More on ISR Experiments, Data Reduction, and Analysis

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SRI International

Outline

1. ISR Pulses and Experiments
   - The Nature of the IS Target
   - *F*-Region Experiments
   - *E*-Region Experiments
   - *D*-Region Experiments
   - System Info
   - Beam Pointing
2. Level-0 Processing
   - General
   - Power Estimation
   - ACF / Spectra Estimation
3. Level-1 Processing
   - N_e Estimation
   - ACF / Spectral Fits
   - ACF / Spectral Fits
4. Level-2 Processing
   - Vector Velocities / Electric Fields
   - E-Region Winds
   - Collision Freqs. / Conductivities / Currents / Joule Heating
   - D-Region Parameters

Overspread Targets

(a.k.a, frequency and range aliased targets)

- For a target with a bandwidth B, you must sample at a rate F exceeding B (e.g., for IS, $B \sim 40\ \mathrm{kHz}$).
- For a target which could be as far away as R_{max}, radar pulses must be at least $2R_{max}/c$ apart.

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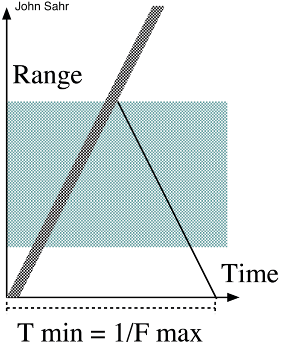
Thus, there is a competition between distance and bandwidth

- $B < F < \frac{c}{2R_{max}}$
- or: $B\frac{2R_{max}}{c} < 1$
- At 450 MHz, $B \sim 40$ kHz, $R \sim 750$ km (5 ms) $\rightarrow$ highly overspread
- Do we get the range right or the spectrum right??



Overspread Targets

How is this resolved?

- Use the fact that the random scattering process from non-overlapping range bins is uncorrelated.
- Construct autocorrelation function estimate, $R(\tau) = \mathcal{F}[P(f)]$

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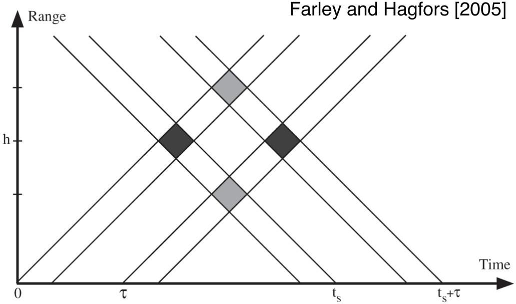
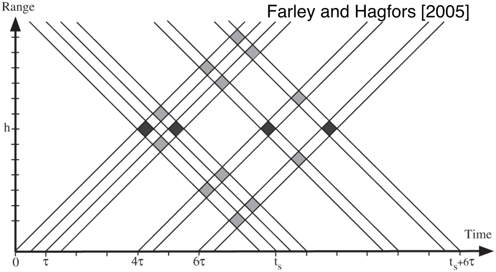
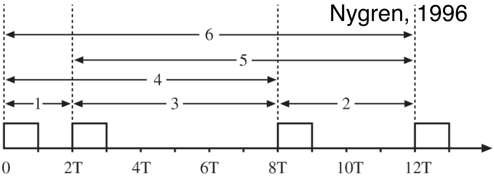
Simplest scheme to measure correlation at a given lag - double pulse:

Farley and Hagfors [2005]



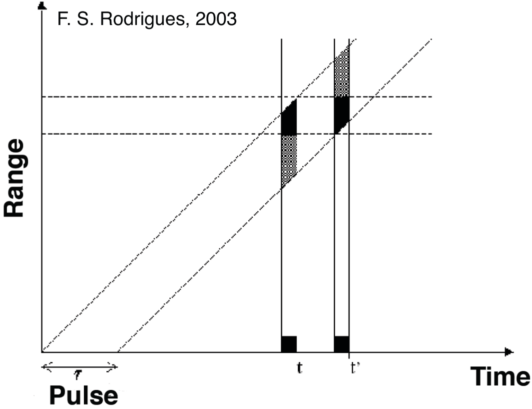
Generalization - Multipulses





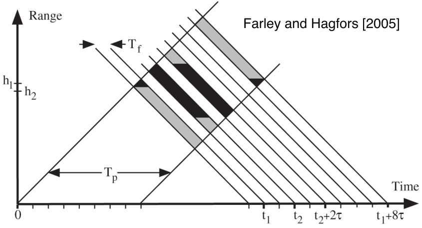
Ambiguity Function

Long-pulse of length τ, sampled at t and t' with a box-car impulse response.



