

Phased Array Radars

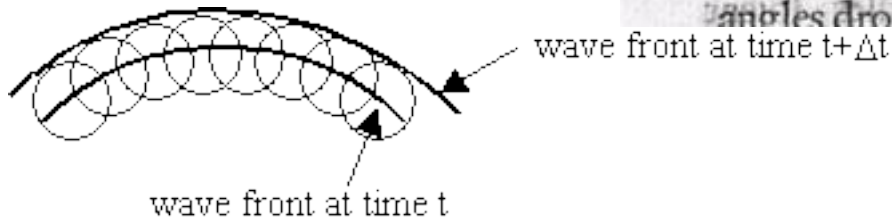
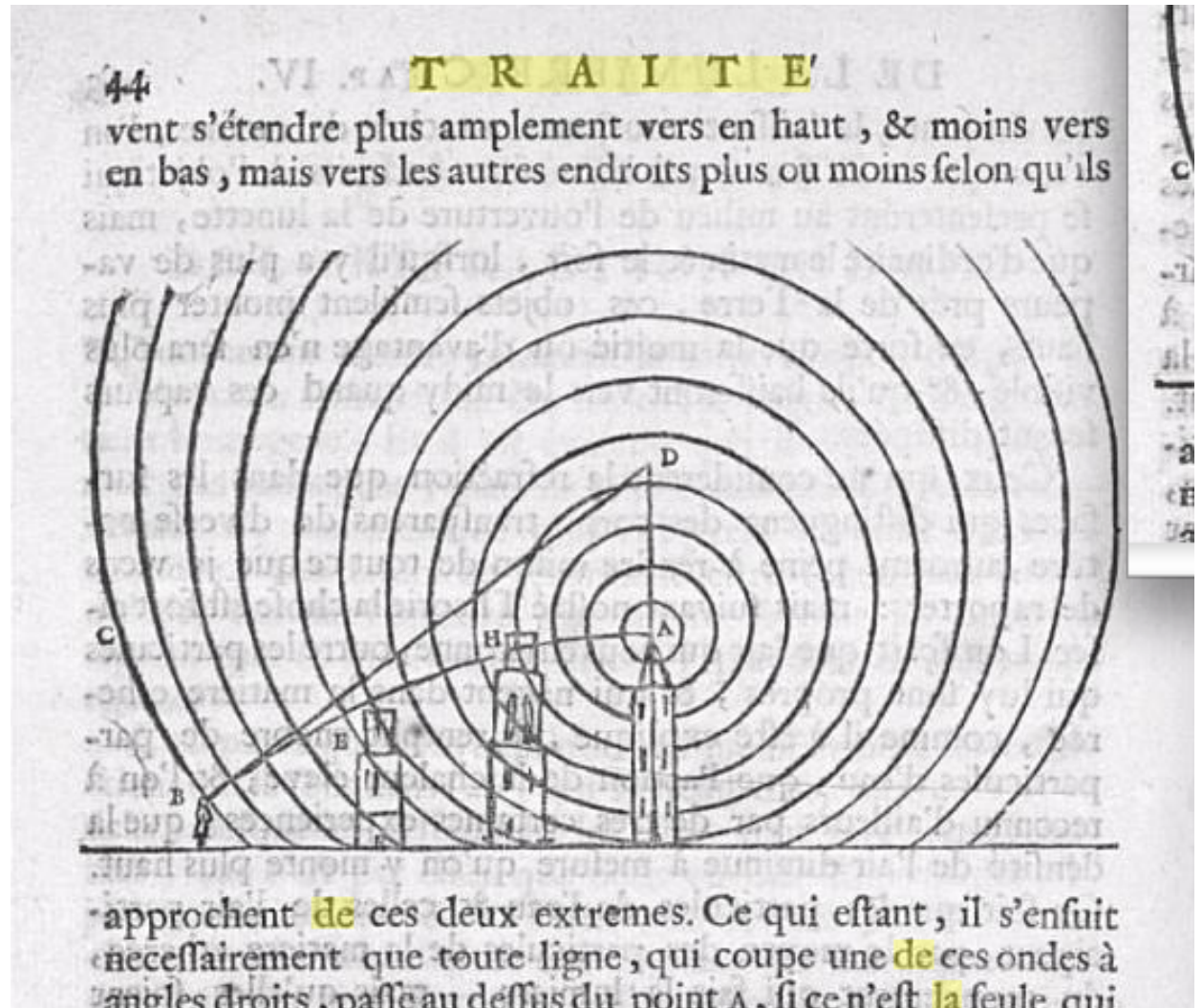
Simple to the Sublime



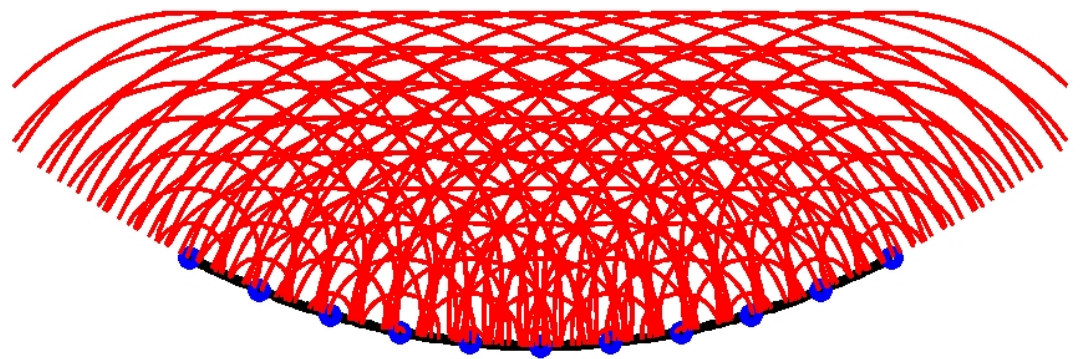
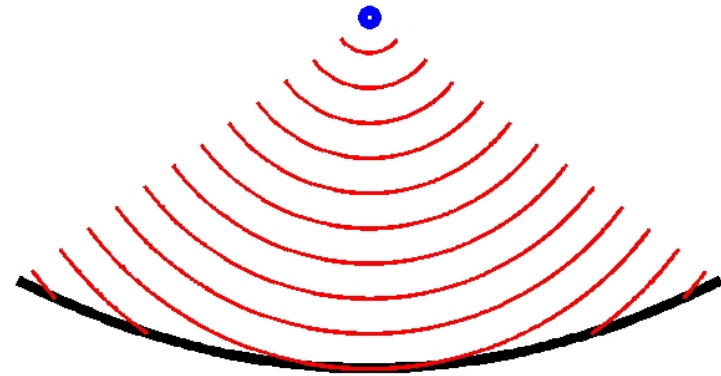
Christiaan Huygens



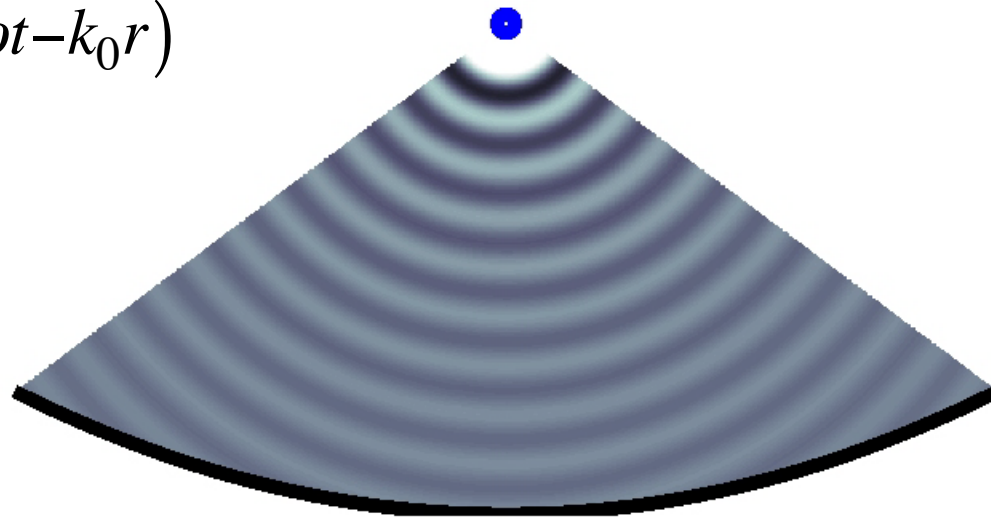
Traité de la Lumiere (Treatise on Light) completed in 1678, published in Leyden in 1690



Dish Antennas



$$E_{\theta} \propto \frac{1}{r} e^{j(\omega t - k_0 r)}$$

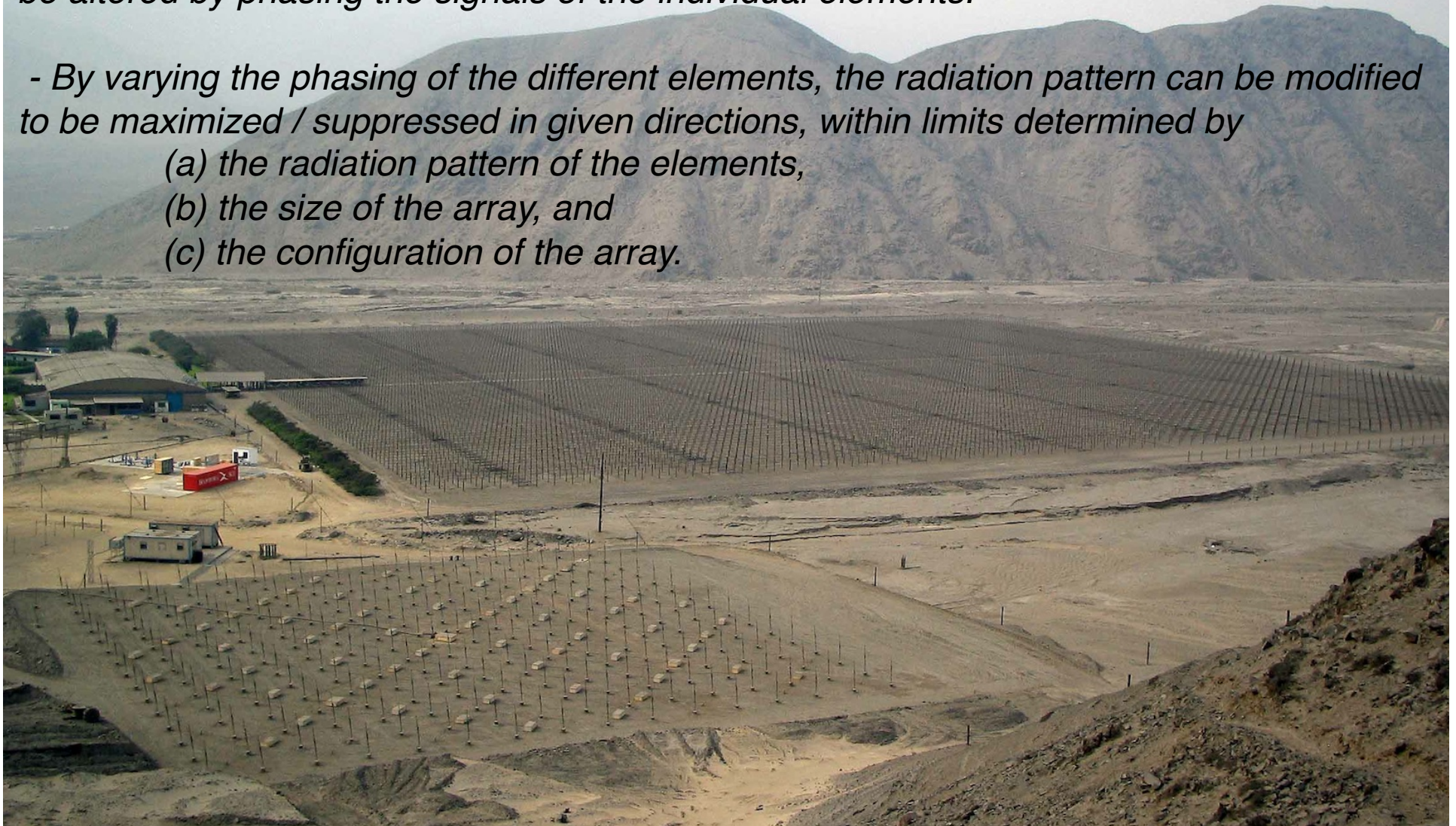


What is a Phased Array?

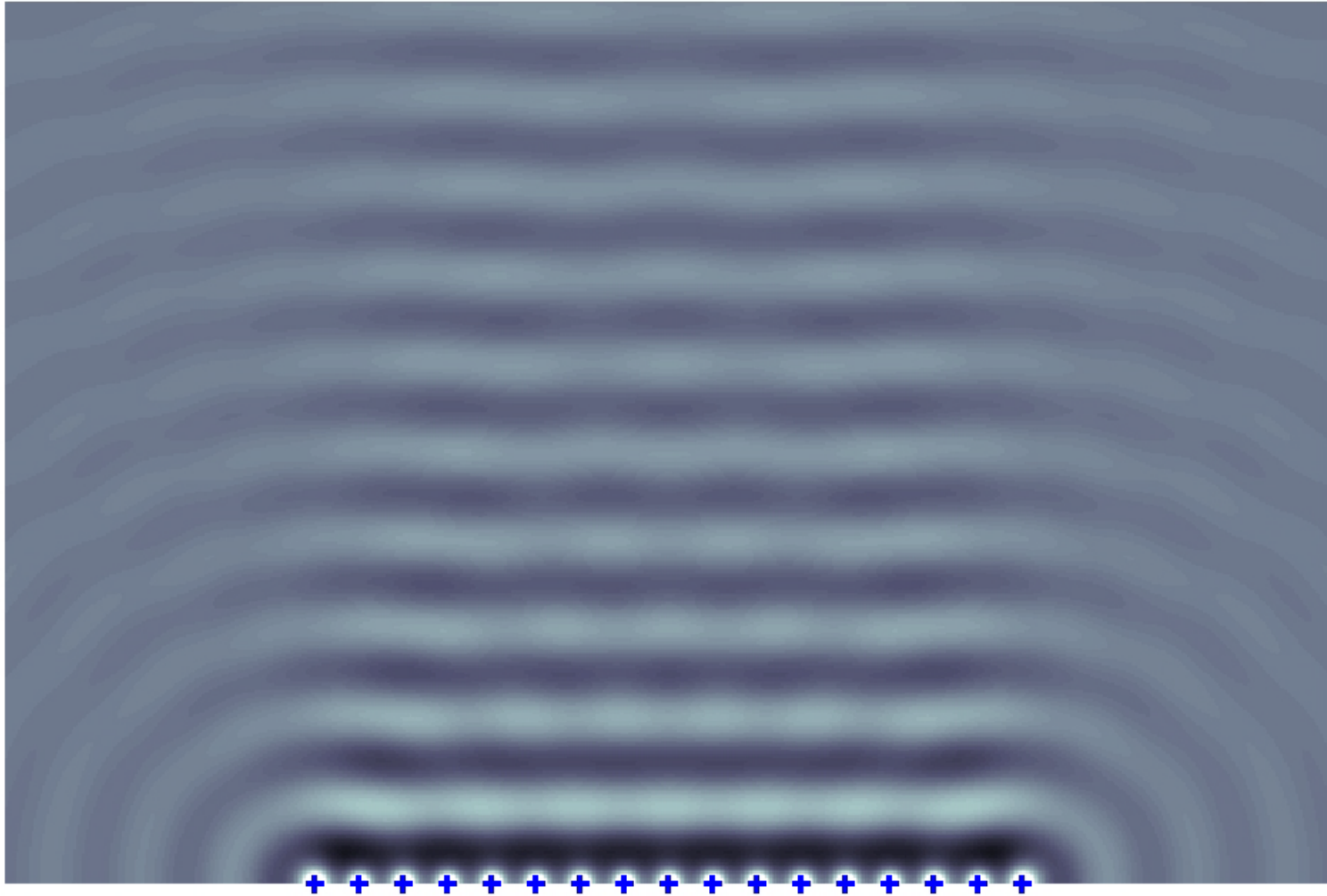
- A phased array is a group of antennas whose effective (summed) radiation pattern can be altered by phasing the signals of the individual elements.

