Appleton had collaborated with the Austrian physicist Wilhelm Altar on magneto-ionic theory in 1925-26. Altar wrote a paper which contains the magneto-ionic equations in so-called “dielectric tensor” form, and in general terms contains what would today be called the dispersion relation in cold plasma theory.

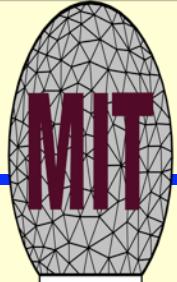


Radar Range Measurement



$$\bullet \text{ Target range} = \frac{c\tau}{2}$$

where c = speed of light
 τ = round trip time

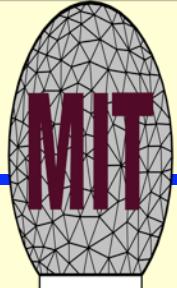


Phase Velocity, Group Velocity, Index of

$$v_p = \frac{\omega}{k}$$

$$v_g \equiv \frac{\partial \omega}{\partial k}$$

$$n = \frac{c}{v_p}.$$



Phase Velocity, Group Velocity, Index of

$$v_p = \frac{\omega}{k}$$

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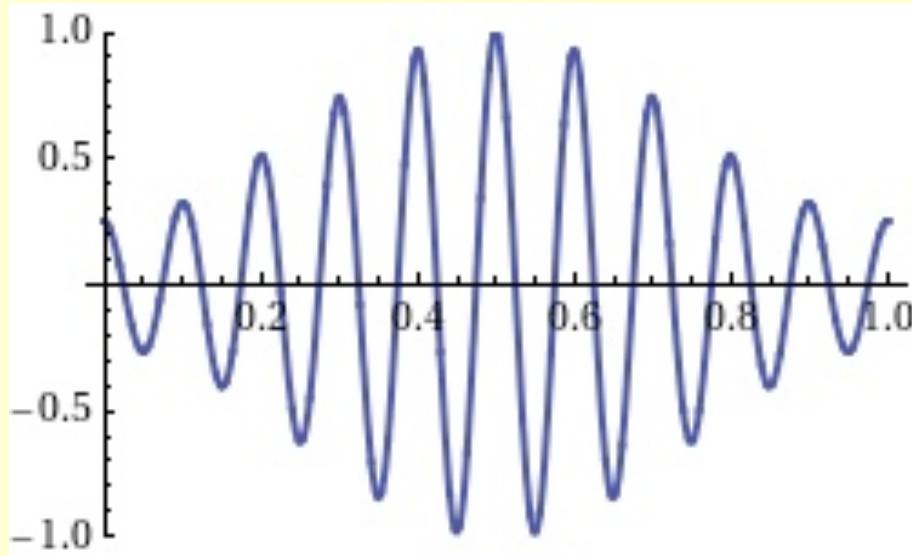
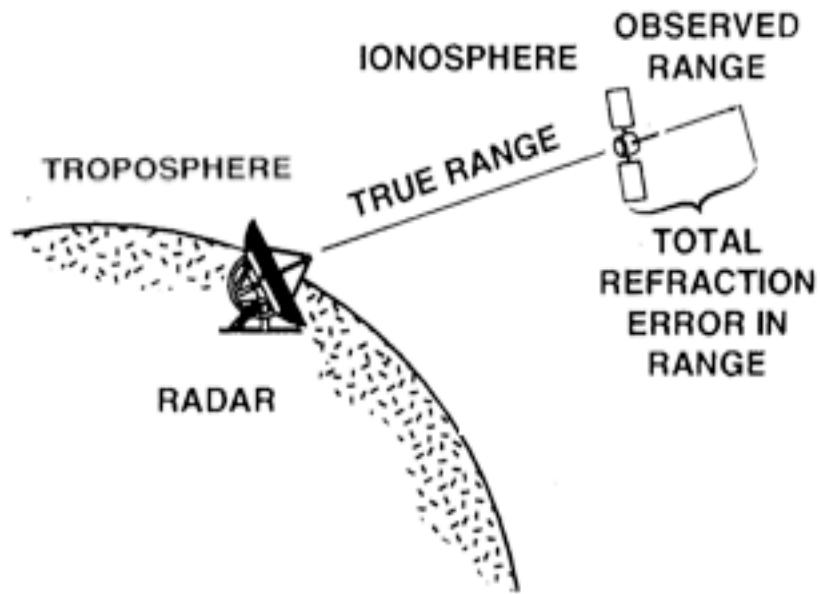
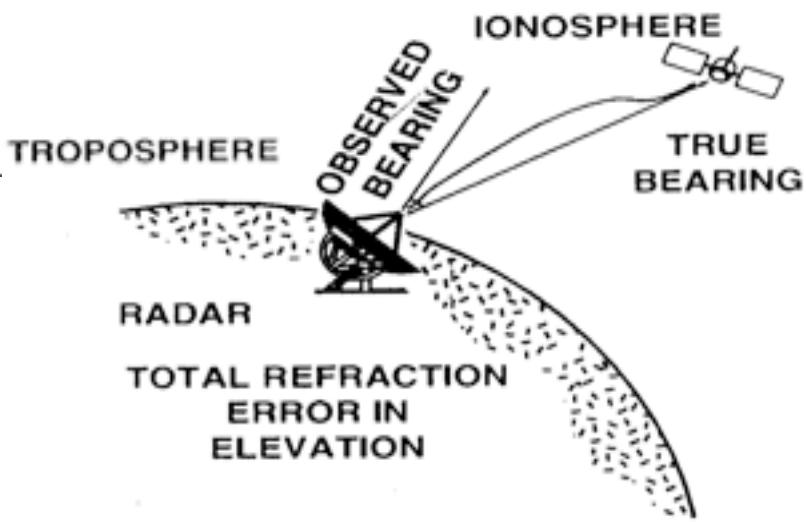
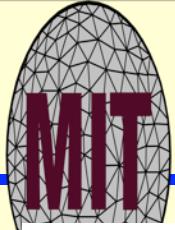


Illustration of Atmospheric Effects





Appleton-Hartree Equation

HAY

$$n^2 = 1 - \frac{X}{1 - iZ - \frac{\frac{1}{2}Y^2 \sin^2 \theta}{1-X-iZ} \pm \frac{1}{1-X-iZ} \left(\frac{1}{4}Y^4 \sin^4 \theta + Y^2 \cos^2 \theta (1 - X - iZ)^2 \right)^{1/2}}$$

or, alternatively^[4]:

$$n^2 = 1 - \frac{X (1 - X)}{1 - X - \frac{1}{2}Y^2 \sin^2 \theta \pm \left(\left(\frac{1}{2}Y^2 \sin^2 \theta \right)^2 + (1 - X)^2 Y^2 \cos^2 \theta \right)^{1/2}}$$

n = complex refractive index

$i = \sqrt{-1}$

$X = \frac{\omega_0^2}{\omega^2}$

$Y = \frac{\omega_H}{\omega}$

$Z = \frac{\nu}{\omega}$

ϵ_0 = permittivity of free space

μ_0 = permeability of free space

B_0 = ambient magnetic field strength

e = electron charge

m = electron mass

θ = angle between the ambient magnetic field vector and the wave vector

ν = electron collision frequency

ω = $2\pi f$ (radial frequency)

f = wave frequency (cycles per second, or Hertz)