- Range ambiguity function is box-car shaped.
- Lag ambiguity is triangular shaped.

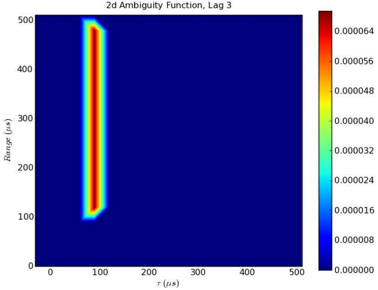
Standard *F*-region Experiment - Long Pulse

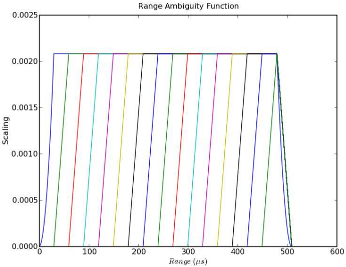


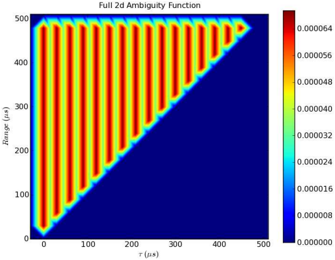
- At high altitudes, use a single long pulse with mismatched filter (oversampled) to measure all lags of the ACF at once
- Sacrifice range resolution
- Typically use a 480 μs pulse (F region) or 1 ms pulse (topside)

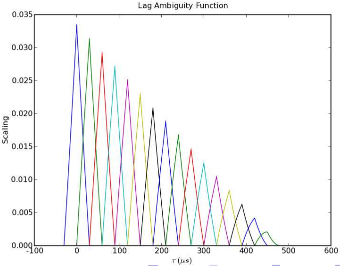
Long Pulse Ambiguity Function

Ambiguity function with a boxcar filter. 480 μs long pulse, 30 μs sampling.



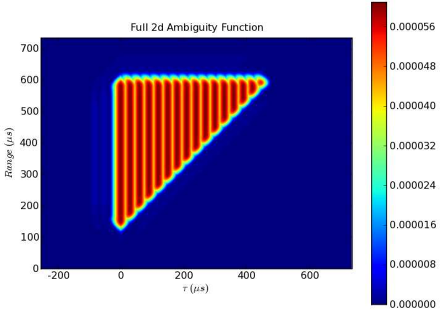


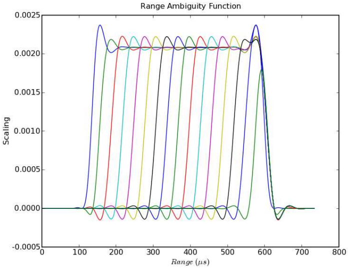


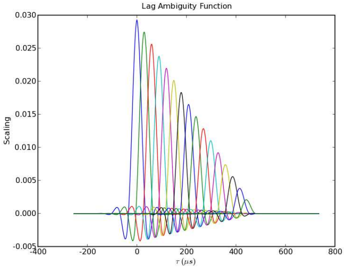


Long Pulse Ambiguity Function

- Ambiguity function including filter effects.
- 480 μs long pulse, 30 μs sampling.
- With filter effects.







Long Pulse Gating

The different lags of the long pulse have very different range ambiguity functions. Is this a problem?

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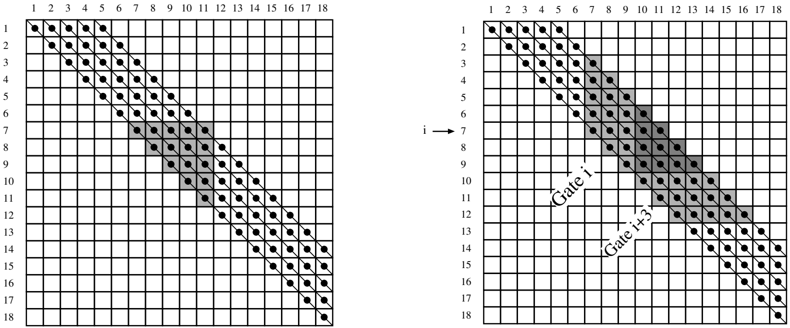
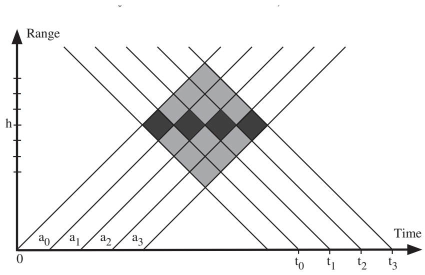
"Simple solution" - Gating using elements of the so-called lag-profile matrix.