- By varying the phasing of the different elements, the radiation pattern can be modified to be maximized / suppressed in given directions, within limits determined by

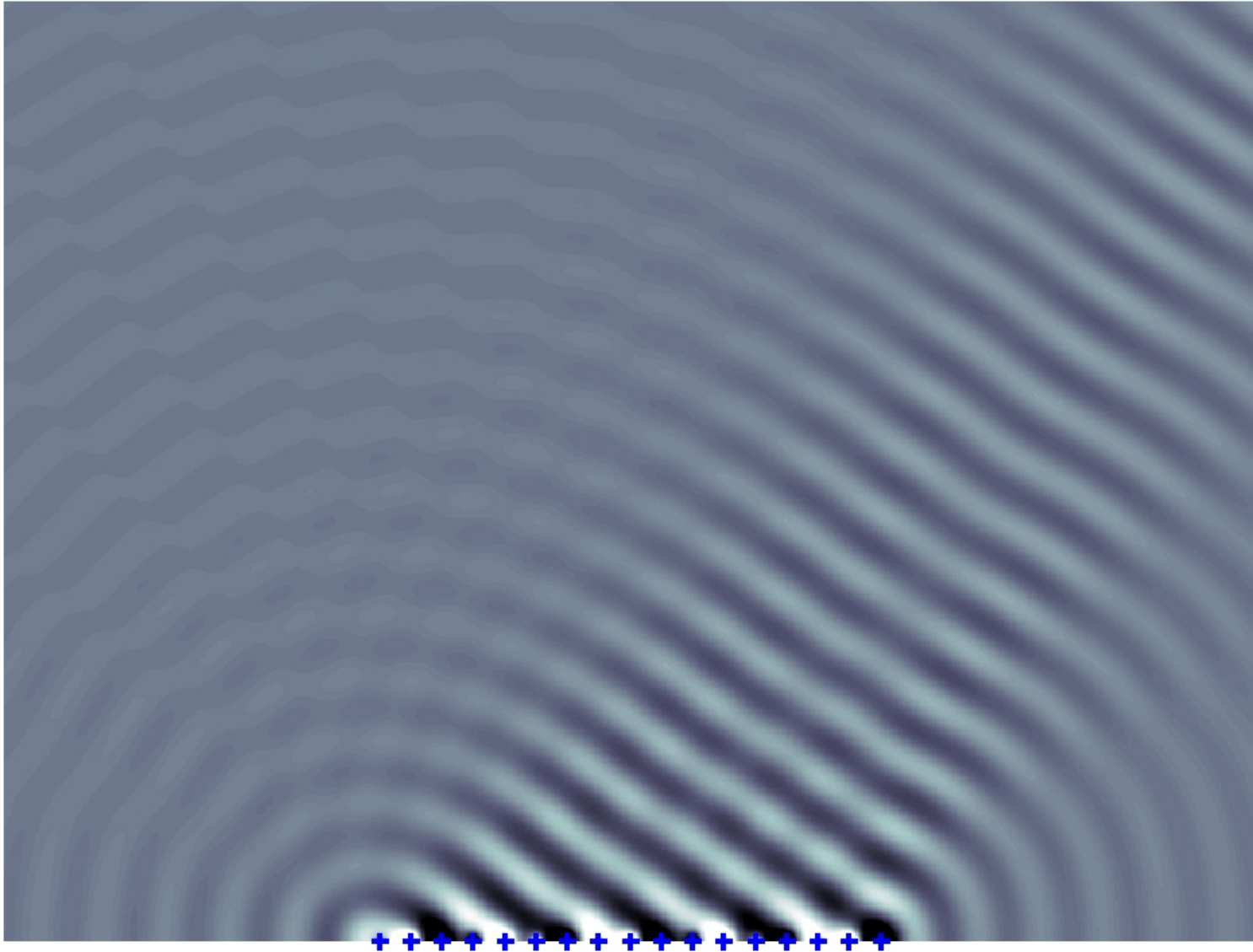
- (a) the radiation pattern of the elements,*
- (b) the size of the array, and*
- (c) the configuration of the array.*



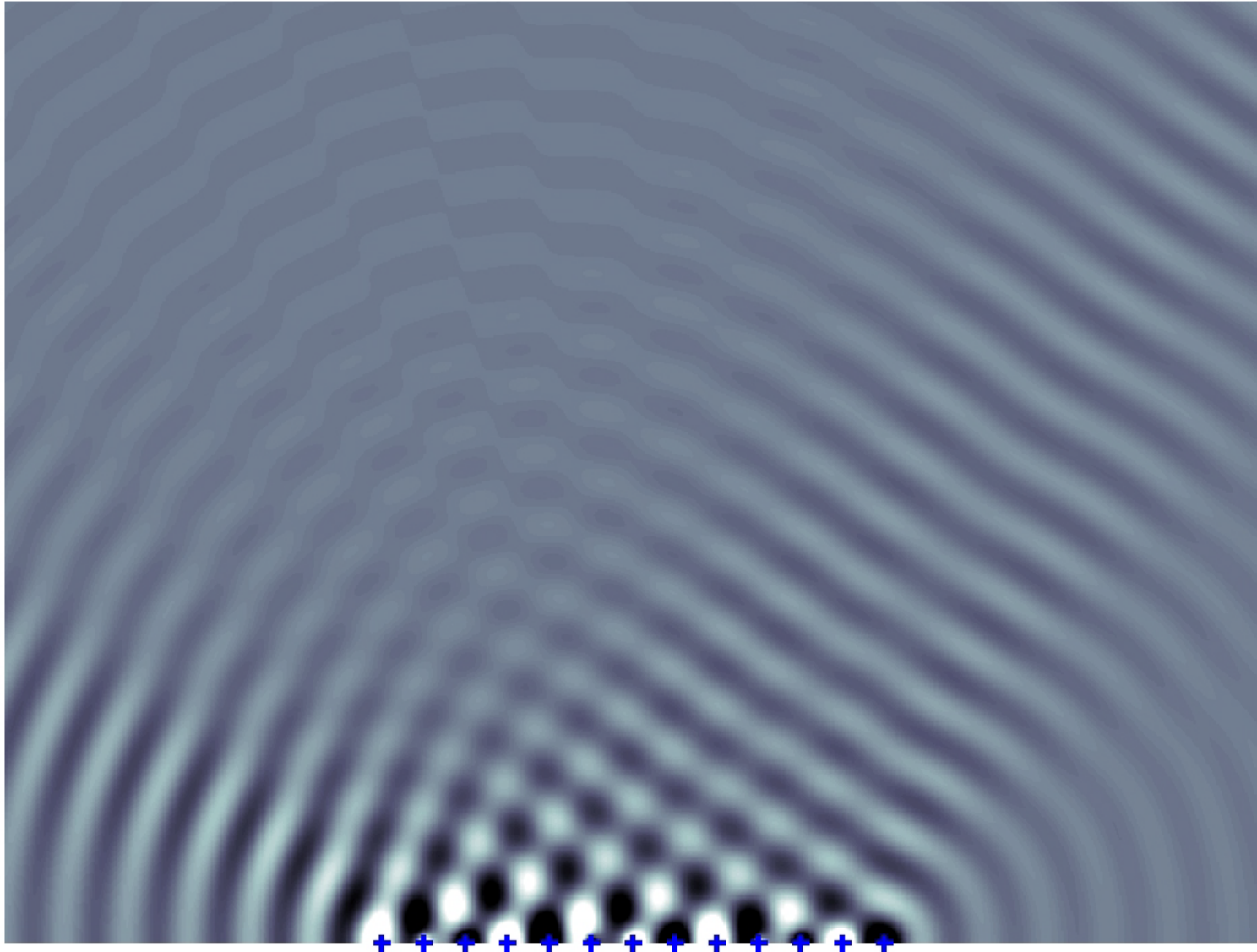
Phased Array, $\lambda/2$ spacing



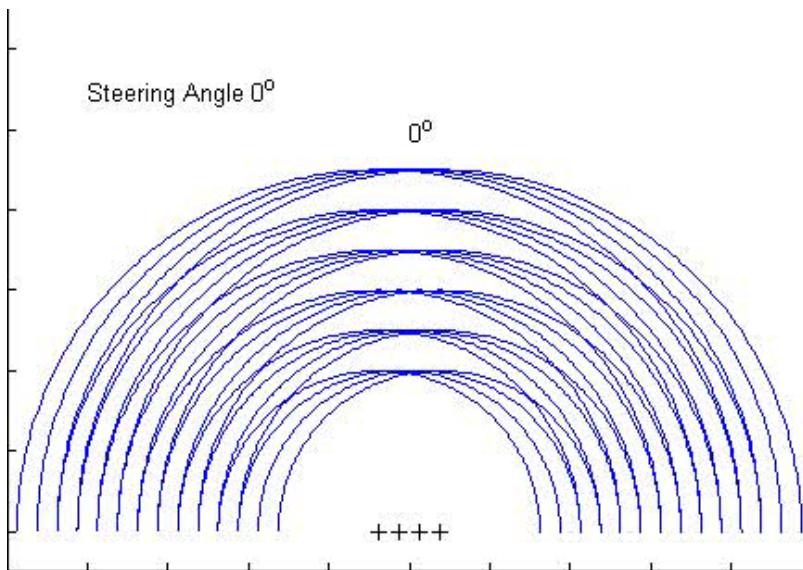
Phased Array, $\lambda/2$ spacing



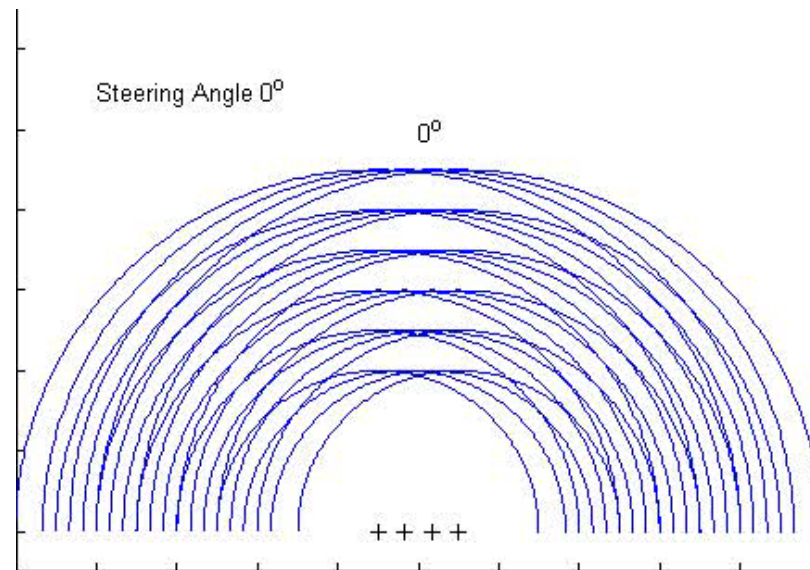
Phased Array, $2\lambda/3$ spacing



Grating Lobes

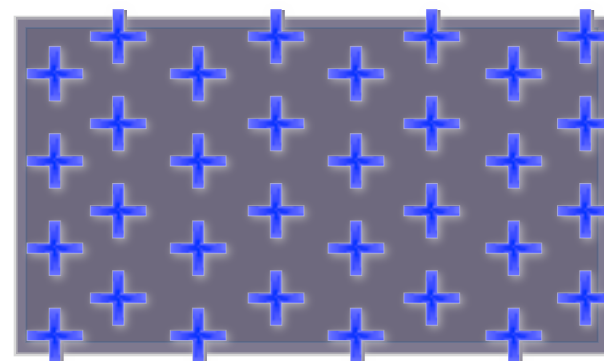
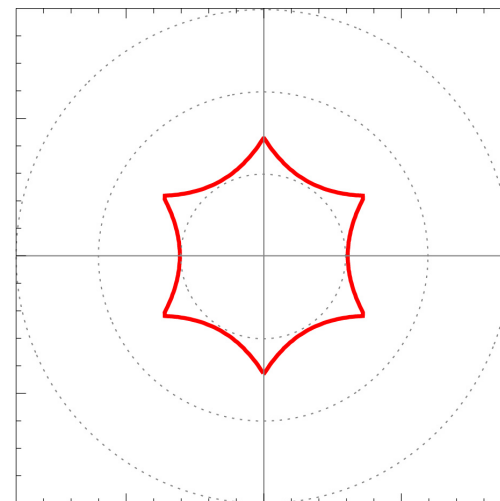
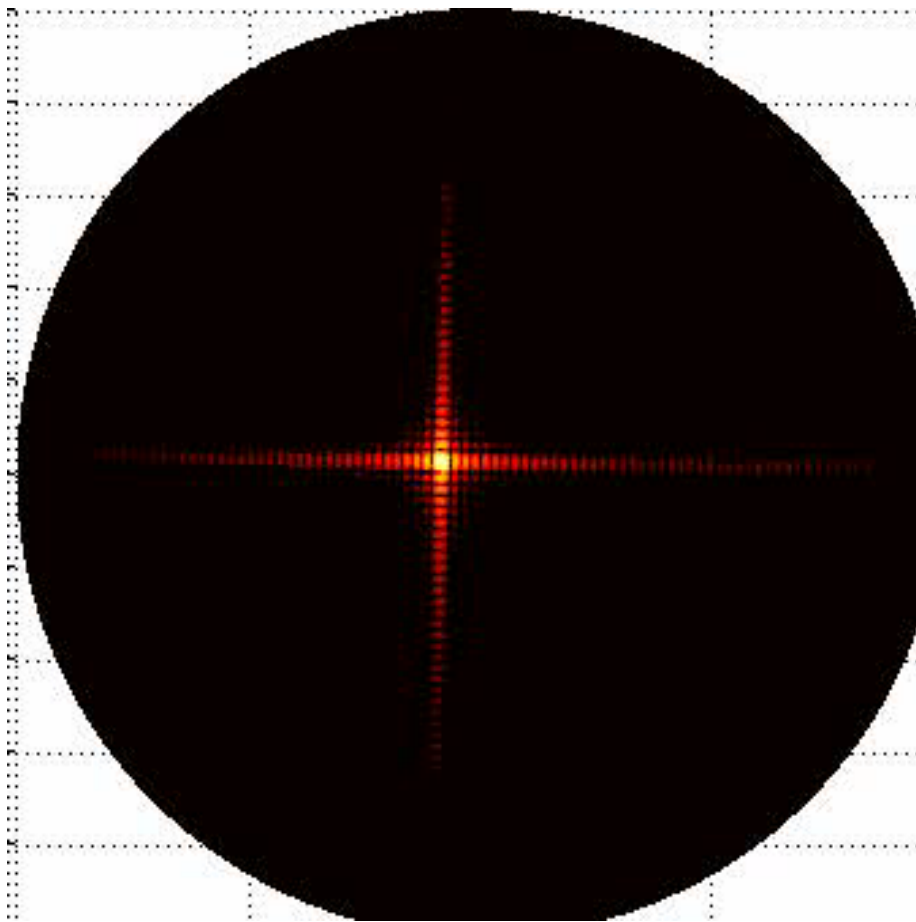


Element Spacing 0.50λ



Element Spacing 0.67λ

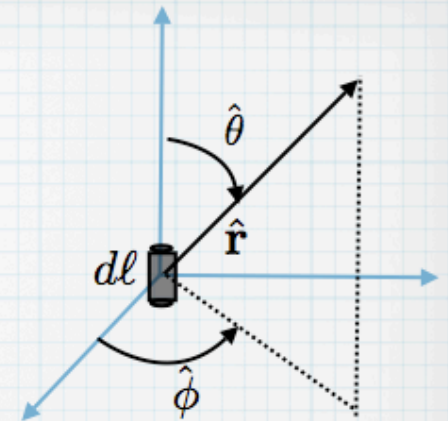
Grating Lobes (AMISR example)



Hertzian Dipole

$$\begin{aligned}
 H_\phi &= Idl \sin \theta \frac{1}{4\pi} \left[\frac{jk_0}{r} + \frac{1}{r^2} + 0 \right] e^{j(\omega t - k_0 r)} \\
 E_r &= Idl \cos \theta \frac{jz_0}{2\pi k_0} \left[0 + \frac{jk_0}{r^2} + \frac{1}{r^3} \right] e^{j(\omega t - k_0 r)} \\
 E_\theta &= Idl \sin \theta \frac{jz_0}{4\pi k_0} \left[\frac{k_0^2}{r} - \frac{jk_0}{r^2} + \frac{1}{r^3} \right] e^{j(\omega t - k_0 r)}
 \end{aligned}$$

Spherically
expanding
wavefront



$$z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \Omega$$

$$k_0 = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \omega\sqrt{\mu_0\epsilon_0}$$

For $r \gg \lambda$, keep terms only linear in r - **far field approximation**.

$$E_\theta \perp H_\phi \perp \hat{\mathbf{r}} \quad \frac{E_\theta}{H_\phi} = z_0$$

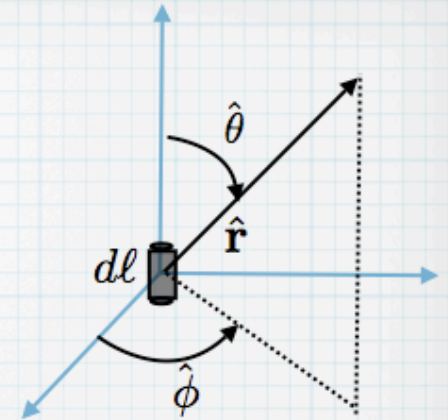
Power flow represented by Poynting vector

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} \quad \langle P_r \rangle = \frac{1}{2} \Re\{P_r\} = \frac{1}{2} \Re\{\mathbf{E} \times \mathbf{H}\} \cdot \hat{\mathbf{r}} \quad \text{W/m}^2$$

Hertzian Dipole (2)

$$\begin{aligned}
 H_\phi &= Idl \sin \theta \frac{1}{4\pi} \left[\frac{jk_0}{r} + \frac{1}{r^2} + 0 \right] e^{j(\omega t - k_0 r)} \\
 E_r &= Idl \cos \theta \frac{jz_0}{2\pi k_0} \left[0 + \frac{jk_0}{r^2} - \frac{1}{r^3} \right] e^{j(\omega t - k_0 r)} \\
 E_\theta &= Idl \sin \theta \frac{jz_0}{4\pi k_0} \left[\frac{k_0^2}{r} - \frac{jk_0}{r^2} + \frac{1}{r^3} \right] e^{j(\omega t - k_0 r)}
 \end{aligned}$$

far field
near field
Spherically expanding



Directivity pattern:

$$D(\theta, \phi) = \frac{\text{Power Density Radiated In } (\theta, \phi) \text{ Direction}}{\text{Average Power Density}} = 4\pi R^2 \frac{\text{Power Density In } (\theta, \phi)}{\text{Total Power Radiated}}$$

$$\langle P_r \rangle = \frac{1}{2} \Re \{ \mathbf{E} \times \mathbf{H} \} \cdot \hat{\mathbf{r}} = I^2 z_0 (dl)^2 k_0^2 \sin^2 \theta \frac{1}{32\pi^2 r^2} \text{ W/m}^2$$

$$P_{total} = \int_0^{2\pi} d\phi \int_0^\pi \langle P_r \rangle r^2 \sin \theta d\theta = z_0 \frac{\pi}{3} \left(\frac{Idl}{\lambda} \right)^2 \text{ W}$$

$$P_{total} = \frac{1}{2} I^2 R_{rad}$$

$$D(\theta, \phi) = \frac{3}{2} \sin^2 \theta$$

Directivity Patterns for Dipoles

Hertzian Dipole

$$D(\theta, \phi) = \frac{3}{2} \sin^2 \theta$$

$$\text{HPBW} = 90^\circ$$

Half-Wave Dipole

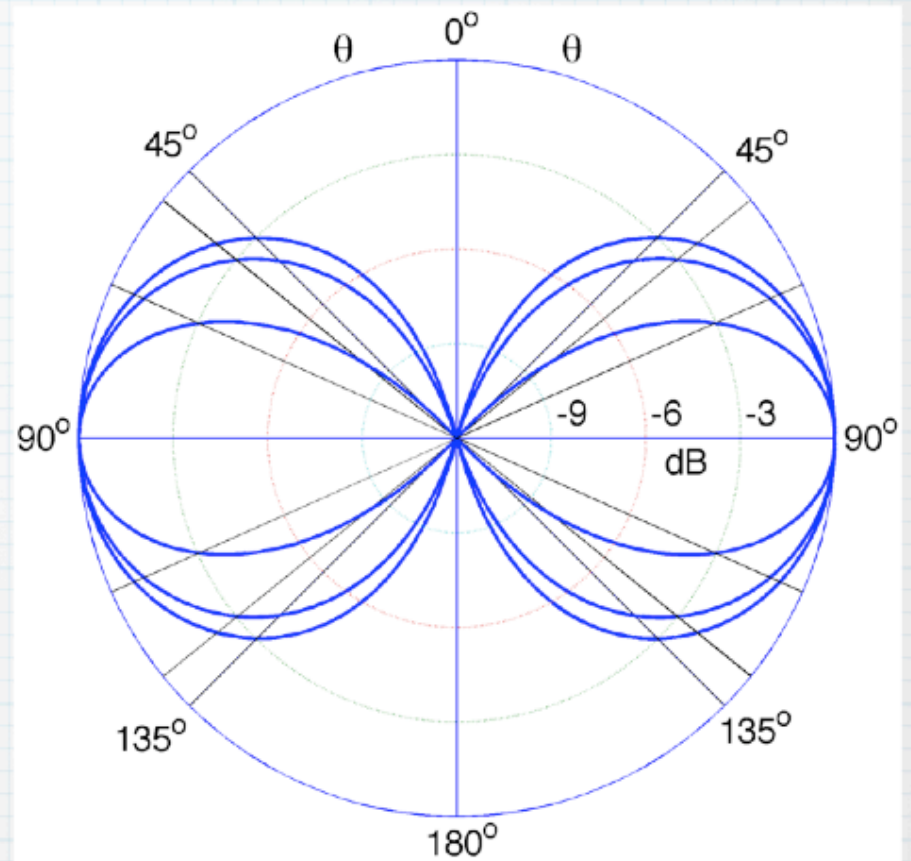
$$D(\theta, \phi) = 1.64 \left[\frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \right]^2$$