$\omega_0 = 2\pi f_0 = \sqrt{\frac{Ne^2}{\epsilon_0 m}}$ = electron plasma frequency

$\omega_H = 2\pi f_H = \frac{B_0 |e|}{m}$ = electron gyro frequency

Index of Refraction in the Ionosphere

$$n^2 = 1 - \frac{X(1-X)}{(1-X) - \frac{1}{2} Y_T^2 \pm \left(\frac{1}{4} Y_T^4 + (1-X)^2 Y_L^2 \right)^{\frac{1}{2}}}$$

$$X = \frac{\omega_N^2}{\omega^2} \quad Y = \frac{\omega_H}{\omega} \quad \omega_N = \left(\frac{Ne^2}{\epsilon_0 m_e} \right)^{\frac{1}{2}} \quad \omega_H = \frac{e|B|}{m_e}$$

ω = the angular frequency of the radar wave,

$Y_L = Y \cos\theta$, $Y_T = Y \sin\theta$,

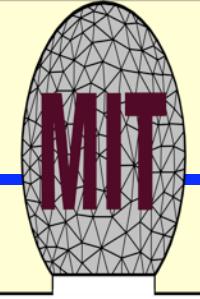
θ = angle between the wave vector \bar{k} and \bar{B} ,

\bar{k} = wave vector of propagating radiation,

\bar{B} = geomagnetic field, N = electron density

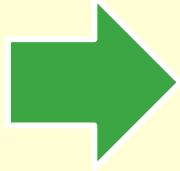
e = electronic charge, m_e = electron mass,

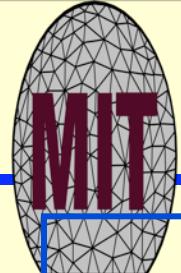
and ϵ_0 = permittivity constant.



Outline

- More History
 - Index of Refraction
- More Basic Radar





RADAR

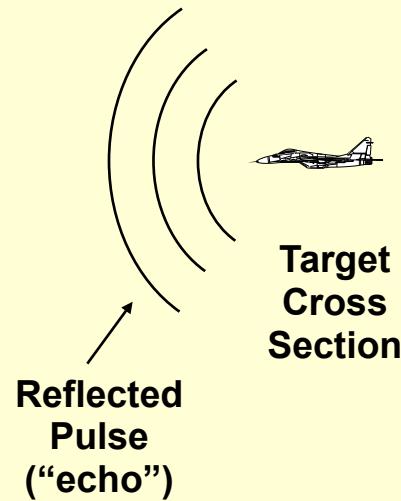
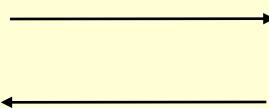
RAdio Detection And Ranging

Antenna



Transmitted
Pulse

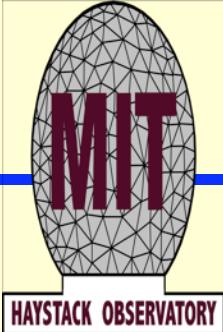
Propagation



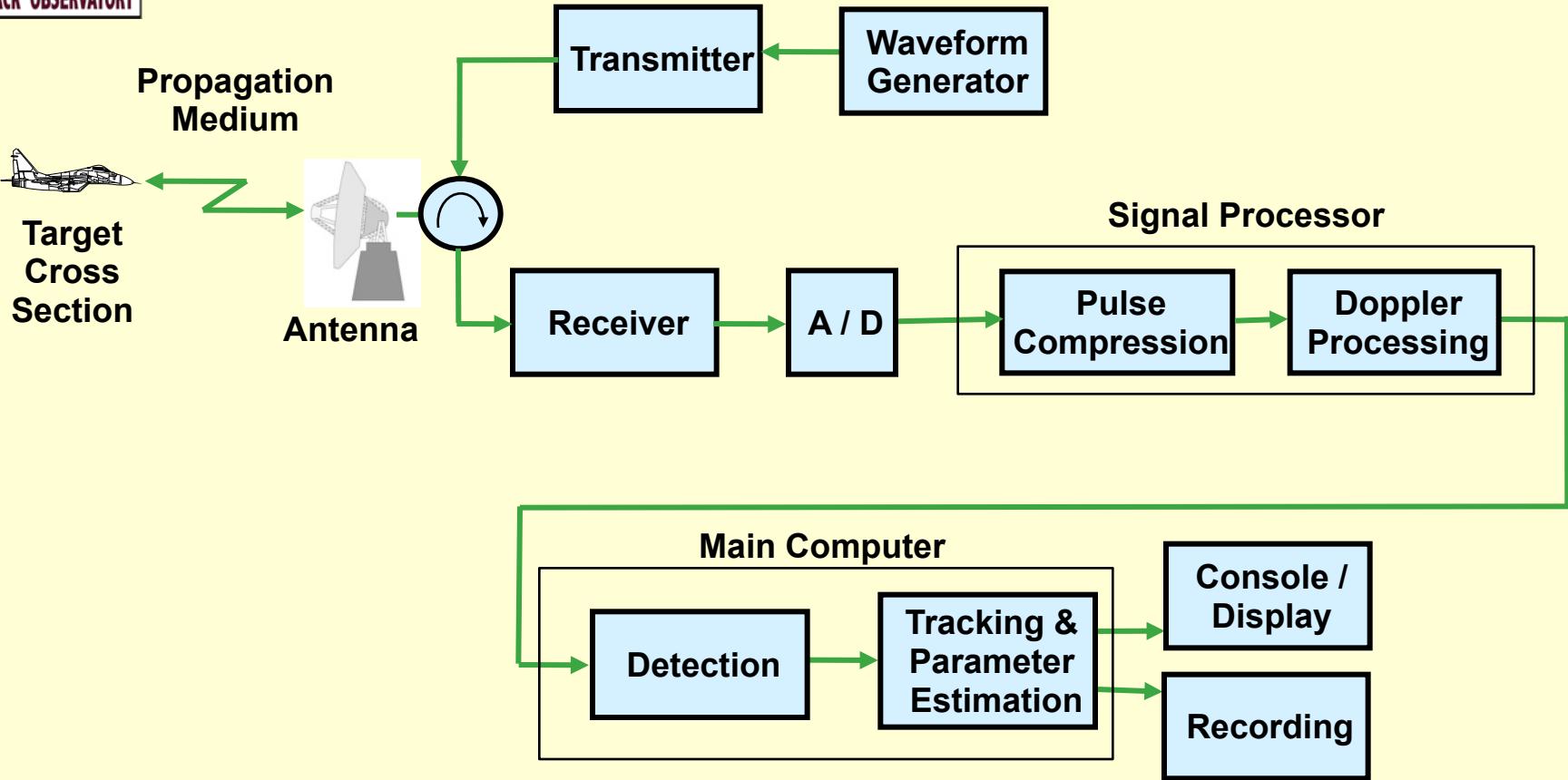
Target
Cross
Section

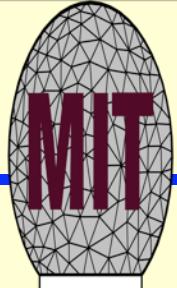
Radar observables:

- Target range
- Target angles (azimuth & elevation)
- Target size (radar cross section)
- Target speed (Doppler)
- Target features (imaging)

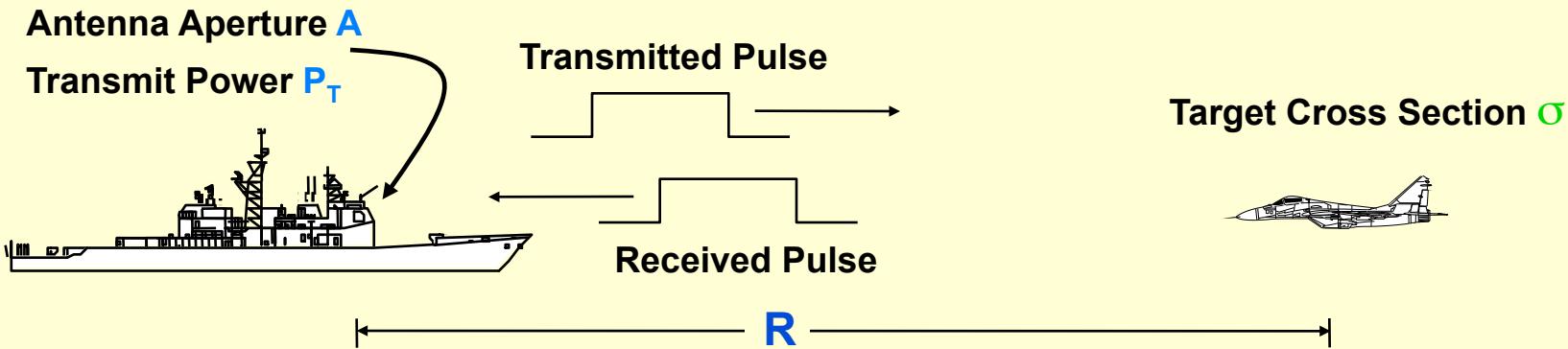


Radar Block Diagram





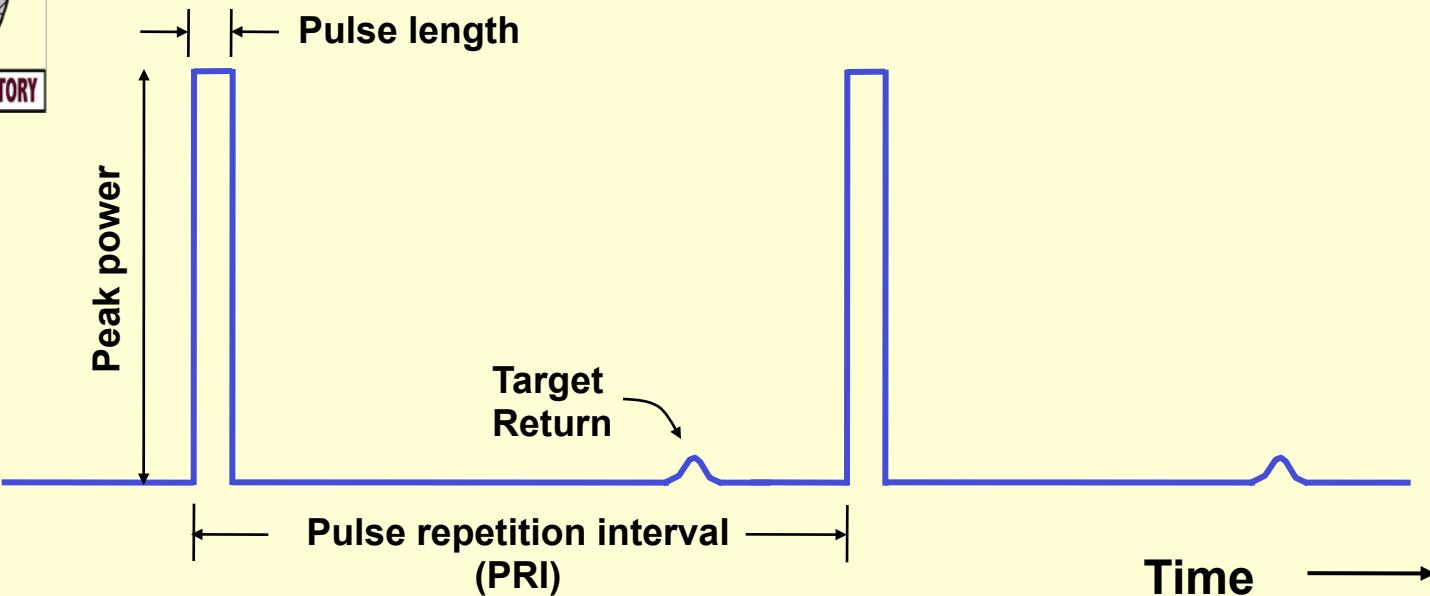
Radar Range Equation



$$\text{Received Signal Energy} = [P_T] \left[\frac{4\pi A}{\lambda^2} \right] \left[\frac{1}{4\pi R^2} \right] \left[\frac{1}{L} \right] [\sigma] \left[\frac{1}{4\pi R^2} \right] [A] [\tau]$$

Legend for units:
Transmit Power: P_T
Transmit Gain: $\frac{4\pi A}{\lambda^2}$
Spread Factor: $\frac{1}{4\pi R^2}$
Losses: $\frac{1}{L}$
Target RCS: σ
Spread Factor: $\frac{1}{4\pi R^2}$
Receive Aperture: A
Dwell Time: τ

Pulsed Radar Terminology and Concepts

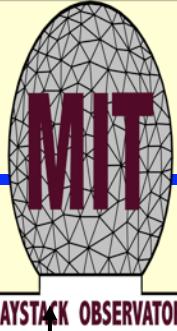


$$\text{Duty cycle} = \frac{\text{Pulse length}}{\text{Pulse repetition interval}}$$