Nygren, 1996

A better method - treat as an inverse problem: deconvolution or full profile methodologies. Active area of research.

Standard *E*-region Experiment - Coded Pulse



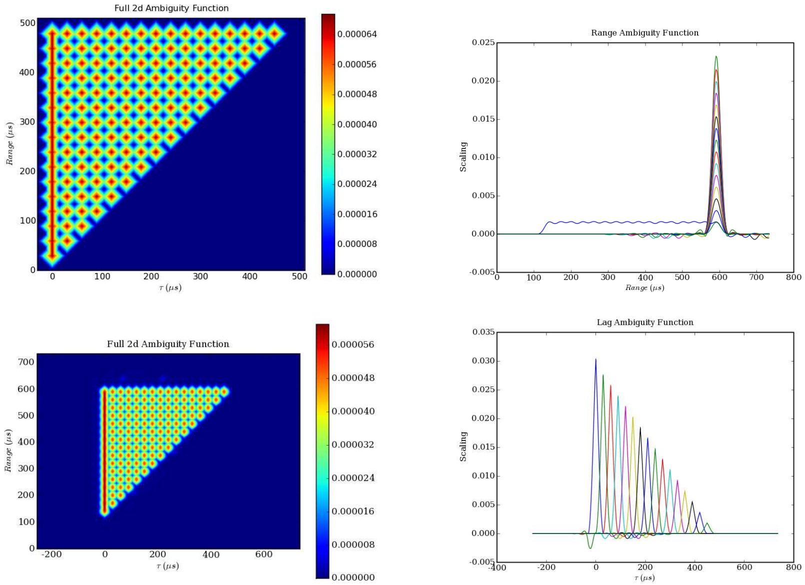
Farley and Hagfors [2005]

E.g., consider lag estimate using $v(t_0)$ and $v^*(t_1)$ - choose a_n such that clutter terms cancel.

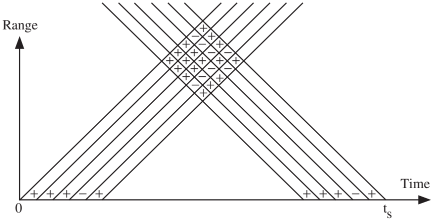
- At lower altitudes, we require better range resolution.
- For this, we utilize binary coded pulse ACF measurements (do not compress pulse or eliminate clutter like BC - eliminate correlation of clutter)
- Random (CLP) or alternating (cyclic codes)
- Standard experiment is 480 μs, 16-baud (4.5 km), randomized strong code.
- Include an uncoded 30 μs pulse for zero-lag normalization.

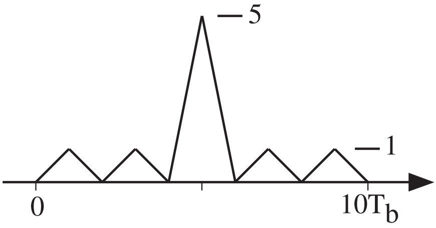
Standard *E*-region Experiment - Ambiguity Function

Ambiguity function including filter effects. 480 μs (16-baud, 30 μs baud, 32 pulse).



Standard E/F-region Power Measurement





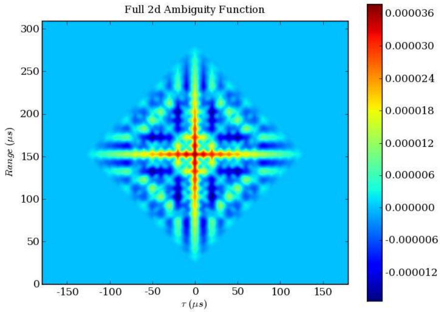
Farley and Hagfors [2005]

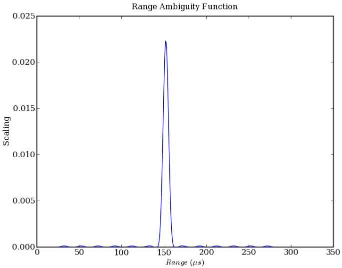
- Pulse compression code allow for high sensitivity, high range resolution power measurements.
- Plasma must remain correlated over pulse length (limits range of use for most systems).
- Typical code is 13-baud Barker code, 130 μs.