$$\text{HPBW} \approx 78^\circ$$

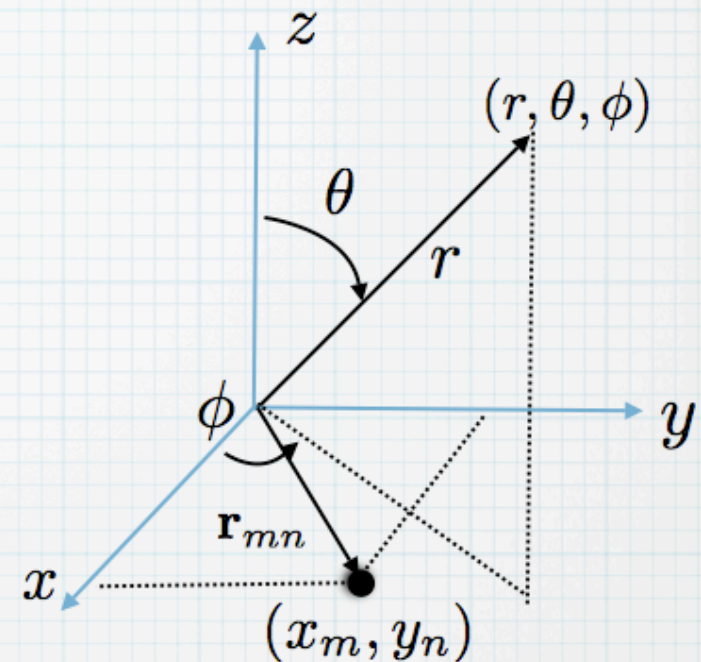
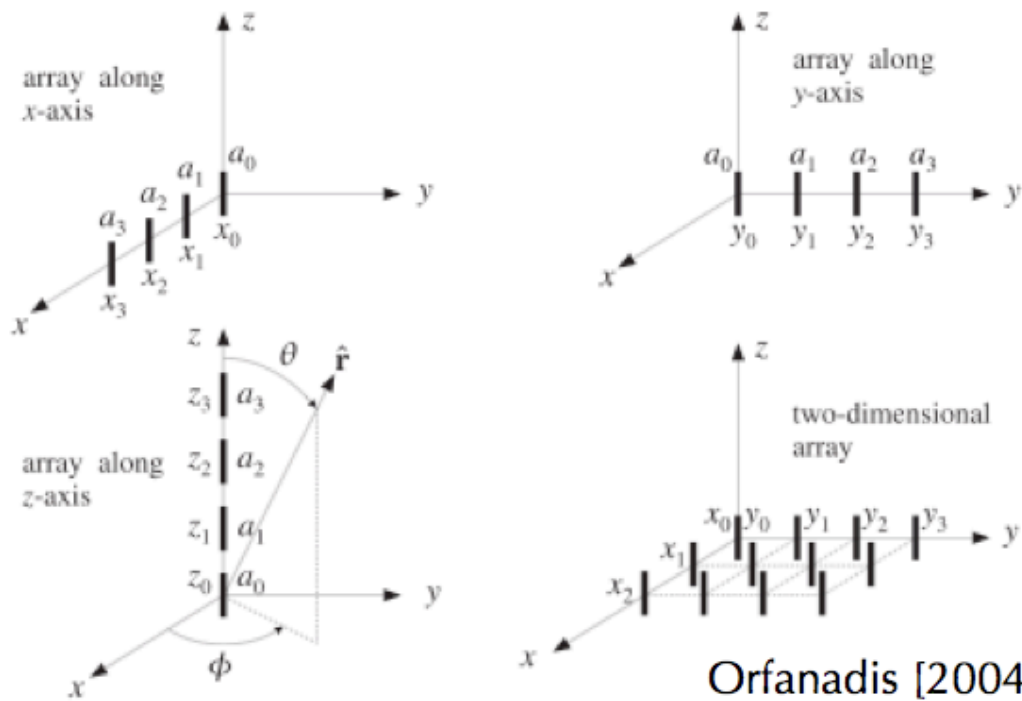
Full-Wave Dipole

$$D(\theta, \phi) = 2.41 \left| \frac{\cos(\pi \cos \theta) - 1}{\sin \theta} \right|^2$$

$$\text{HPBW} \approx 48^\circ$$



Antenna Arrays



$$\mathbf{r}_{mn} = x_m \hat{x} + y_n \hat{y}$$

Assumptions:

1. Far field

- parallel rays, $1/r$ amplitude dependence

2. No mutual coupling between elements (will discuss later)

3. A "reference" element radiates from the origin

4. All elements/radiators are identical, max radiation in z direction (broadside)

Antenna Arrays

$$\mathbf{r}_{mn} = x_m \hat{x} + y_n \hat{y}$$

Reference element at origin will produce a vector electric field at point (r, θ, ϕ)

$$\mathbf{E}_{00} = I_{00} (E_\theta \hat{\theta} + E_\phi \hat{\phi})$$

↑
Constant

Fields due to m th element is:

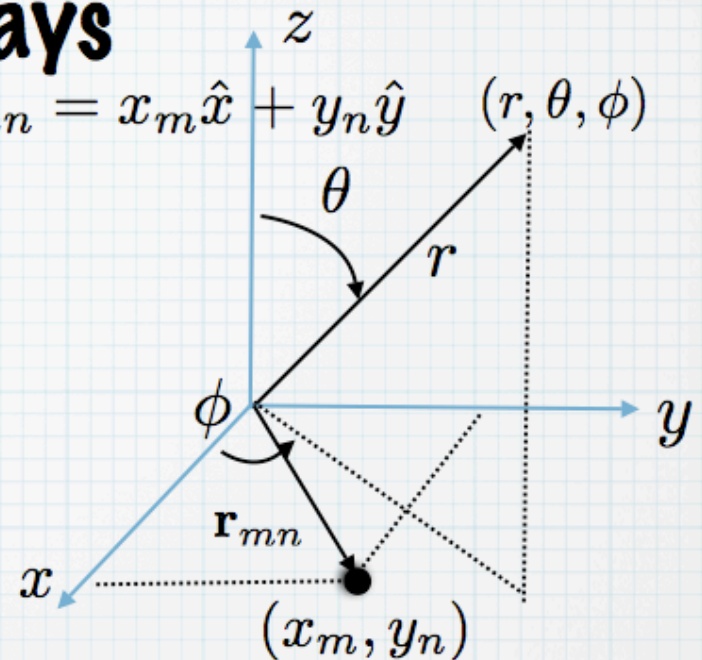
$$\begin{aligned} \mathbf{E}_{mn} &= I_{mn} (E_\theta \hat{\theta} + E_\phi \hat{\phi}) e^{jk \mathbf{r}_{mn} \cdot \hat{\mathbf{r}}} \\ &= I_{mn} (E_\theta \hat{\theta} + E_\phi \hat{\phi}) e^{jk(x_m \sin \theta \cos \phi + y_n \sin \theta \sin \phi)} \end{aligned}$$

Total vector field at (r, θ, ϕ)

$$\mathbf{E} = (E_\theta \hat{\theta} + E_\phi \hat{\phi}) \sum_m \sum_n I_{mn} e^{jk \mathbf{r}_{mn} \cdot \hat{\mathbf{r}}}$$

↑
Element Factor

↑
Array Factor



Antenna Arrays

$$F_{array}(\theta, \phi) = \sum_m \sum_n I_{mn} e^{jk \mathbf{r}_{mn} \cdot \hat{\mathbf{r}}}$$

Poynting vector

$$\begin{aligned} \mathbf{P} &= \frac{1}{2} \Re\{\mathbf{E} \times \mathbf{H}\} = \frac{1}{2z_0} |\mathbf{E}|^2 \hat{\mathbf{r}} \\ &= \frac{1}{2z_0} (|E_\theta|^2 + |E_\phi|^2) |F_{array}|^2 \hat{\mathbf{r}} \end{aligned}$$

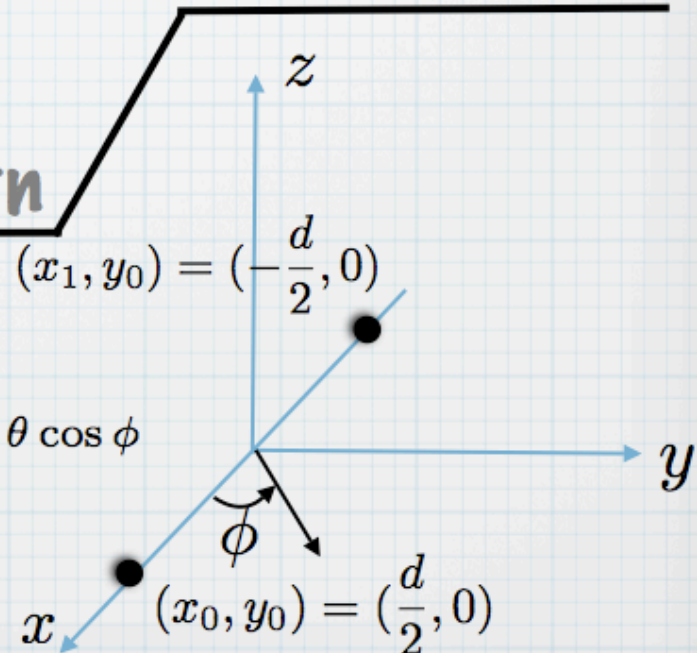
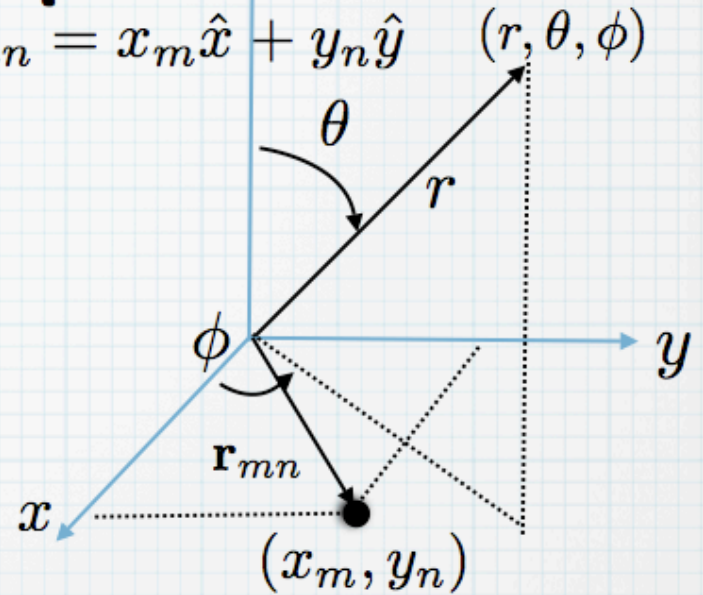
Element Pattern

Array Pattern

Simple Two Element Array

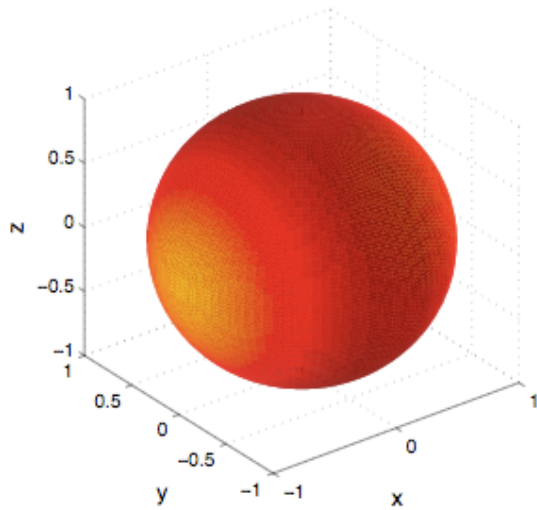
$$F_{array} = I_{00} e^{jk(d/2) \sin \theta \cos \phi} + I_{10} e^{-jk(d/2) \sin \theta \cos \phi}$$

$$\mathbf{r}_{mn} = x_m \hat{x} + y_n \hat{y}$$

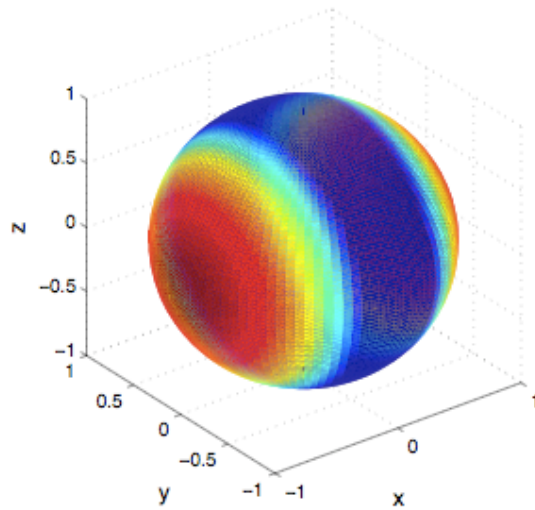


Two-element Array

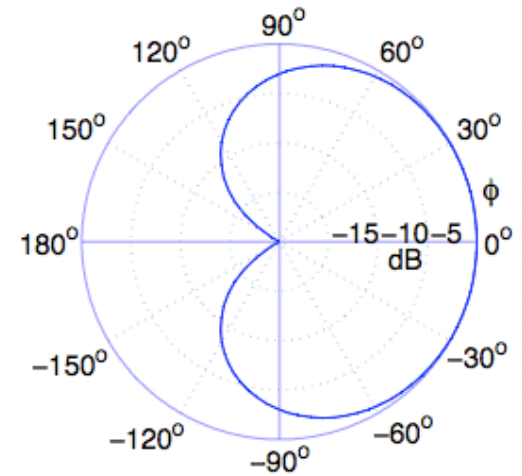
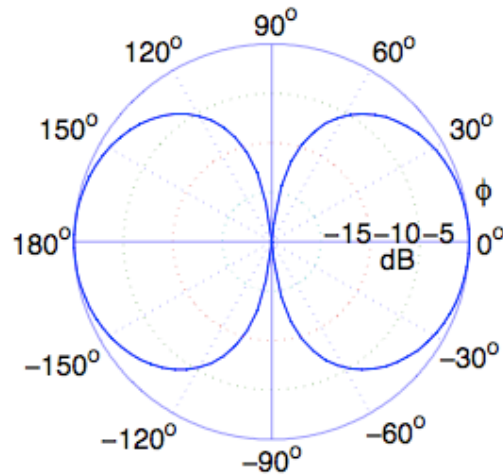
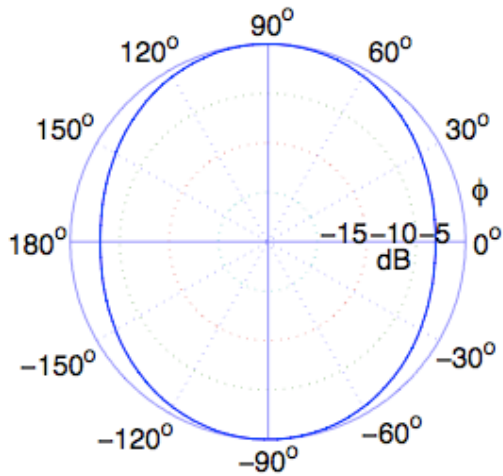
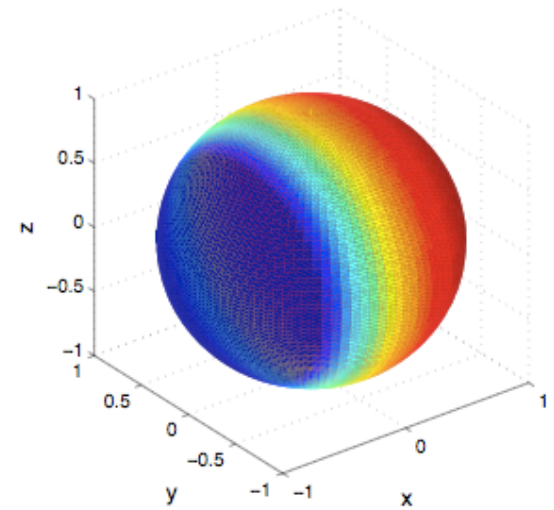
$$d=0.25\lambda, I_{00}=1, I_{10}=1$$