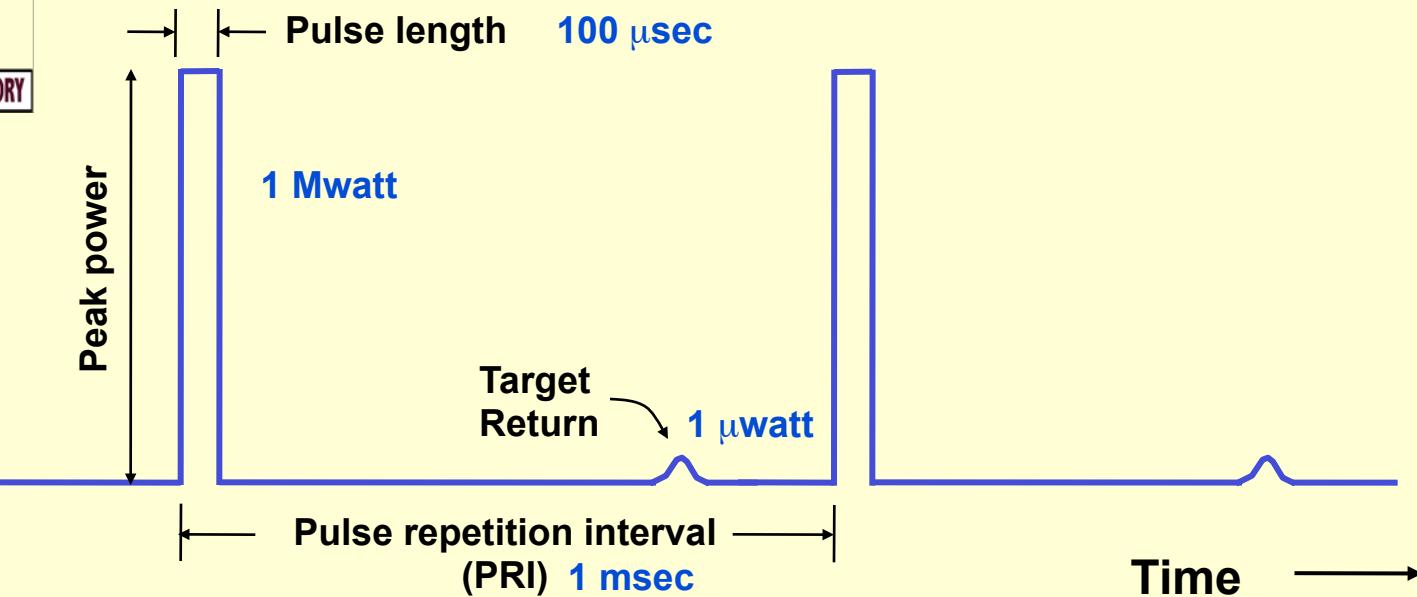
$$\text{Average power} = \text{Peak power} * \text{Duty cycle}$$

$$\text{Pulse repetition frequency (PRF)} = 1/(\text{PRI})$$

Continuous wave (CW) radar: Duty cycle = 100% (always on)



Pulsed Radar Terminology and Concepts

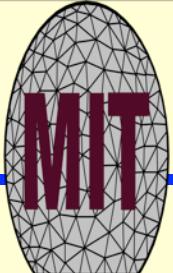


$$\text{Duty cycle} = \frac{\text{Pulse length}}{\text{Pulse repetition interval}} \quad 10\%$$

$$\text{Average power} = \text{Peak power} * \text{Duty cycle} \quad 100 \text{ kWatt}$$

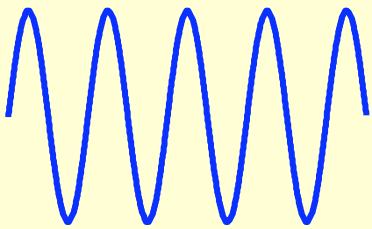
$$\text{Pulse repetition frequency (PRF)} = 1/(\text{PRI}) \quad 1 \text{ kHz}$$

Continuous wave (CW) radar: Duty cycle = 100% (always on)

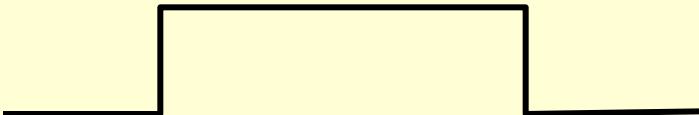


Radar Waveforms

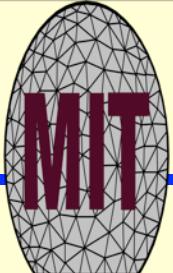
What do radars transmit?



Waves?

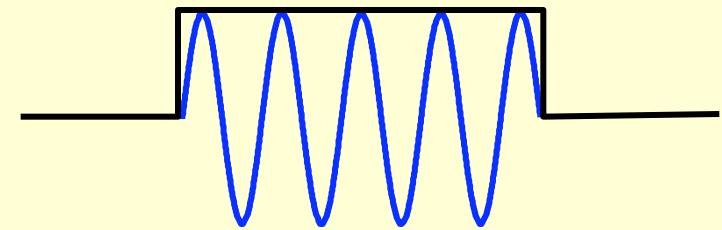
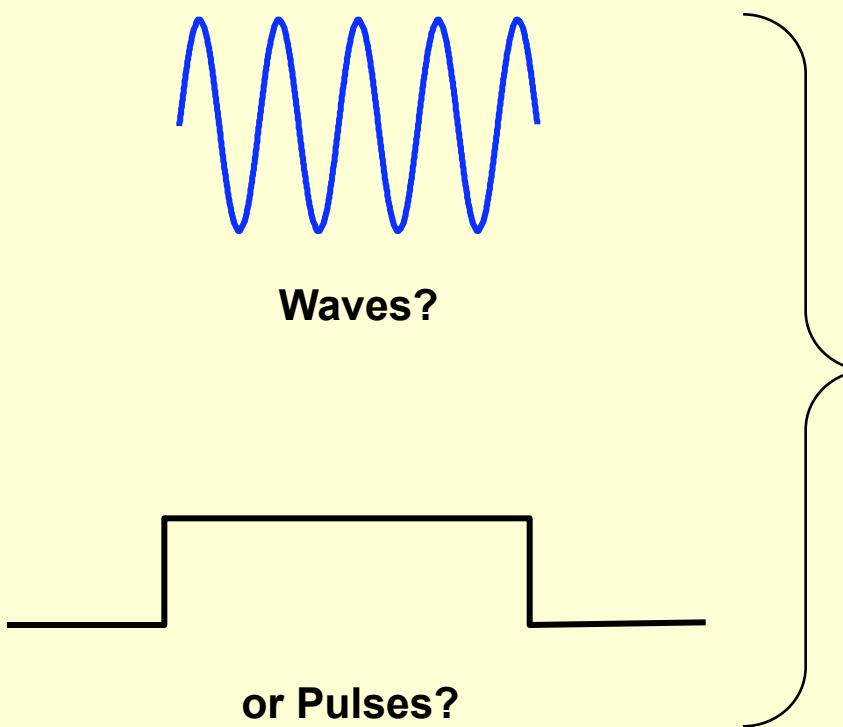


or Pulses?

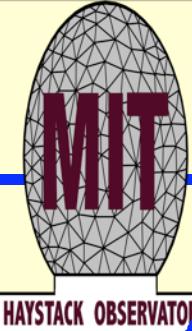


Radar Waveforms

What do radars transmit?

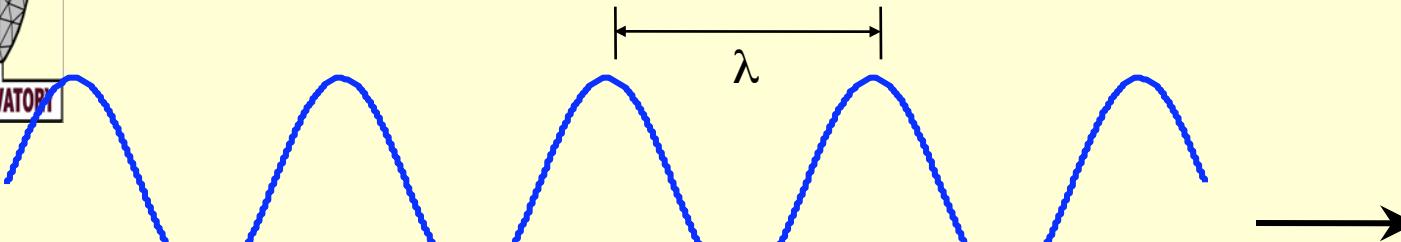


**Waves, modulated
by “on-off” action of
pulse envelope**



Properties of Waves

Relationship Between Frequency and Wavelength



Speed of light, c

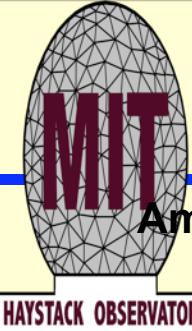
$$c = 3 \times 10^8 \text{ m/sec}$$

$$= 300,000,000 \text{ m/sec}$$

$$\text{Frequency (1/s)} = \frac{\text{Speed of light (m/s)}}{\text{Wavelength } \lambda \text{ (m)}}$$