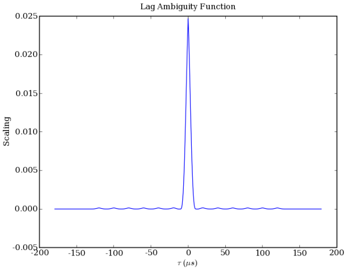
E/F-region Power Measurement - Ambiguity Function

Ambiguity function including filter effects.

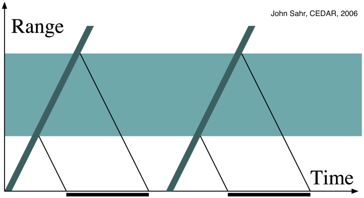
130 μs (13-baud, 10 μs baud, 5 μs sampling).







Standard *D*-region Experiments



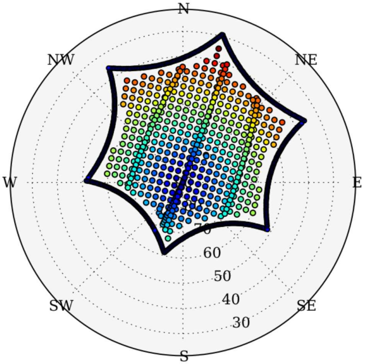
- Long correlation times (narrow spectral widths) in the *D* region require pulse-to-pulse techniques
- We employ coded double-pulse techniques that give range resolutions up to 600 m and spectral resolutions up to 1 Hz.

Mode	Pulse	Baud	δR	τ	IPP	δf	Nyquist	δt
0	130 μs	10 μs	1.5 km	5 μs (0.75 km)	2 ms	2 Hz	250 Hz	1 s
1	260 μs	10 μs	1.5 km	5 μs (0.75 km)	4 ms	1 Hz	125 Hz	2.5 s
2	130 μs	10 μs	1.5 km	5 μs (0.75 km)	2 ms	2 Hz	250 Hz	1.8 s
3	280 μs	10 μs	1.5 km	5 μs (0.75 km)	3 ms	1.3 Hz	167 Hz	2.7 s
4	112 μs	4 μs	0.6 km	2 μs (0.3 km)	3 ms	1.3 Hz	167 Hz	2.7 s

System Information

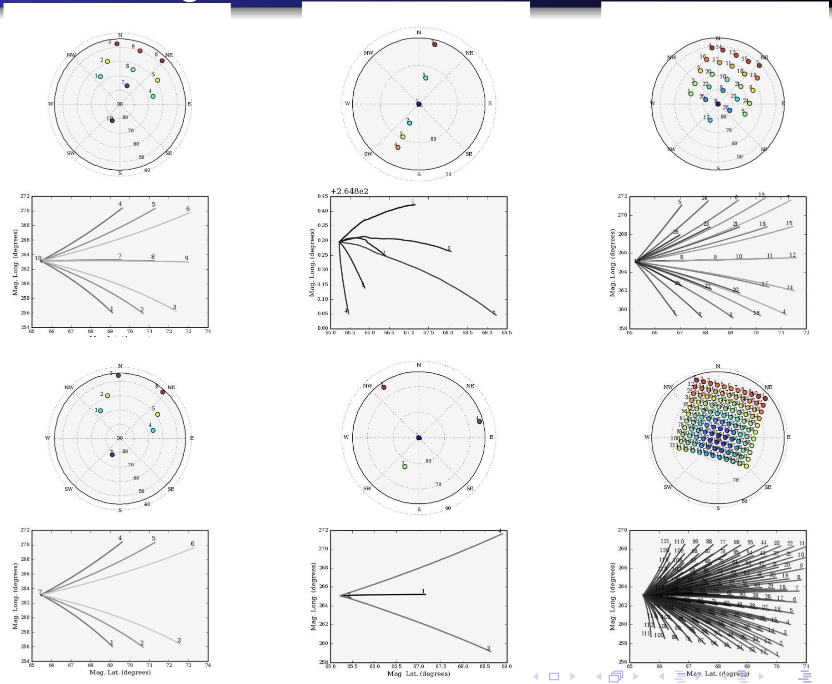
- 128-panel AMISR system (upgraded from 96 in Sep. 07)
- Pulse-to-pulse phase capability
- $\sim$1.6 MW peak Tx (upgraded from $\sim$1.3 MW)
- $\sim$10% max duty cycle
- 4 reception channels
- Tx band 449-450 MHz
- 3.5 MHz max Rx bandwidth
- 4 μs min pulsewidth (freq. allocation limitation)
- Fully programmable, remotely operable/ted
- Graceful degradation - reliable operations