$$d=0.25\lambda, I_{00}=1, I_{10}=-1$$

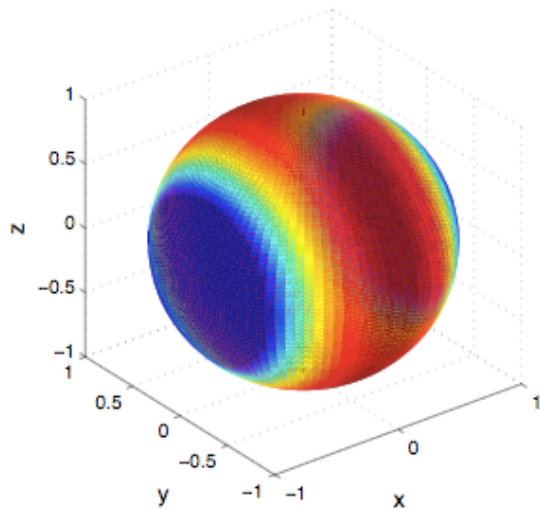


$$d=0.25\lambda, I_{00}=1, I_{10}=0+1j$$

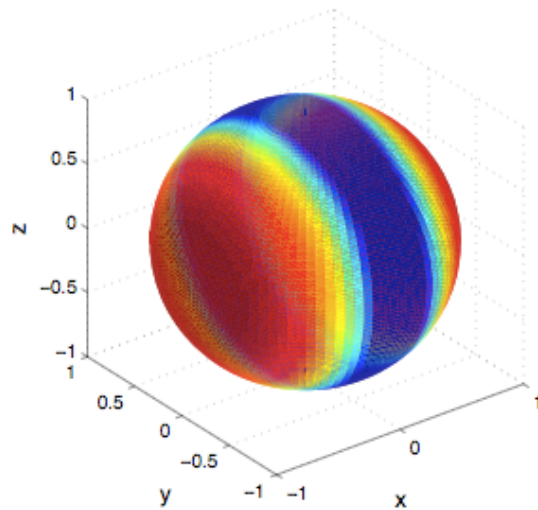


Two-element Array

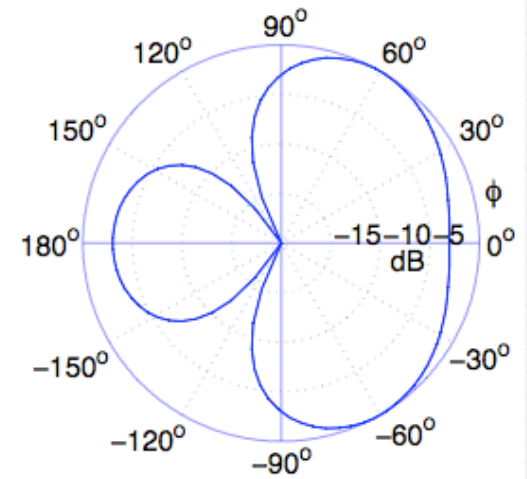
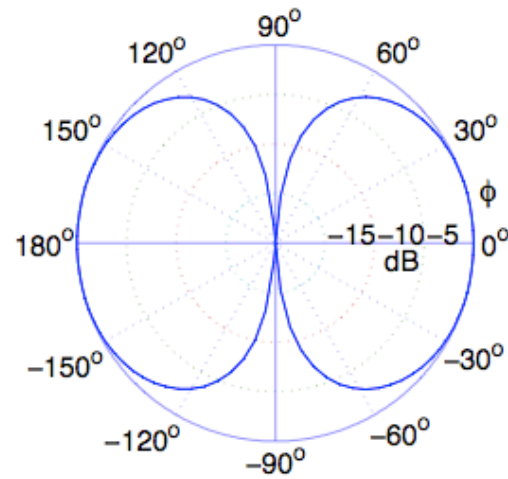
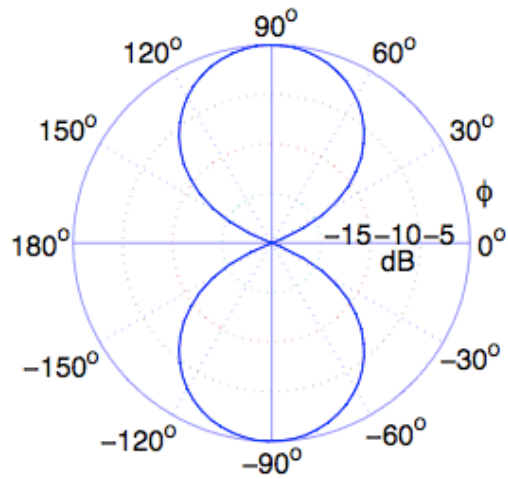
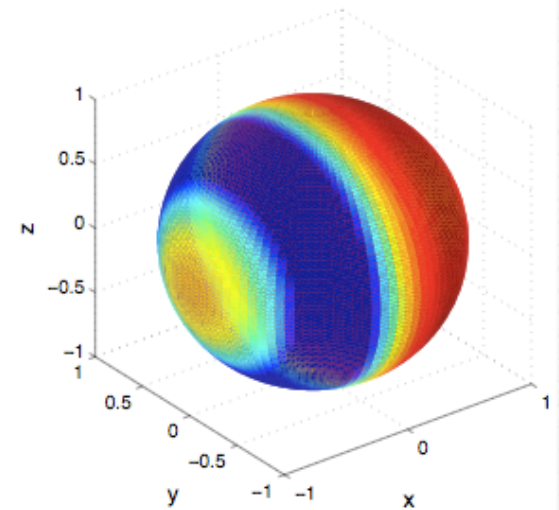
$$d=0.5\lambda, I_{00}=1, I_{10}=1$$



$$d=0.5\lambda, I_{00}=1, I_{10}=-1$$

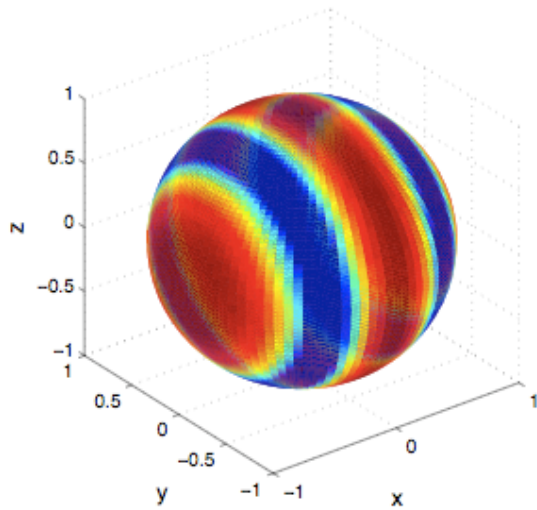


$$d=0.5\lambda, I_{00}=1, I_{10}=0+1i$$

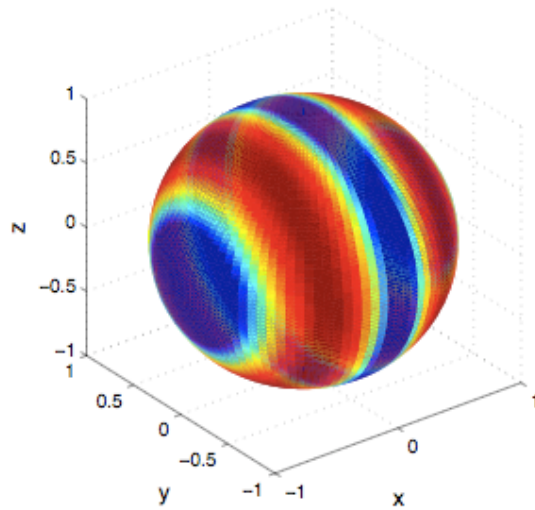


Two-element Array

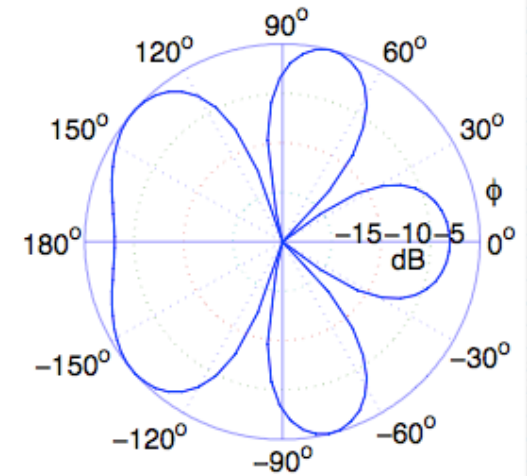
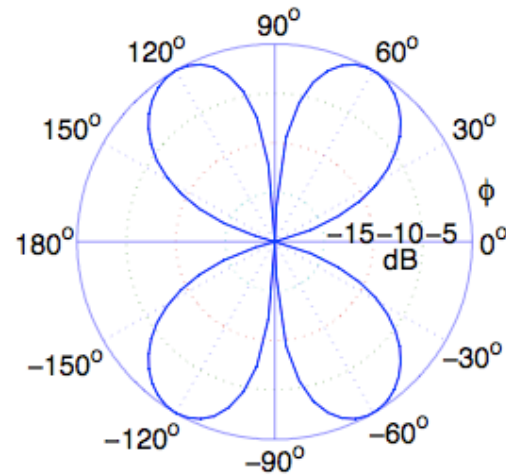
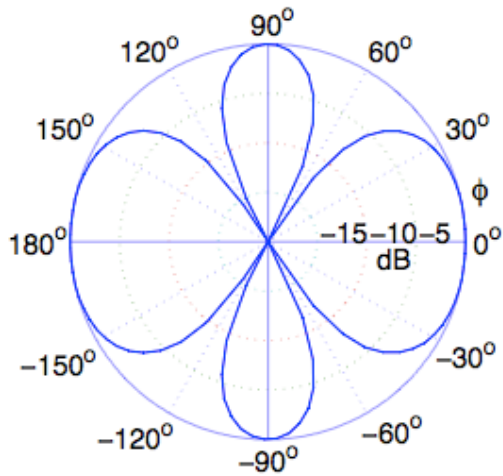
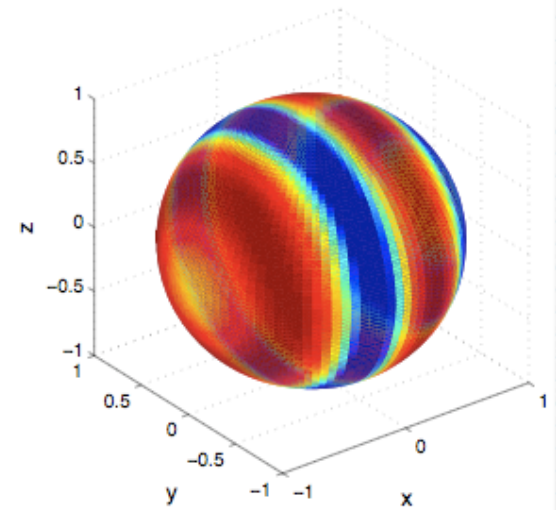
$$d=1\lambda, I_{00}=1, I_{10}=1$$



$$d=1\lambda, I_{00}=1, I_{10}=-1$$



$$d=1\lambda, I_{00}=1, I_{10}=0+1i$$



Array Steering

$$F_{array}(\theta, \phi) = \sum_m \sum_n I_{mn} e^{jk(x_m \sin \theta \cos \phi + y_n \sin \theta \sin \phi)}$$

If the element constants have no phase angles, beam maximum will be in direction:

$$x_m \sin \theta \cos \phi + y_n \sin \theta \sin \phi = 0 \longrightarrow \theta = 0$$

Say we want to point in direction (θ_0, ϕ_0)

$$x_m \sin \theta_0 \cos \phi_0 + y_n \sin \theta_0 \sin \phi_0 + (\psi_m + \psi_n) = 0$$

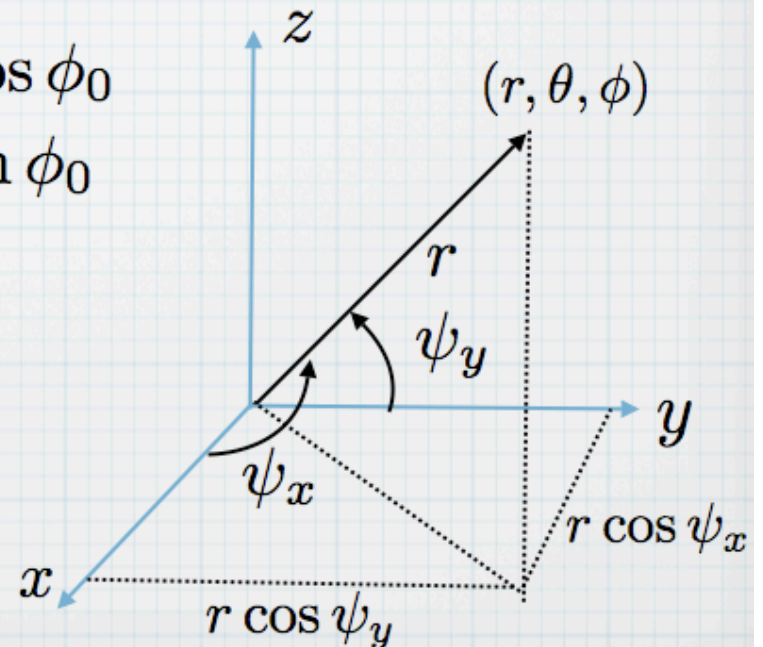
$$\psi_m = -kx_m \sin \theta_0 \cos \phi_0$$

$$\psi_n = -ky_n \sin \theta_0 \sin \phi_0$$

Direction cosines:

$$\cos \psi_{x0} = \sin \theta_0 \cos \phi_0$$

$$\cos \psi_{y0} = \sin \theta_0 \sin \phi_0$$



Array Steering

$$F_{array}(\theta, \phi) = \sum_m \sum_n I_{mn} e^{jk(x_m \sin \theta \cos \phi + y_n \sin \theta \sin \phi)}$$

$$F_{array} = \sum_{m,n} I_{mn} e^{jkx_m (\cos \psi_x - \cos \psi_{x0})} e^{jky_n (\cos \psi_y - \cos \psi_{y0})}$$

Say we want to point in direction (θ_0, ϕ_0)

$$x_m \sin \theta_0 \cos \phi_0 + y_n \sin \theta_0 \sin \phi_0 + (\psi_m + \psi_n) = 0$$

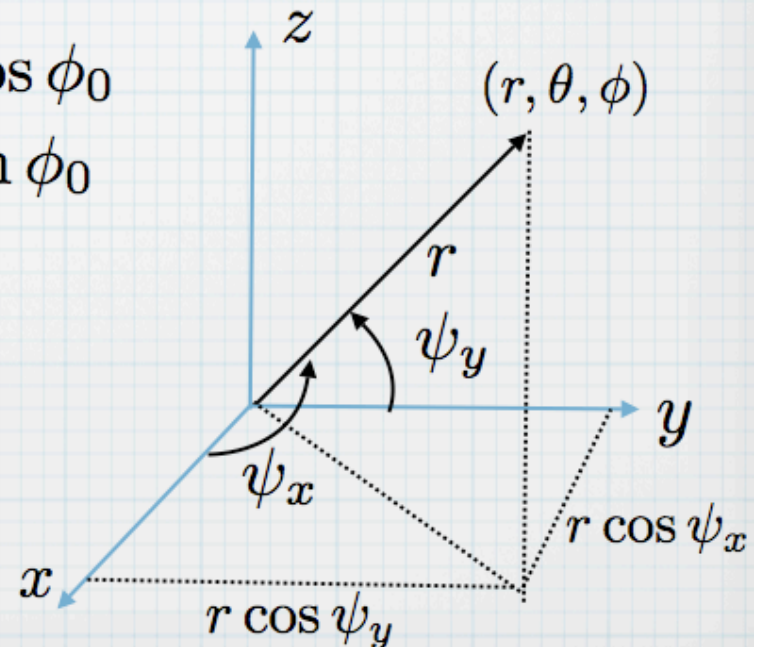
$$\psi_m = -kx_m \sin \theta_0 \cos \phi_0$$

$$\psi_n = -ky_n \sin \theta_0 \sin \phi_0$$

Direction cosines:

$$\cos \psi_{x0} = \sin \theta_0 \cos \phi_0$$

$$\cos \psi_{y0} = \sin \theta_0 \sin \phi_0$$



Directive Gain of Antenna Array

Recall:

$$D(\theta, \phi) = \frac{\text{Power Density Radiated In } (\theta, \phi) \text{ Direction}}{\text{Average Power Density}} = 4\pi R^2 \frac{\text{Power Density In } (\theta, \phi)}{\text{Total Power Radiated}}$$

$$\langle P_r \rangle = \frac{1}{2} \Re \{ \mathbf{E} \times \mathbf{H} \} \cdot \hat{\mathbf{r}} = \frac{1}{2z_0} |\mathbf{E}|^2 |F_{array}|^2 = P_{el} |F_{array}|^2$$

$$P_{total} = \int_0^{2\pi} d\phi \int_0^\pi P_{el} |F_{array}|^2 r^2 \sin \theta d\theta$$

$$D(\theta, \phi) = 4\pi r^2 \frac{P_{el} |F_{array}|^2}{\int_0^{2\pi} d\phi \int_0^\pi P_{el} |F_{array}|^2 r^2 \sin \theta d\theta}$$

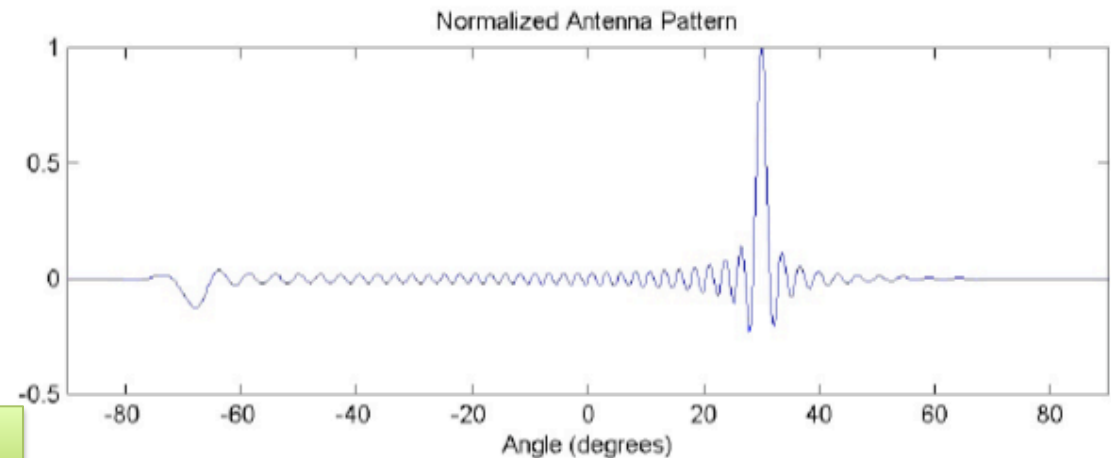
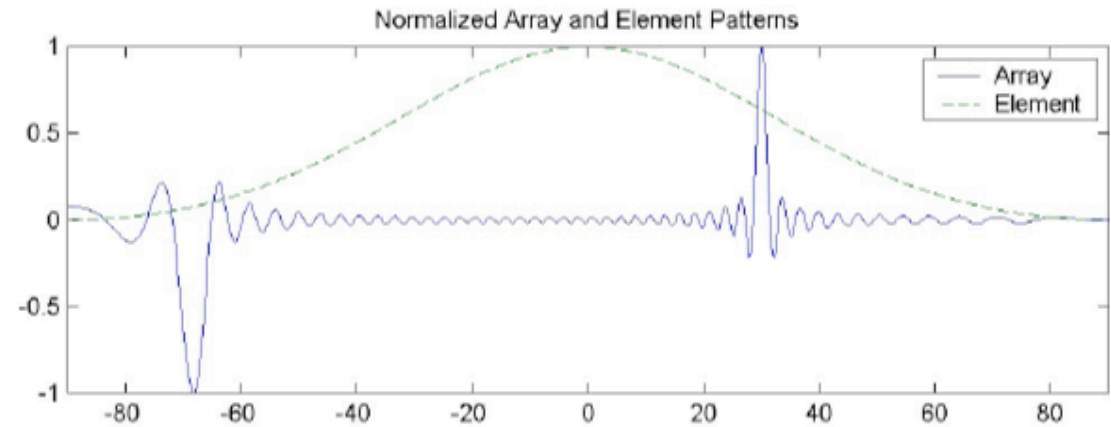
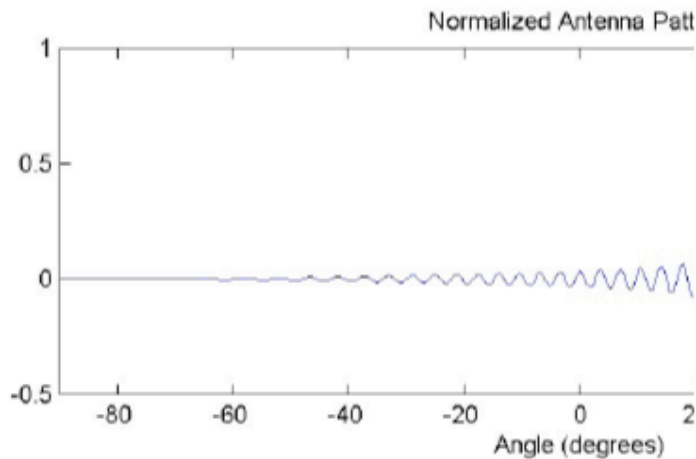
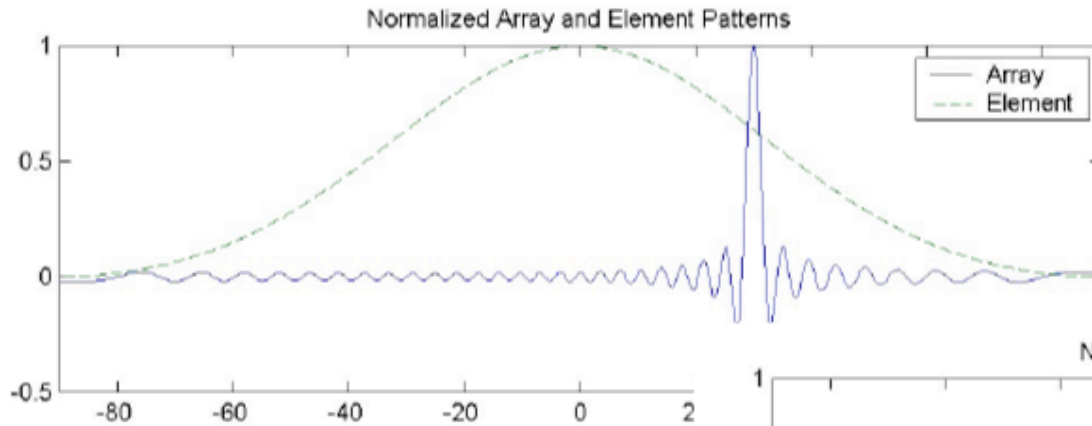
If element pattern is much broader than array pattern,