Examples:

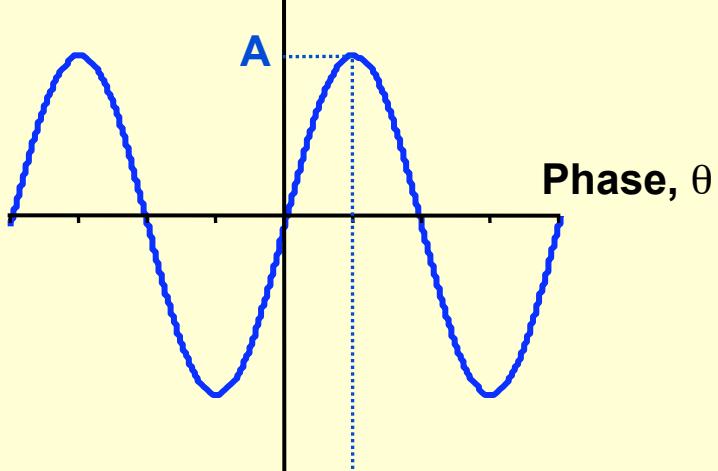
<u>Frequency</u>	<u>Wavelength</u>
100 MHz	3 m
1 GHz	30 cm
3 GHz	10 cm
10 GHz	3 cm



Properties of Waves

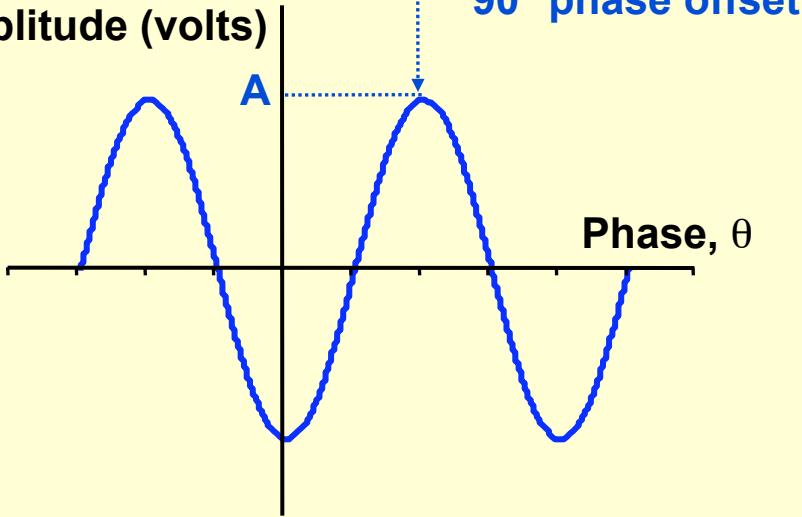
Phase and Amplitude

Amplitude (volts)

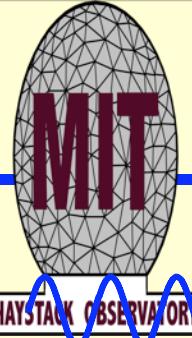


$$A \sin(\theta)$$

Amplitude (volts)

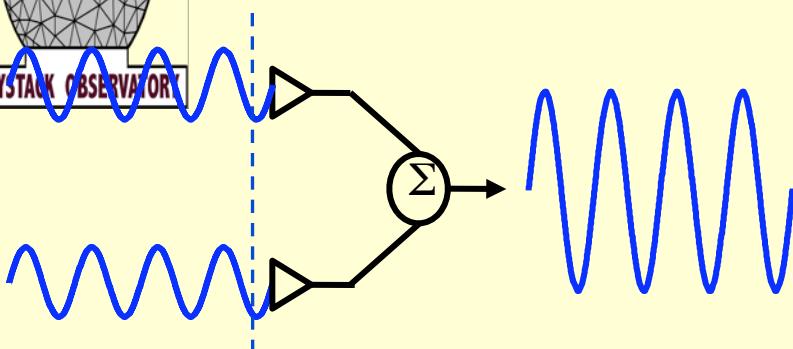


$$A \sin(\theta - 90^\circ)$$

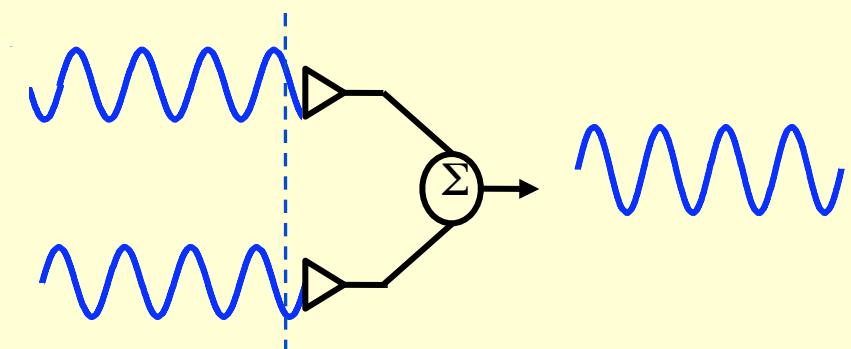


Properties of Waves

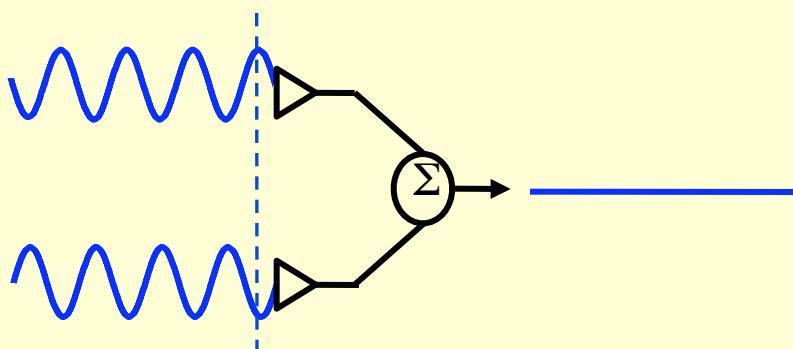
Constructive vs. Destructive Addition



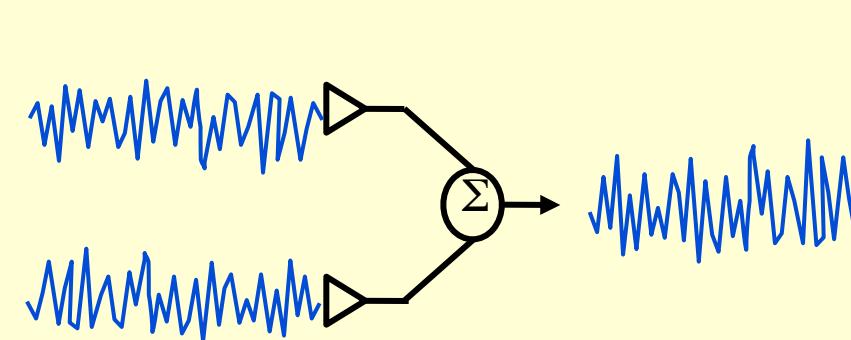
**Constructive
(in phase)**



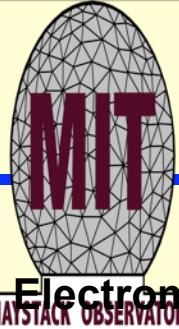
**Partially Constructive
(somewhat out of phase)**