Beam Pointing



- Range of pointing positions within grating lobe limits
- "Normal" experiments include ~1-10 beams
- Main limitation is integration time / sensitivity

Beam Pointing



General

A typical experiment consists of:

- Data samples
- Noise samples
- Cal pulse samples

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- Interleaving of pulses (possibly on different frequencies)
- Clutter considerations, Noise & Cal sample placement
- Maximization of duty cycle
- Beam pointing, Distribution of pulses, Integration time considerations
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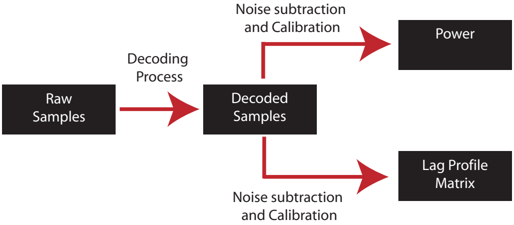

Power Estimation

Received power can be written as

$$P_r = \frac{P_t \tau_p}{r^2} K_{sys} \frac{N_e}{(1 + k^2\lambda_D^2)(1 + k^2\lambda_D^2 + T_r)} \text{ Watts}$$

where

P_r - received power (Watts)
P_t - transmit power (Watts)
τ_p - pulse length (seconds)
r - range (meters)
N_e - electron density (m^{-3})
k - Bragg scattering wavenumber (rad/m)
λ_D - Debye length (m)
T_r - electron to ion temperature ratio
K_{sys} - system constant (m^5/s)

Power Estimation

Received signal power needs to be calibrated to absolute units of Watts. To do this, we in general (a) take noise samples and (b) inject a calibration pulse at each AEU, which is then summed in the same way as the signal. The absolute calibration power in Watts is:

$$P_{cal} = k_B T_{cal} B \quad \text{Watts}$$

where
k_B - Boltzmann constant (J/kg K)
T_{cal} - temperature of calibration source (K)
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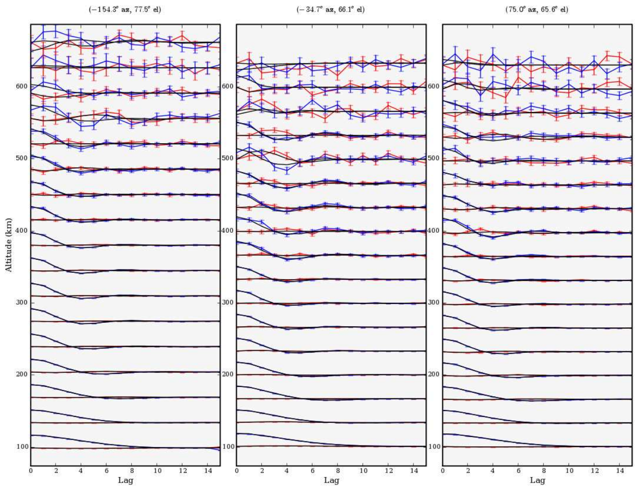
The measurement of the calibration power (after noise subtraction) can then be used as a yardstick to convert the received power to Watts. This is done as,