**Element pattern
↙ doesn't matter.**

$$D(\theta, \phi) = 4\pi r^2 \frac{|F_{array}|^2}{\int_0^{2\pi} d\phi \int_0^\pi |F_{array}|^2 r^2 \sin \theta d\theta}$$

Directive Gain of Antenna Array

$$D = 4\pi R^2 \frac{\text{Power Density In } (\theta, \phi)}{\text{Total Power Radiated}}$$



$$D(\theta, \phi) = 4\pi r$$

The Fourier Analogy

Array factor can be interpreted as DFT of weighting factors

$$F_{array} = \sum_m I_m e^{j k d m (\cos \psi_x - \cos \psi_{x0})}$$

$$= \sum_m I_m e^{j m \gamma}$$

Array factor in spatial z domain

$$= \sum_m I_m z^m$$

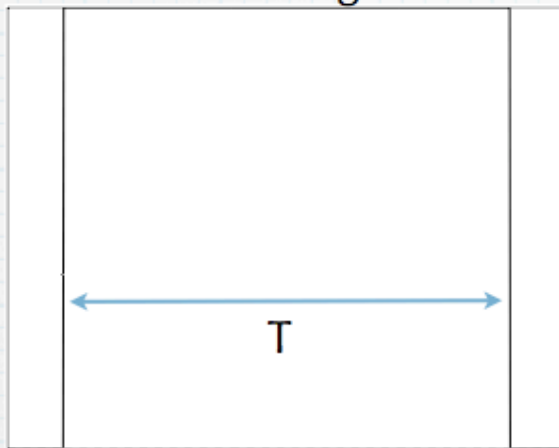
Inverse DFT - principle of many array design methods (analogous to FIR filter design)

$$I_m = \frac{1}{2\pi} \int_{-\pi}^{\pi} F_{array}(\gamma) e^{-j \gamma m} d\gamma$$

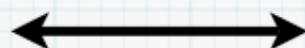
The Fourier Analogy (2)

One application - beam broadening

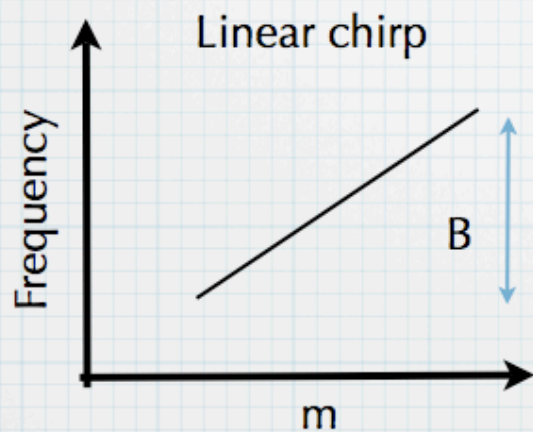
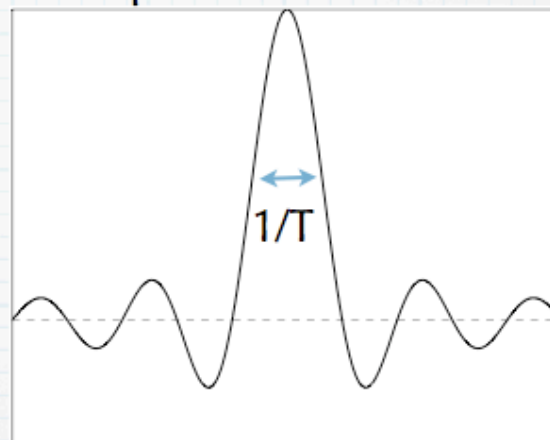
uniform weights



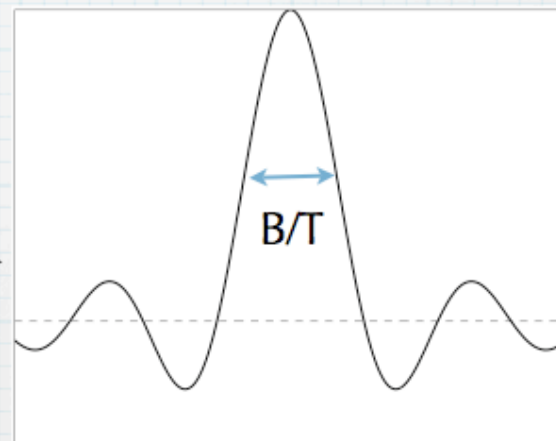
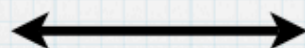
DFT



spatial sinc function



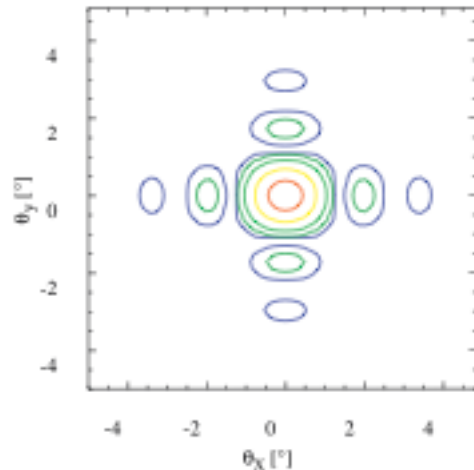
DFT



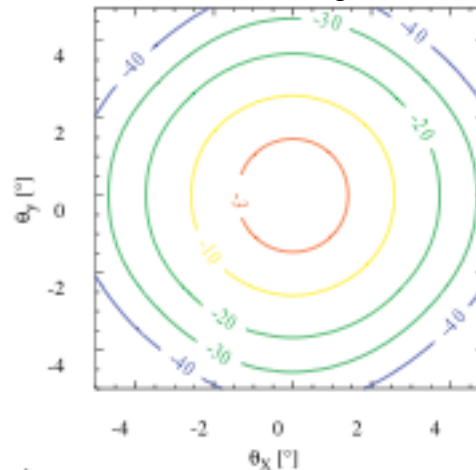
Tx/Rx Beam Pattern Control

Chau et al., [2009]

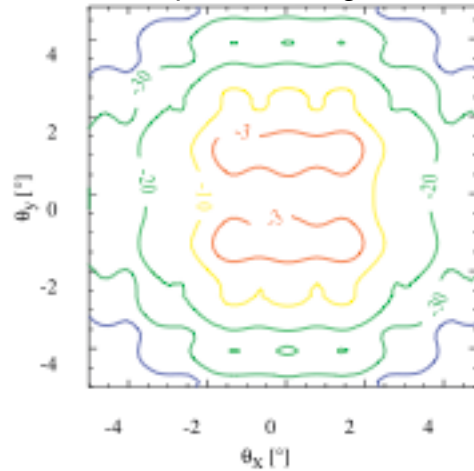
a *Narrow Beam*



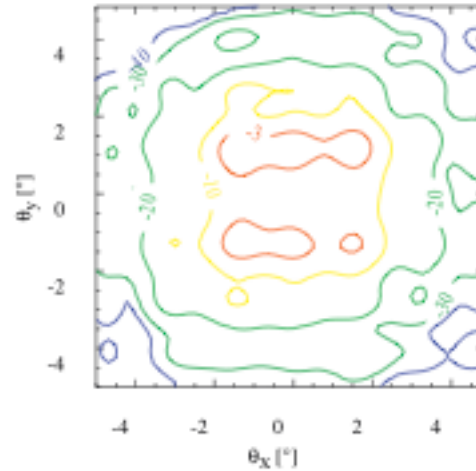
b *Amplitude and Phase Changes for beam broadening*



c *Just phase changes*

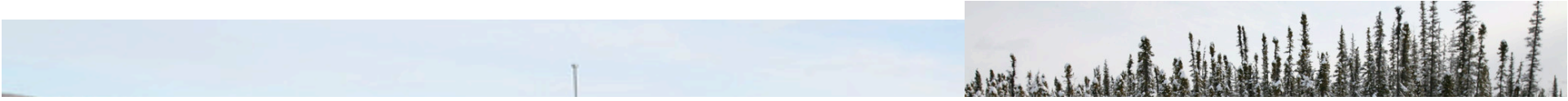


d *With AEU data*



- Determine which meteors in the narrow beam are coming from sidelobes ($\sim 15\%$)
- Increase number of large cross-section meteor detections

Method of Moments (mutual coupling)



RADIO SCIENCE, VOL. 46, RS2012, doi:10.1029/2010RS004518, 2011

A review on array mutual coupling analysis

C. Craeye¹ and D. González-Ovejero¹

Received 8 September 2010; revised 14 December 2010; accepted 6 January 2011; published 8 April 2011.

[1] An overview about mutual coupling analysis in antenna arrays is given. The relationships between array impedance matrix and embedded element patterns, including beam coupling factors, are reviewed while considering general-type antennas; approximations resulting from single-mode assumptions are pointed out. For regular arrays, a common Fourier-based formalism is employed, with the array scanning method as a key tool, to explain various phenomena and analysis methods. Relationships between finite and infinite arrays are described at the physical level, as well as from the point of view of numerical analysis, considering mainly the method of moments. Noise coupling is also briefly reviewed.

Citation: Craeye, C., and D. González-Ovejero (2011), A review on array mutual coupling analysis, *Radio Sci.*, 46, RS2012, doi:10.1029/2010RS004518.

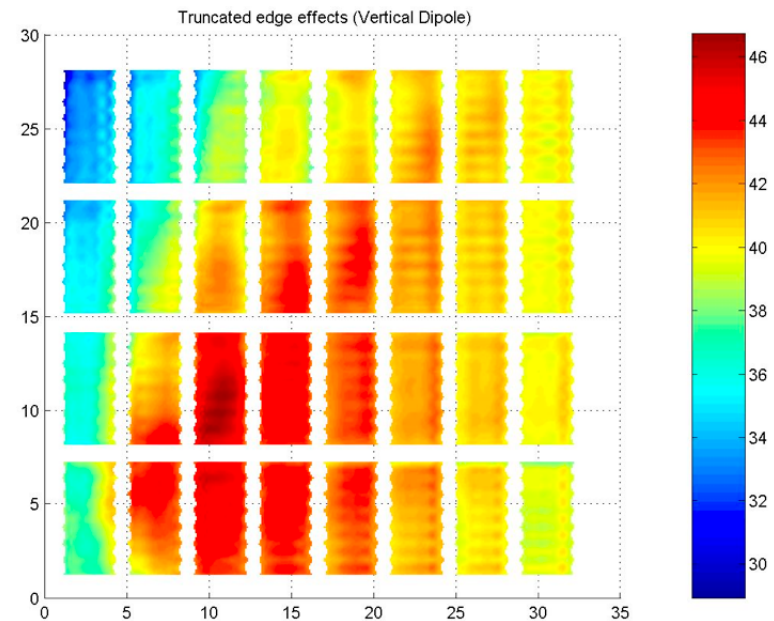
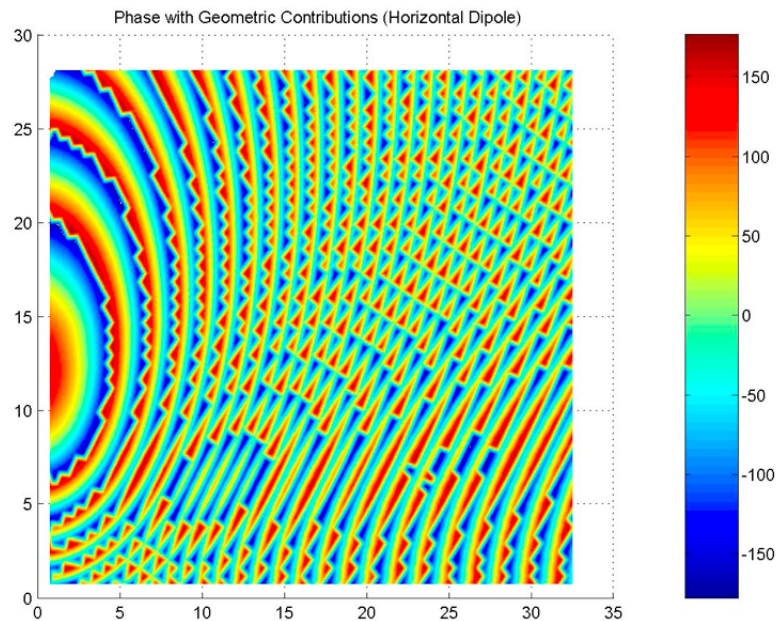


Method of Moments (NEC)

Significant errors are introduced if mutual coupling contributions are neglected. Standard MoM code scales poorly, making it impractical to model a full array. Great research project for engineering-minded student: develop sparse MoM code for an entire array!

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PROCEEDINGS OF THE IEEE, VOL. 55, NO. 2, FEBRUARY, 1967

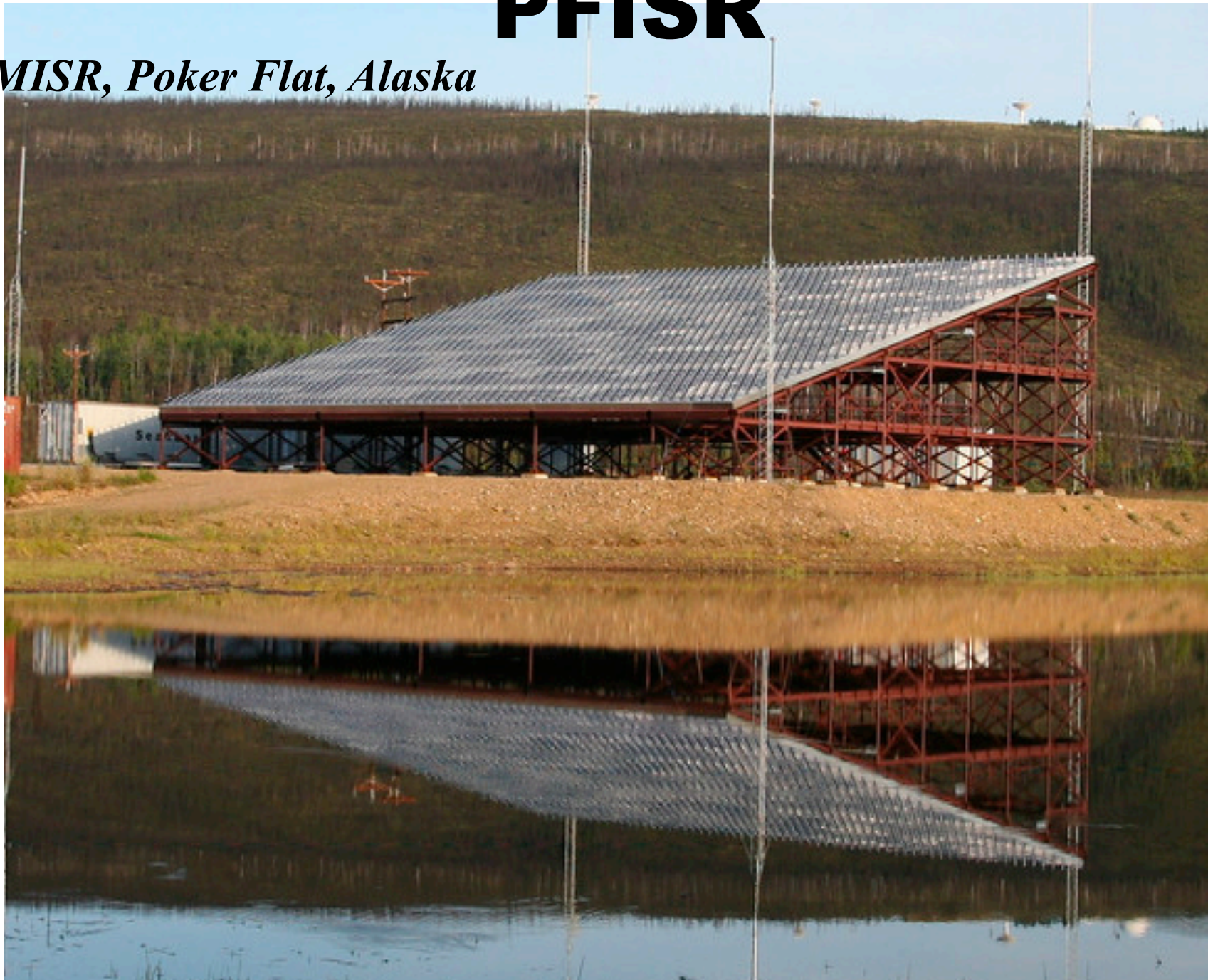


THE USE of high-speed digital computers not only allows one to make more computations than was

specified. This paper deals only with analysis.