**Destructive
(180° out of phase)**

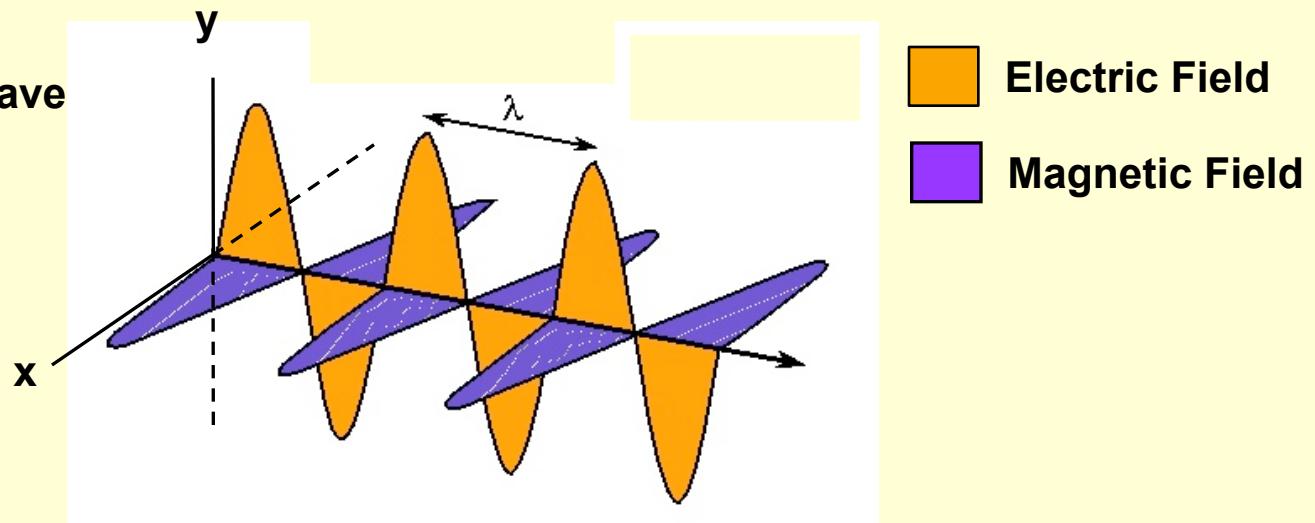


**Non-coherent signals
(noise)**



Polarization

Electromagnetic Wave

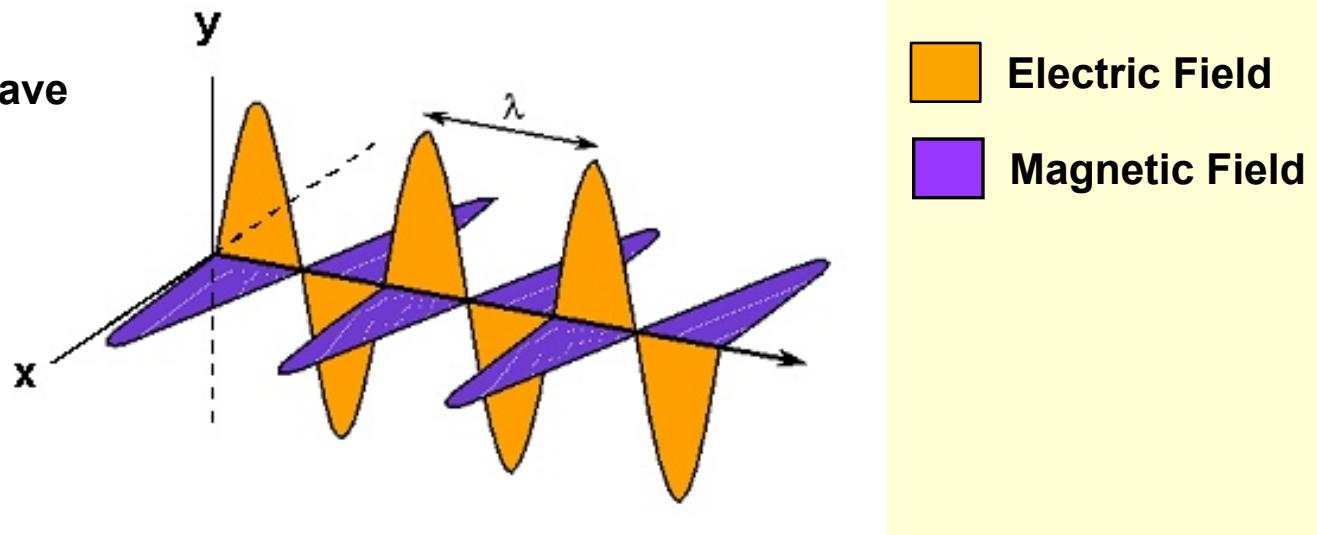


- Electric Field
- Magnetic Field

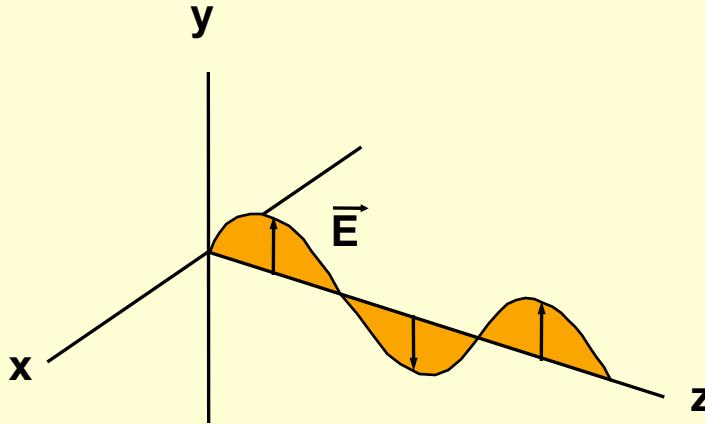


Polarization

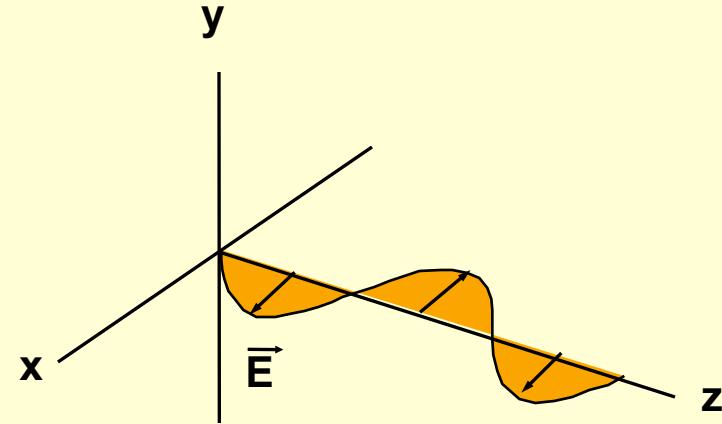
Electromagnetic Wave

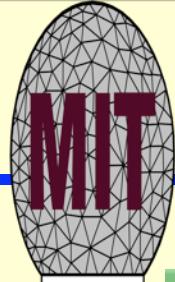


Vertical Polarization



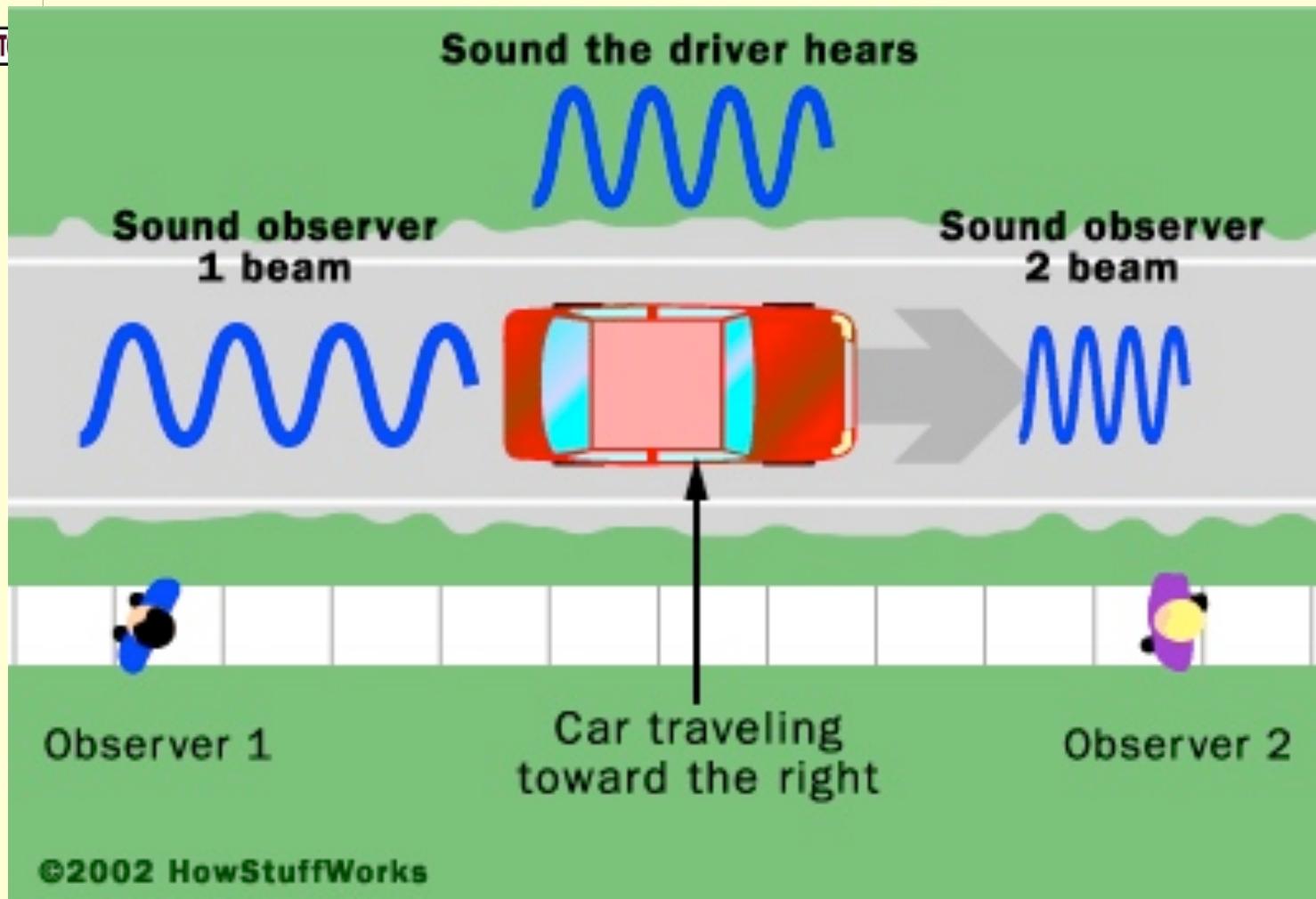
Horizontal Polarization

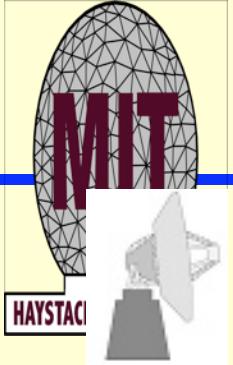




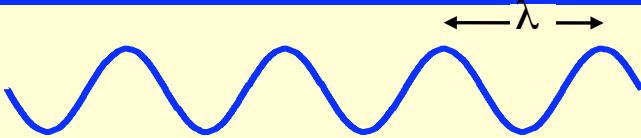
HAYSTACK OBSERVATORY

Doppler Effect

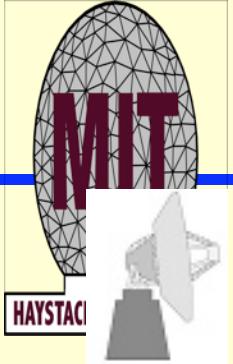




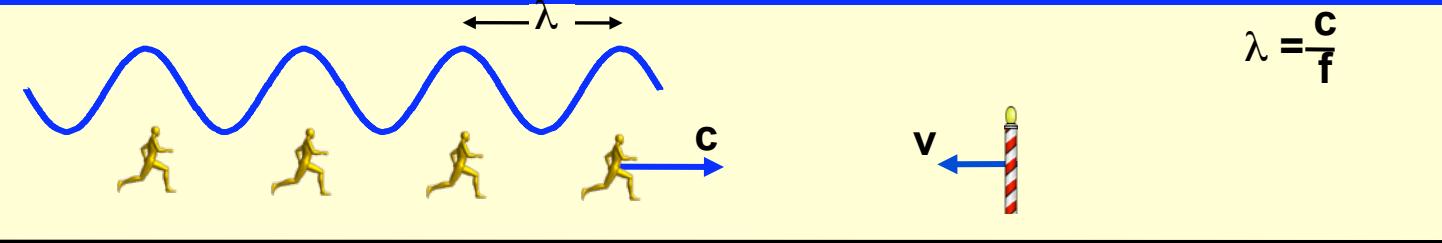
Doppler Shift Concept



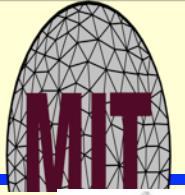