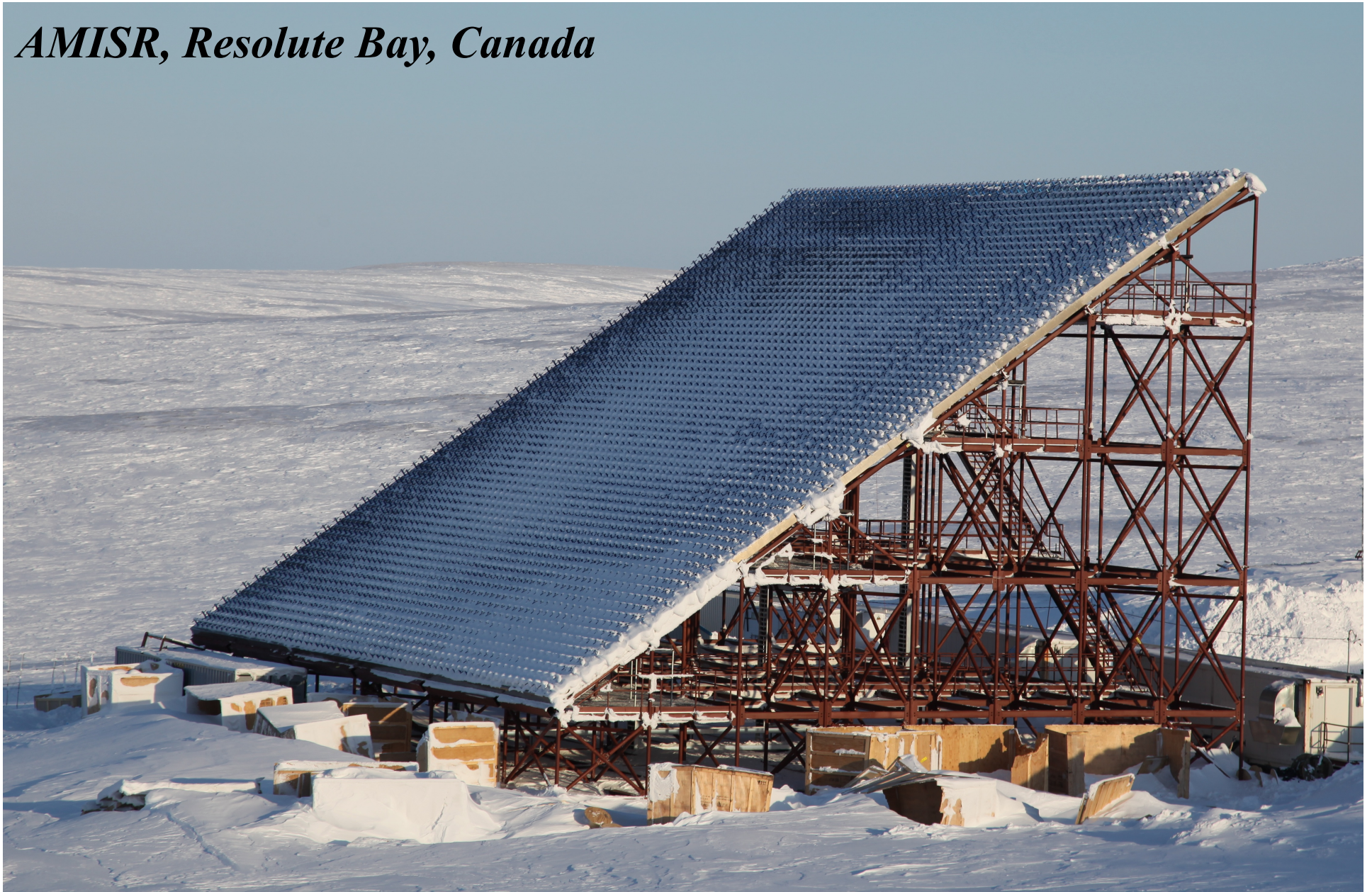
PFISR

AMISR, Poker Flat, Alaska

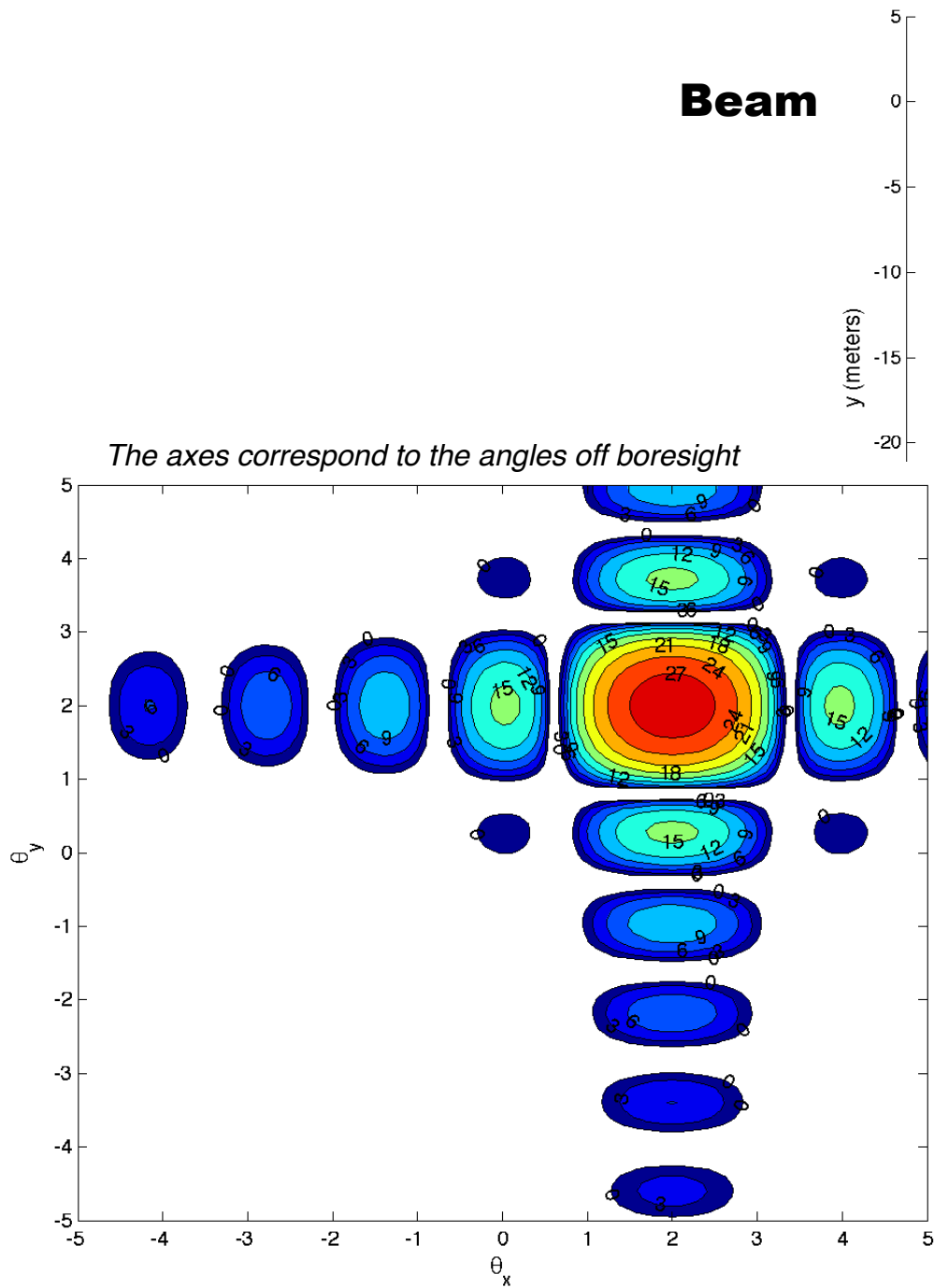


RISR-N

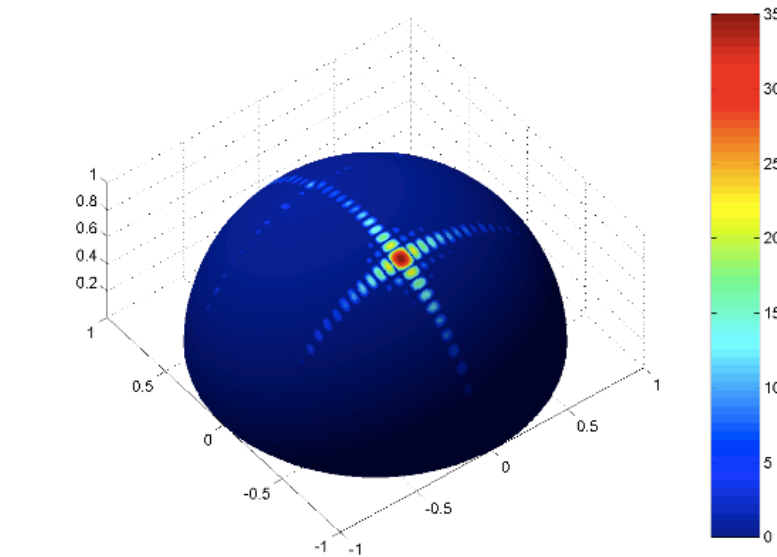
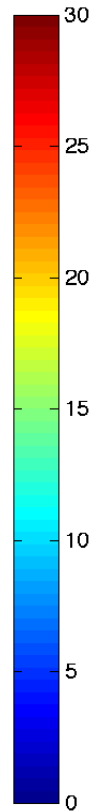
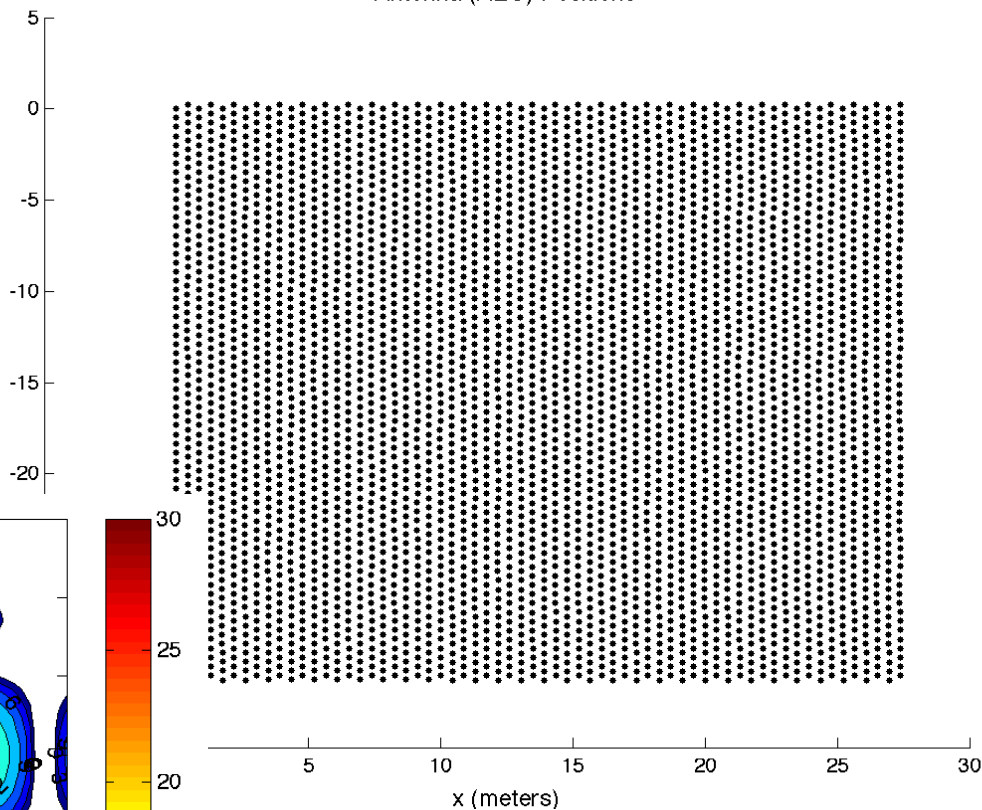
AMISR, Resolute Bay, Canada



Beam



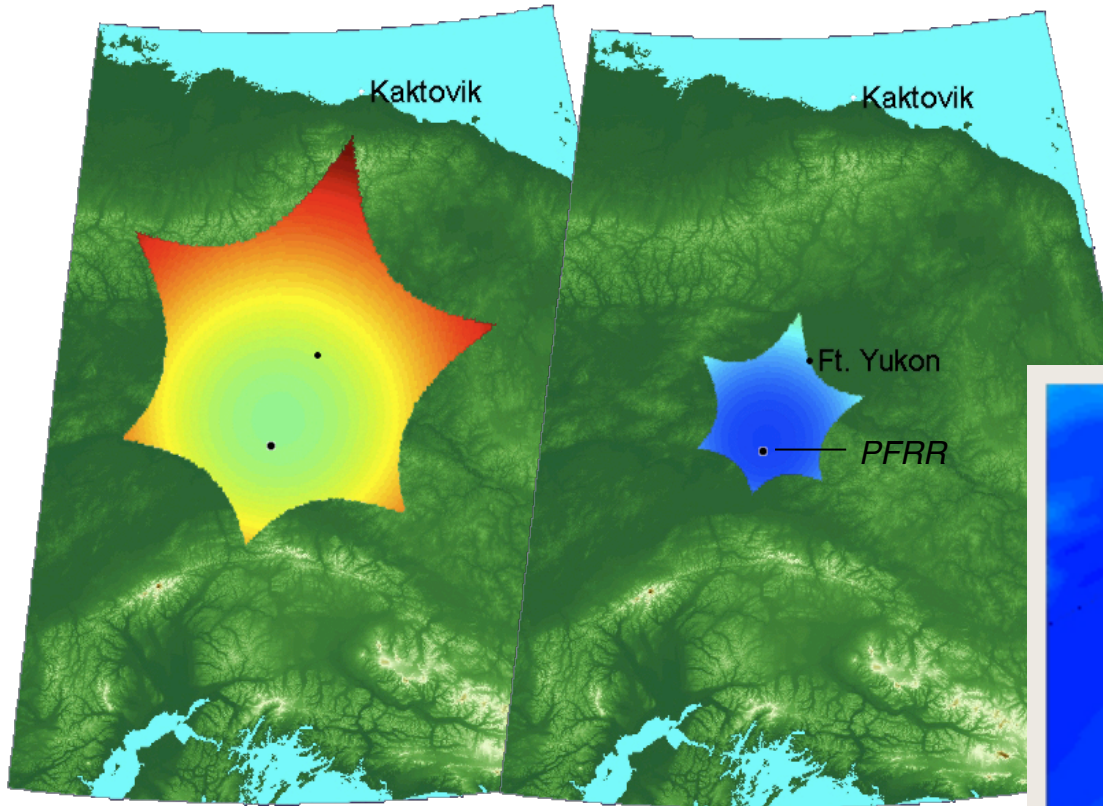
Antenna (AEU) Positions



AMISR Coverage – Poker Flat

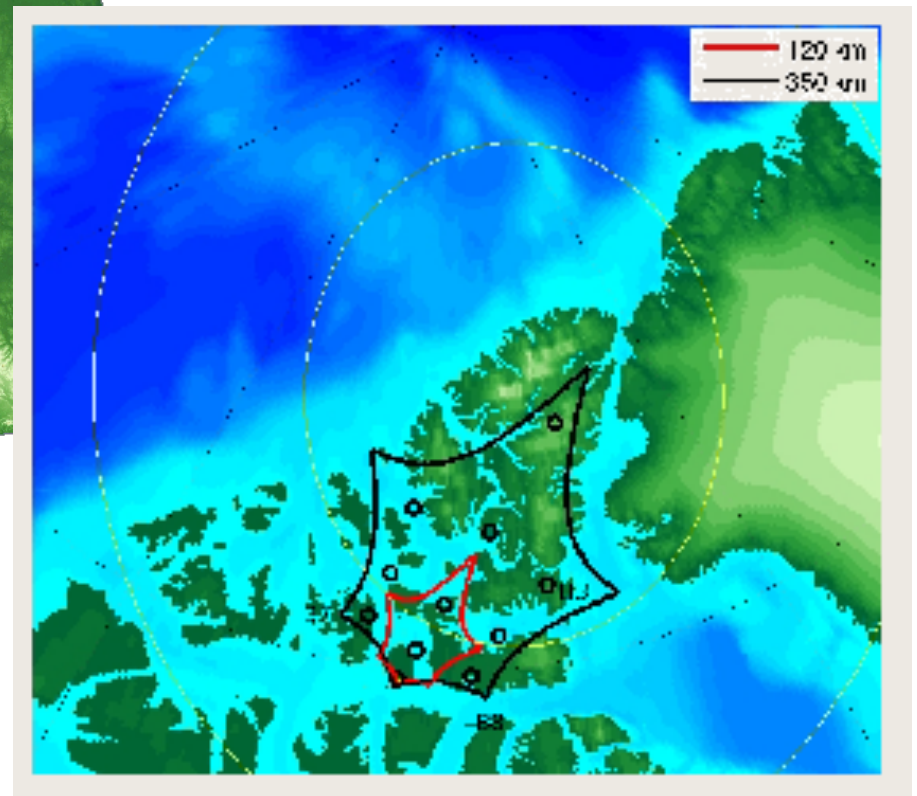
350 km, 1×10^{11} , 10%

150 km, 1×10^{11} , 10%

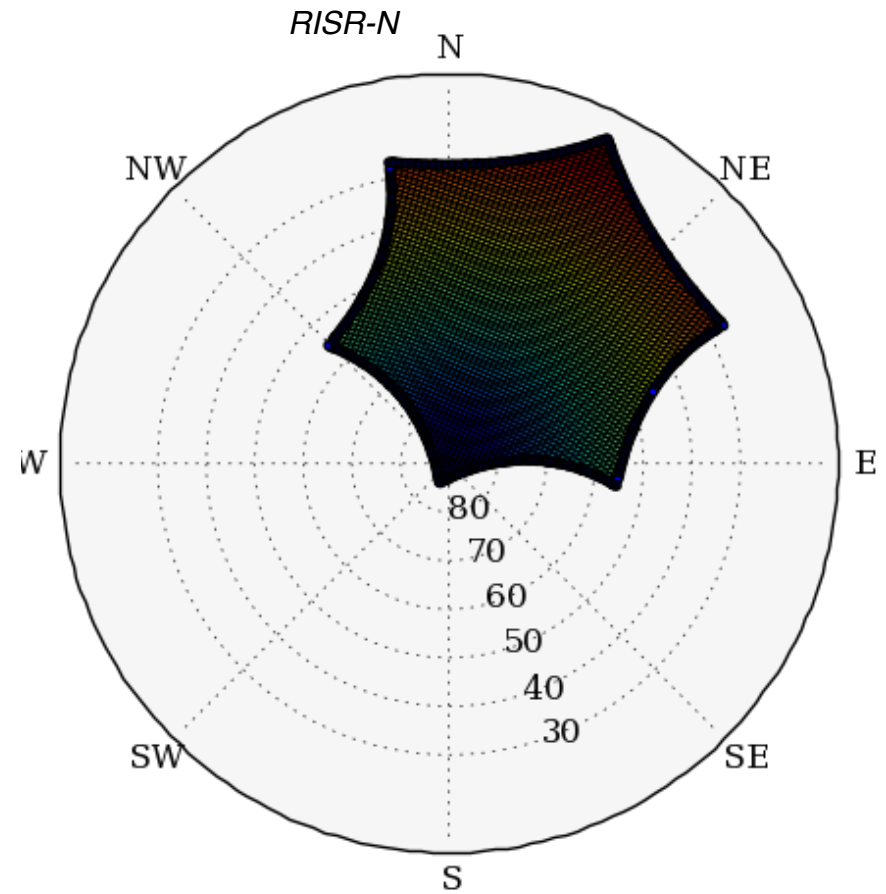
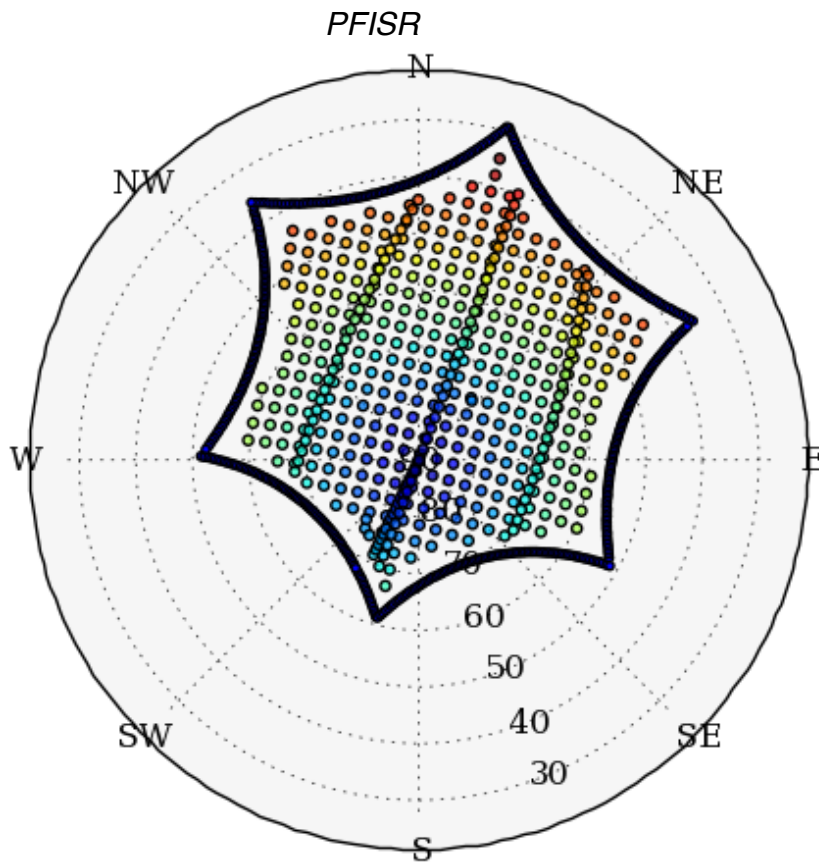


PFISR Coverage

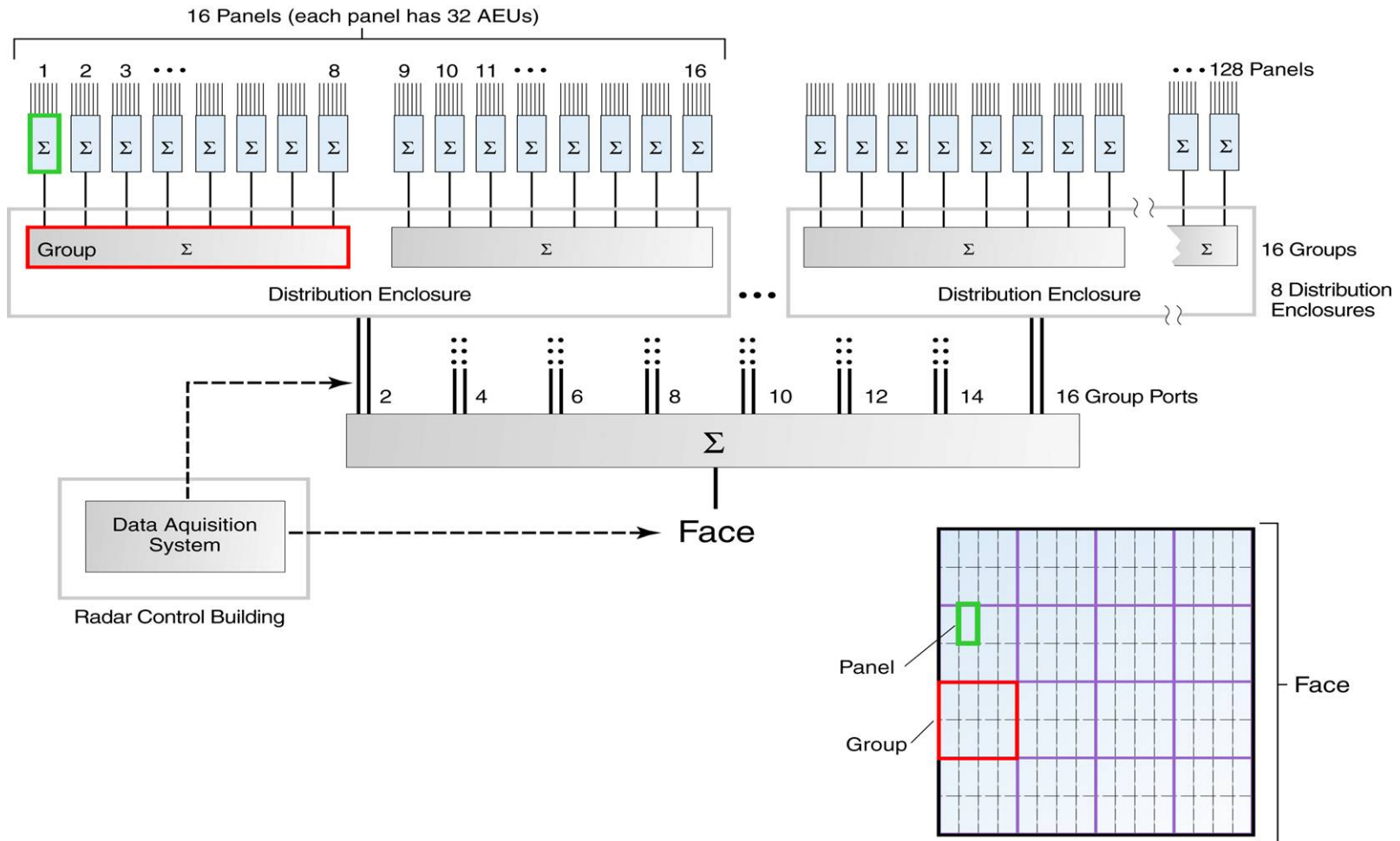
RISR Coverage



Discrete look directions

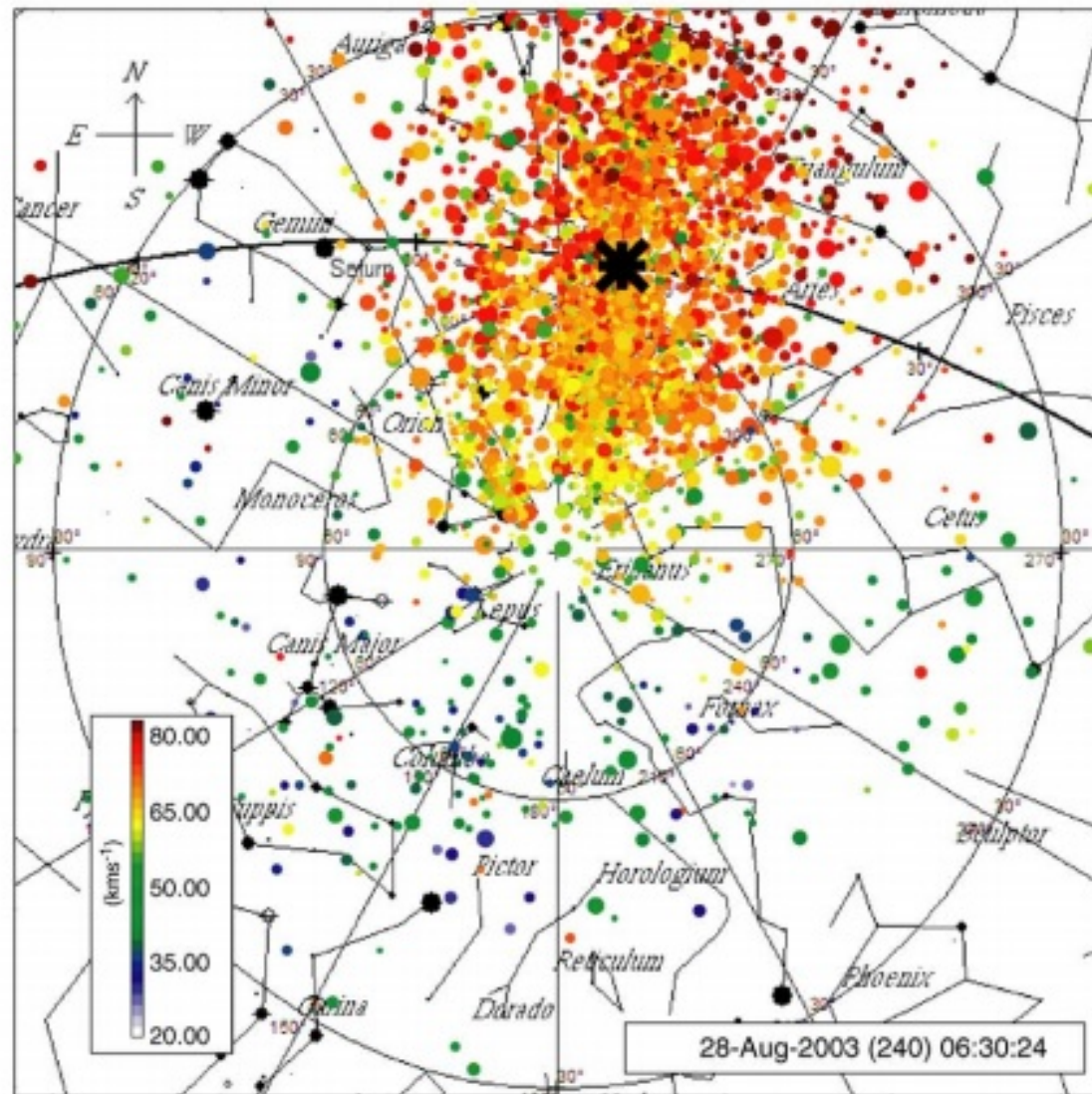


Interferometry



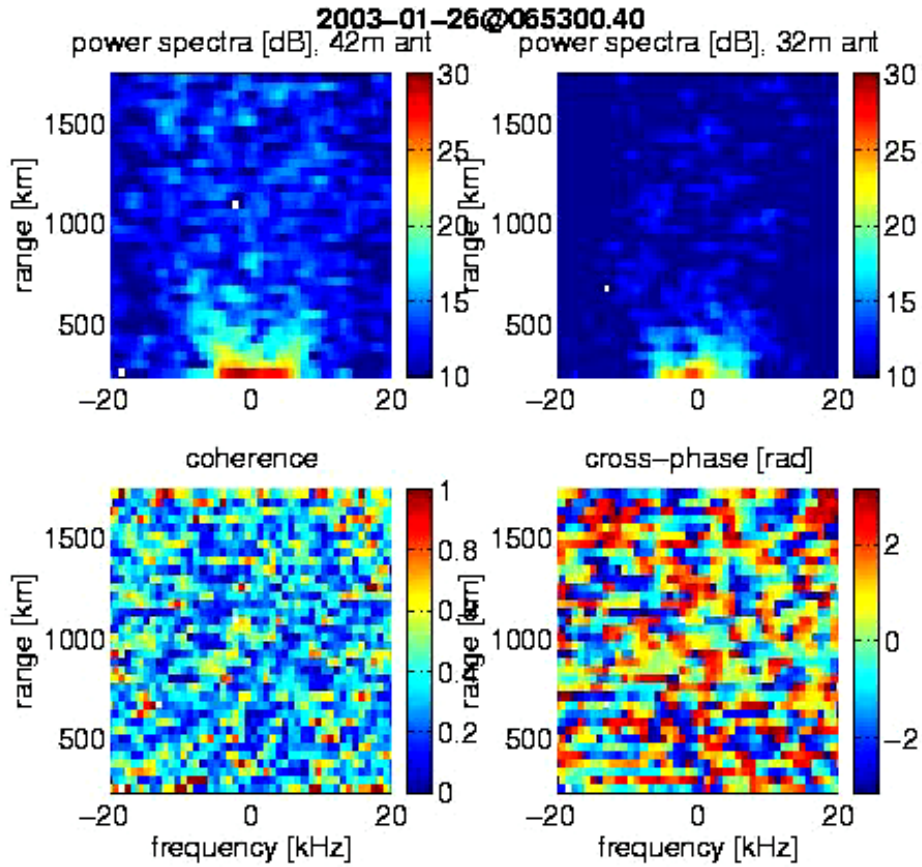
orbit around the Sun), within $\pm 18^\circ$ transverse to the ecliptic and narrow ($\pm 8.5^\circ$) in heliocentric longitude in the ecliptic plane (Fig. 2). Absolute instantaneous

geocentric velocity distribution results from these observations owing to the interferometric capabilities of the JRO system. The observed velocity distribution is



(a)

Interferometry



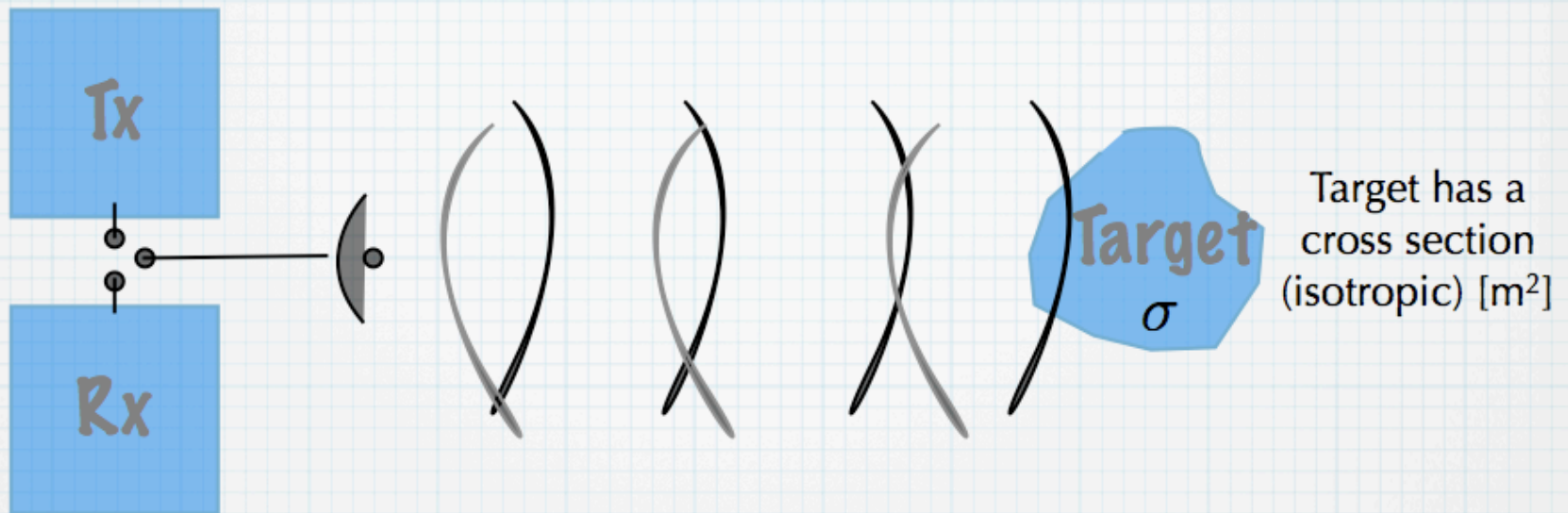
NEIAL Interferometry
ESR



T. Grydeland, 2004

"Equations are the devil's sentences"
- Stephen Colbert

Basic Radar



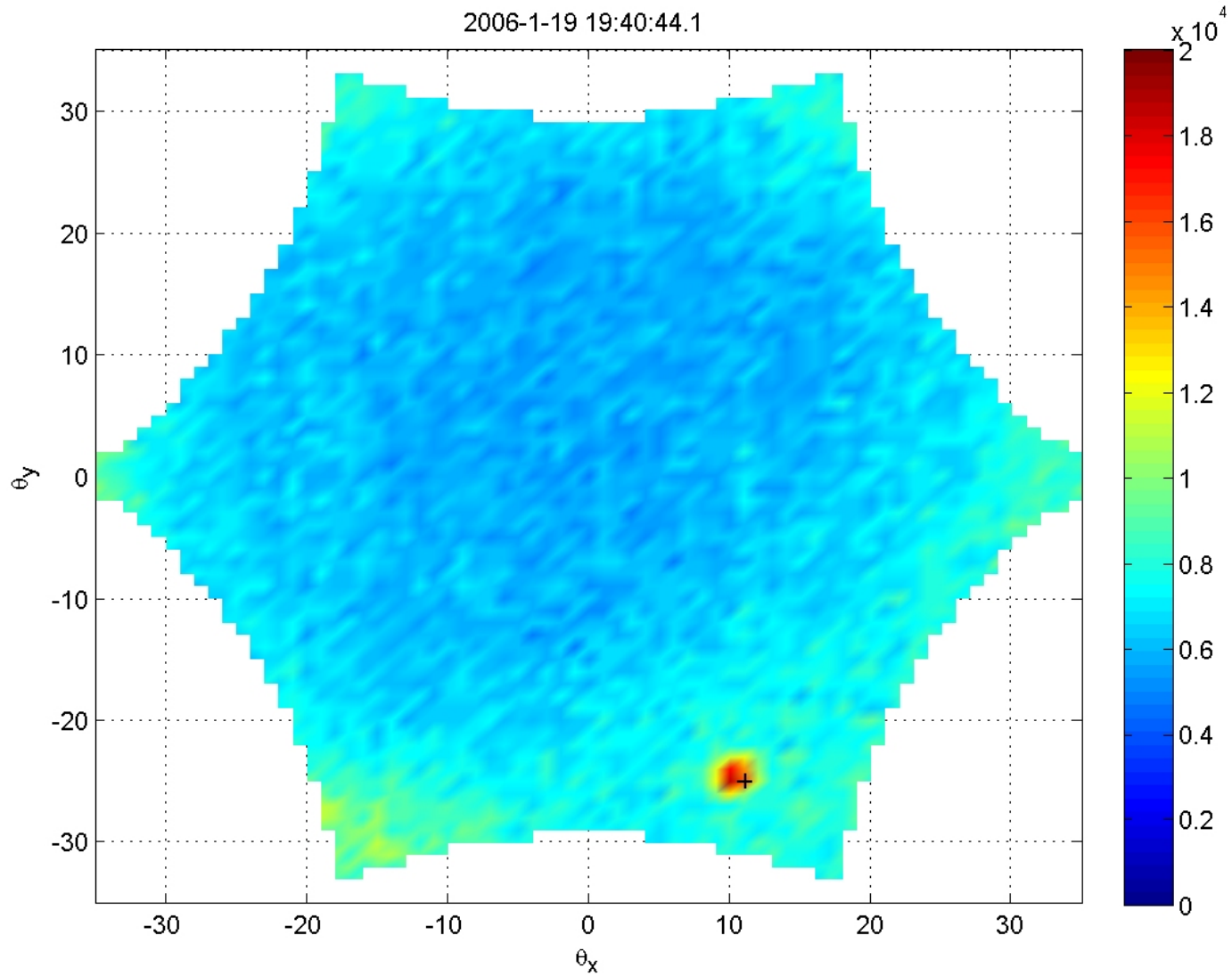
$$P_{inc} = P_t \frac{G_{tx}}{4\pi R^2} \quad \text{W/m}^2 \quad \text{Power incident on target}$$

$$P_{scat} = P_{inc} \sigma_{radar} \quad \text{W} \quad \text{Scattered power}$$