$$\lambda = \frac{c}{f}$$



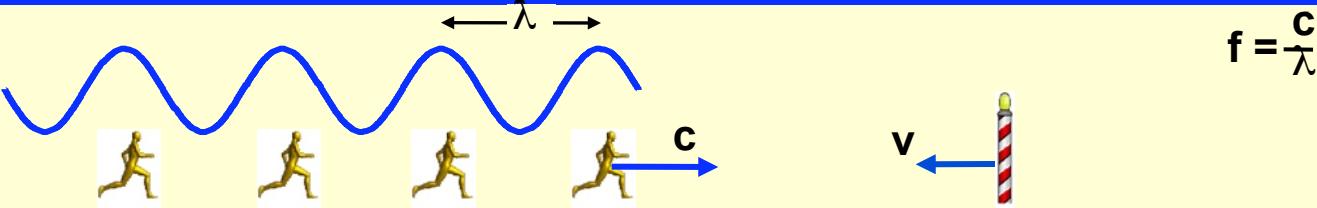
Doppler Shift Concept



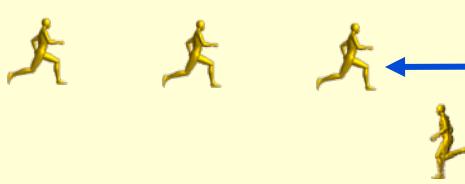
$$\lambda = \frac{c}{f}$$

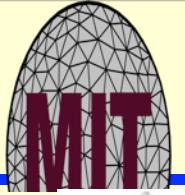


Doppler Shift Concept

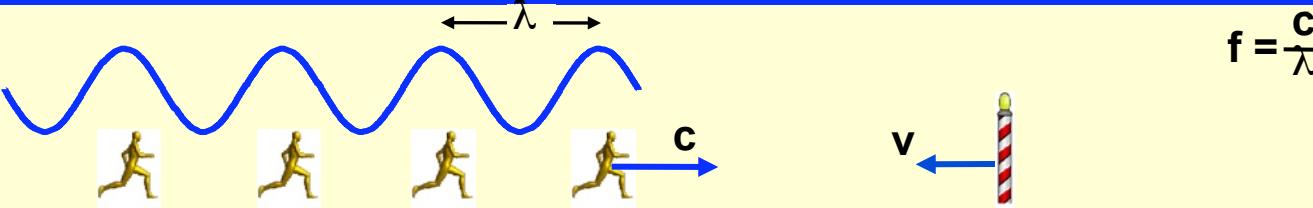


$$f = \frac{c}{\lambda}$$

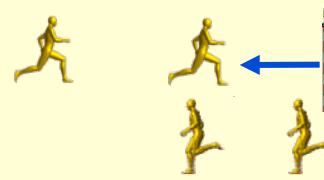
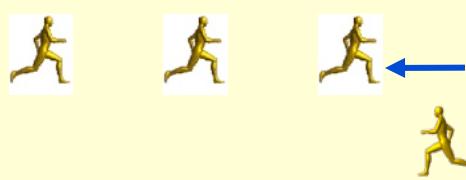


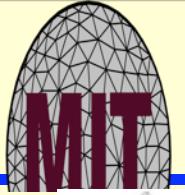


Doppler Shift Concept

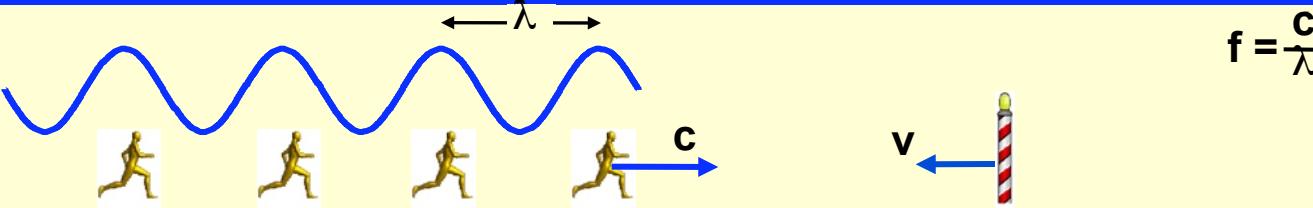


$$f = \frac{c}{\lambda}$$

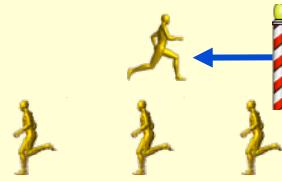
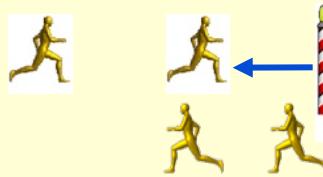


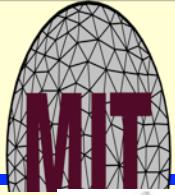


Doppler Shift Concept



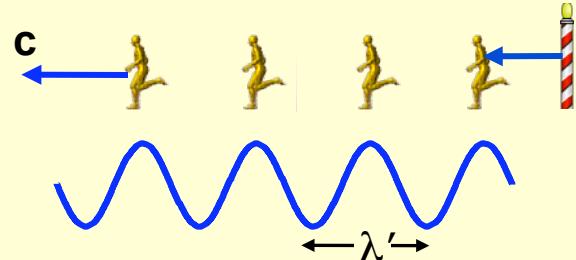
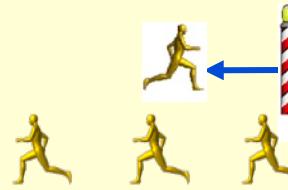