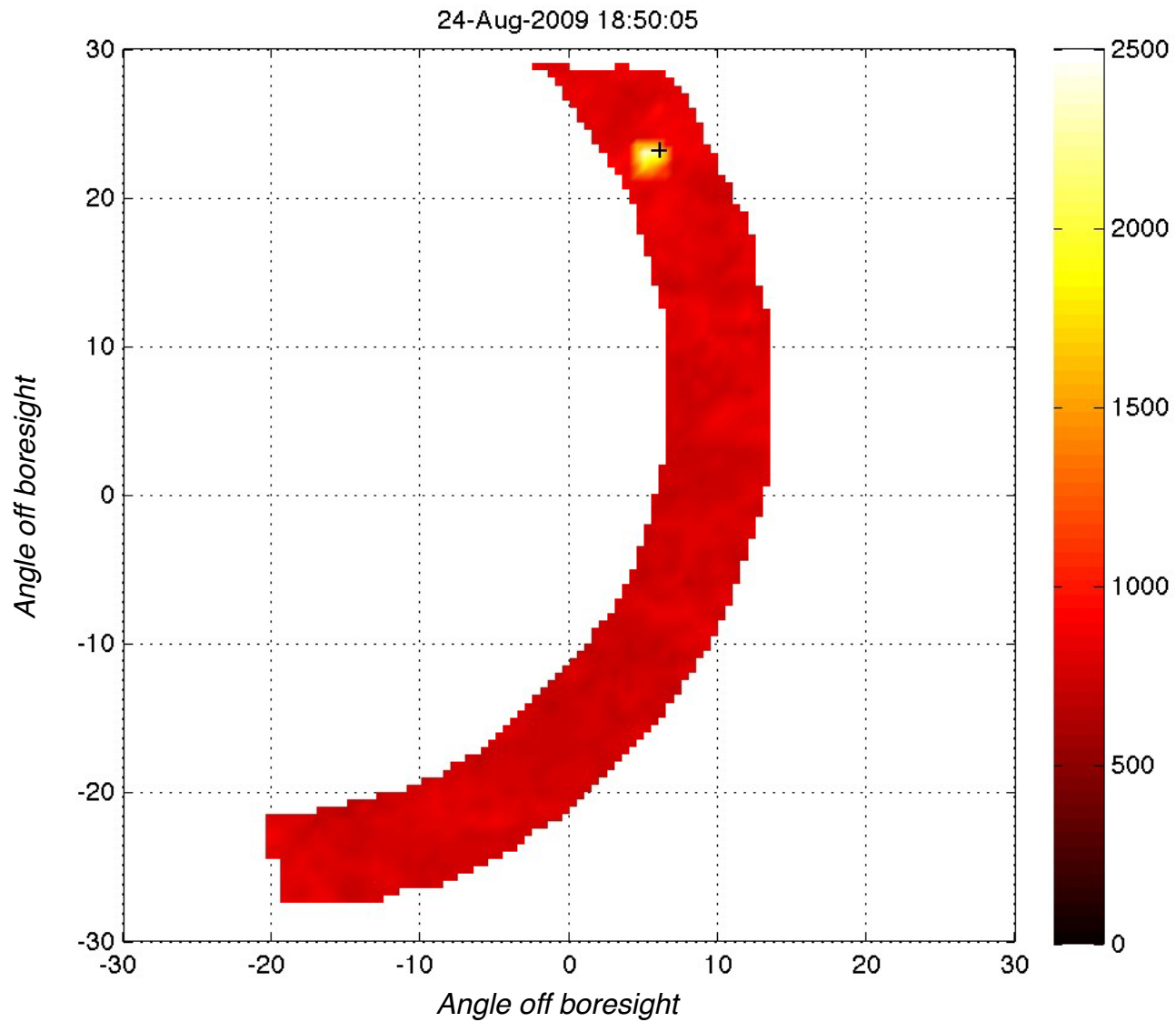
$$P_{rec} = P_{scat} \frac{A_{eff}}{4\pi R^2} \quad \text{W} \quad \text{Received power}$$

$$= P_t \frac{G_{tx} A_{eff} \sigma_{radar}}{16\pi^2 R^4} \quad \text{W} \quad \text{Radar equation}$$

Receive-only Imaging – Beam-Forming Test



Receive-only Imaging – Beam-Forming Test

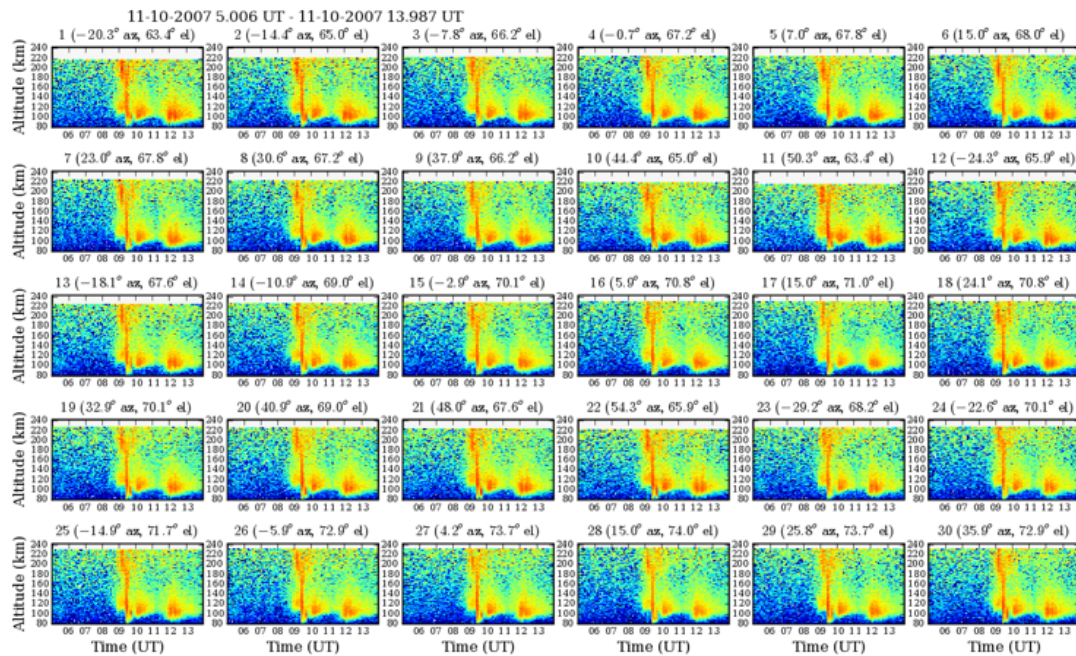
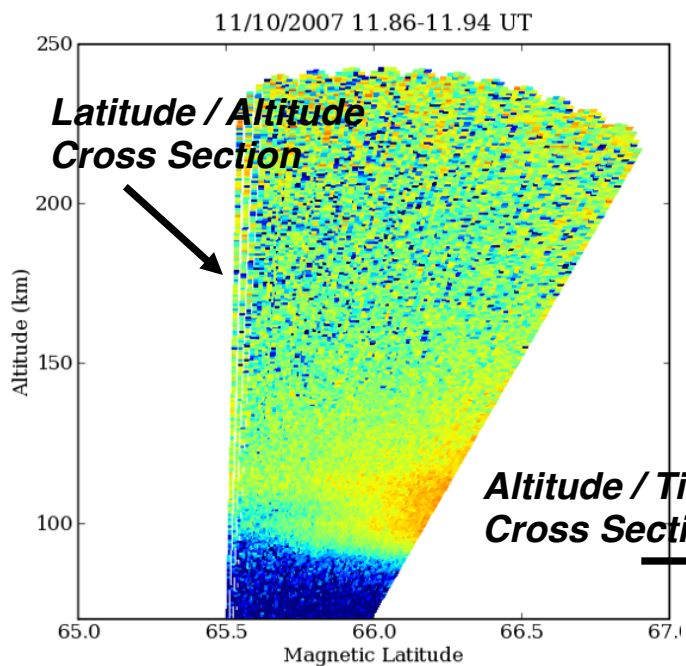


What are the Measurement Improvements

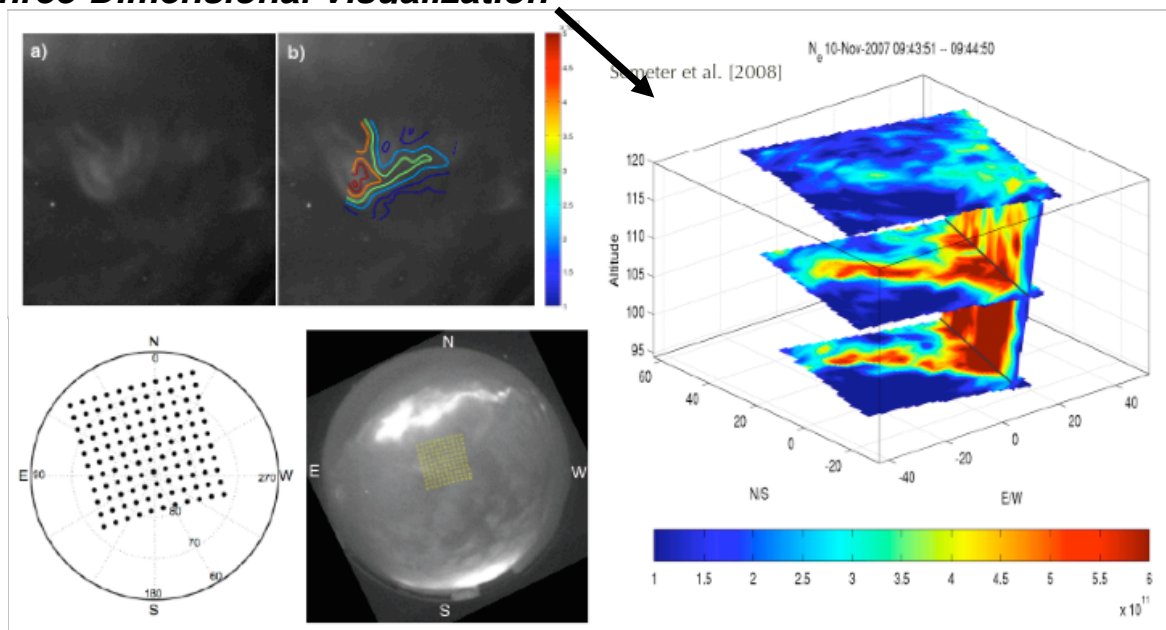
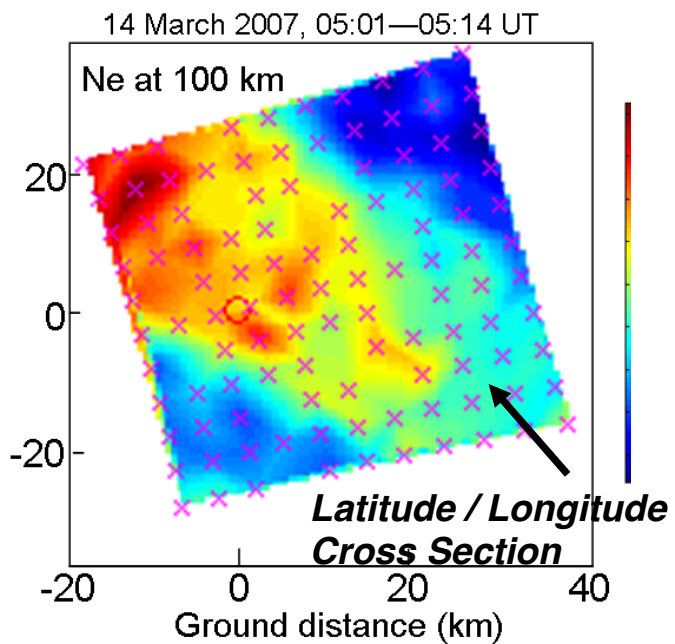


- Inertia-less antenna pointing
 - Pulse-to-pulse beam positioning
 - Supports great flexibility in spatial sampling
 - Helps remove spatial/temporal ambiguities
 - Eliminates need for predetermined integration (dish antenna dwell time)
 - Opens possibilities for in-beam imaging through, e.g., interferometry

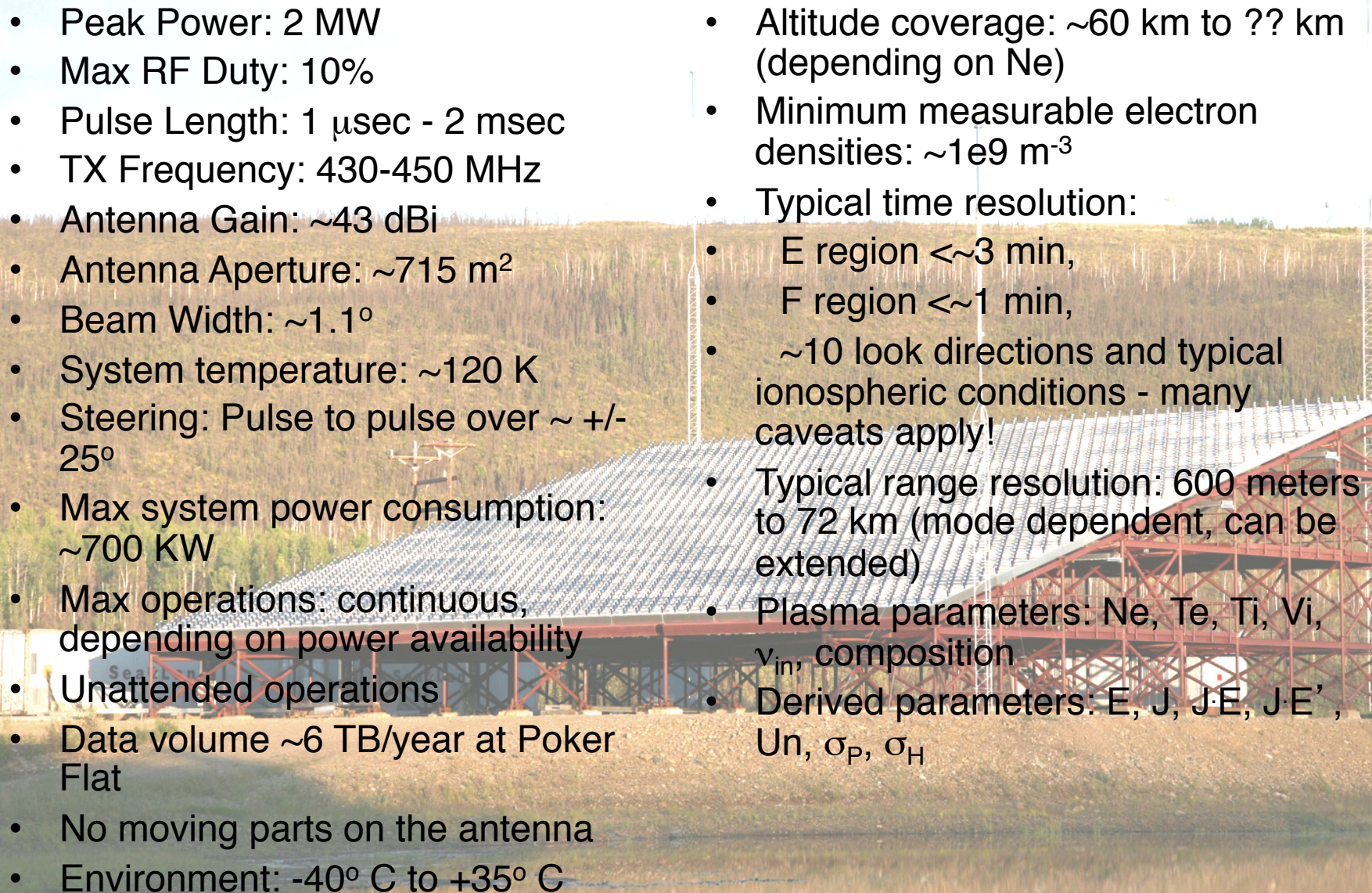
PFISR: Images of the Aurora in 4-Dimensions (3-D images v. time)



Three-Dimensional Visualization



AMISR Technical Specifications

- Peak Power: 2 MW
 - Max RF Duty: 10%
 - Pulse Length: 1 μ sec - 2 msec
 - TX Frequency: 430-450 MHz
 - Antenna Gain: ~ 43 dBi
 - Antenna Aperture: ~ 715 m²
 - Beam Width: $\sim 1.1^\circ$
 - System temperature: ~ 120 K
 - Steering: Pulse to pulse over $\sim \pm 25^\circ$
 - Max system power consumption: ~ 700 KW
 - Max operations: continuous, depending on power availability
 - Unattended operations
 - Data volume ~ 6 TB/year at Poker Flat
 - No moving parts on the antenna
 - Environment: -40° C to $+35^\circ$ C
 - Altitude coverage: ~ 60 km to ?? km (depending on Ne)
 - Minimum measurable electron densities: $\sim 1e9$ m⁻³
 - Typical time resolution:
 - E region $< \sim 3$ min,
 - F region $< \sim 1$ min,
 - ~ 10 look directions and typical ionospheric conditions - many caveats apply!
 - Typical range resolution: 600 meters to 72 km (mode dependent, can be extended)
 - Plasma parameters: Ne, Te, Ti, Vi, v_{in} , composition
 - Derived parameters: E, J, J·E, J·E', Un, σ_p , σ_H
- 

Peak Power 1778 KW (3557/476/31)

2010-07-23 21:57 UTC