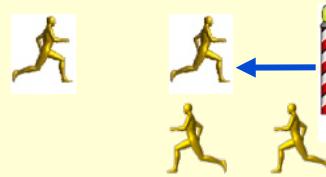
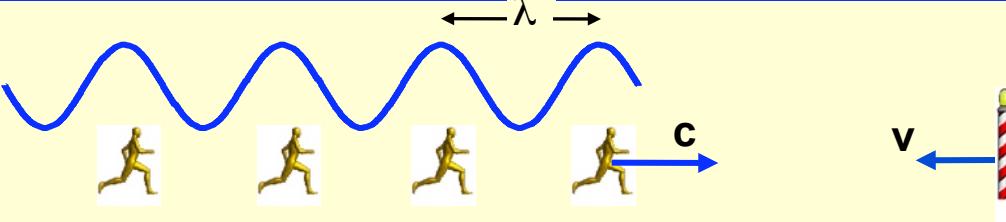
$$f = \frac{c}{\lambda}$$





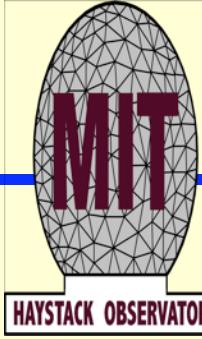
Doppler Shift Concept

$$f = \frac{c}{\lambda}$$



$$f' = f \pm (2v/\lambda)$$

Doppler shift



Resolving Doppler

Tx signal: $\cos(2\pi f_o t)$

Doppler shifted: $\cos[2\pi(f_o + f_D)t]$

Multiply by $\cos(2\pi f_o t)$ -> Low pass filter -> $\cos(2\pi f_D t)$

BUT, the sign of f_D is lost (cosine is an even function)

So, instead use

$$\exp(j2\pi f_D t) = \cos(2\pi f_D t) + j\sin(2\pi f_D t)$$

Generate this signal by mixing cos and sin via two oscillators (same frequency, 90° out of phase)

Components are called I (In phase) and Q
(Quadrature): $A\exp(j2\pi f_D t) = I + jQ$