$$P_r = P_{cal} * (Signal - Noise)/(Cal - Noise) \quad \text{Watts}$$

ACF / Spectra Estimation - E/F region



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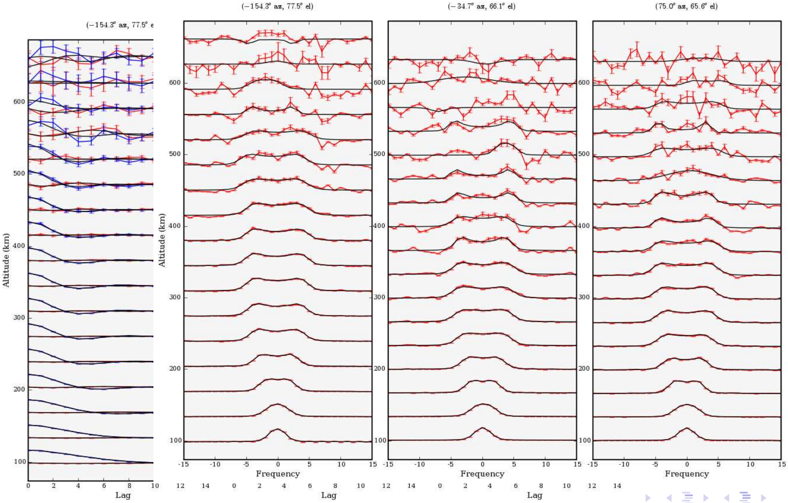
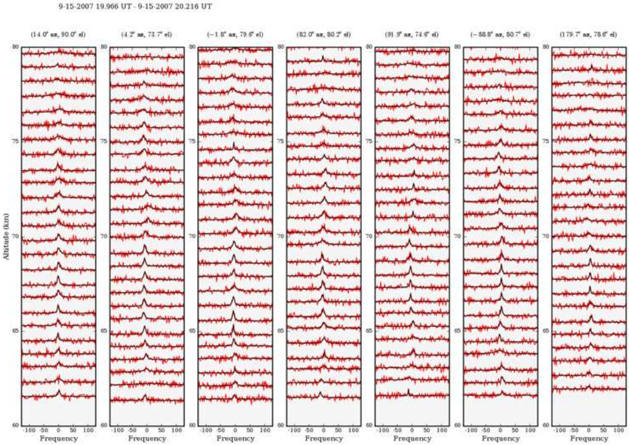


ACF / Spectra Estimation - *D* region



Electron Density

Recall,

$$P_r = \frac{P_t \tau_p}{r^2} K_{sys} \frac{N_e}{(1 + k^2\lambda_D^2)(1 + k^2\lambda_D^2 + T_r)} \text{ Watts}$$

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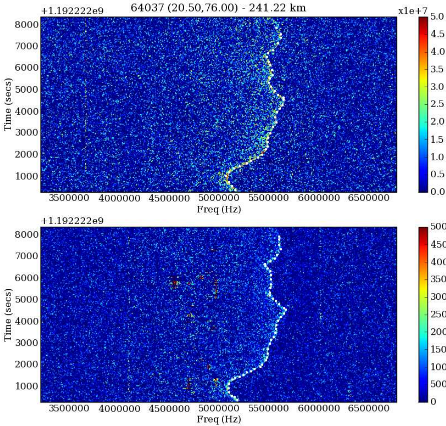
Within K_{sys} is embedded information on the gain, which for a phased-array varies with the look-angle off boresight, as well as the proximity to the grating lobe limits.

Electron Density

$$f_r^2 \approx f_p^2 + \frac{3k^2}{4\pi^2}\frac{k_B T_e}{m_e} + f_c^2 \sin^2 \alpha$$

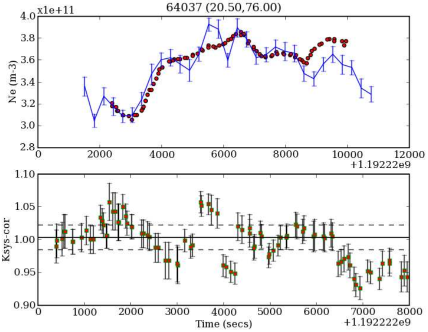
where

f_r - plasma line frequency (Hz)
f_p - plasma frequency (Hz)
T_e - electron temperature (K)
m_e - electron mass (kg)
f_c - electron cyclotron frequency (Hz)
α - magnetic aspect angle



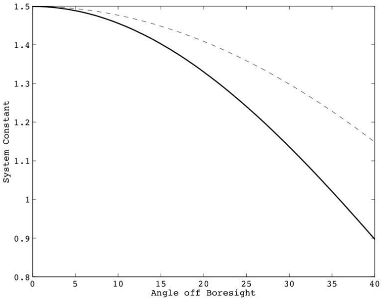
Electron Density

$$K_{sys} = A\cos^{B}(\theta_{BS}) \quad \mathrm{m}^5/\mathrm{s}$$

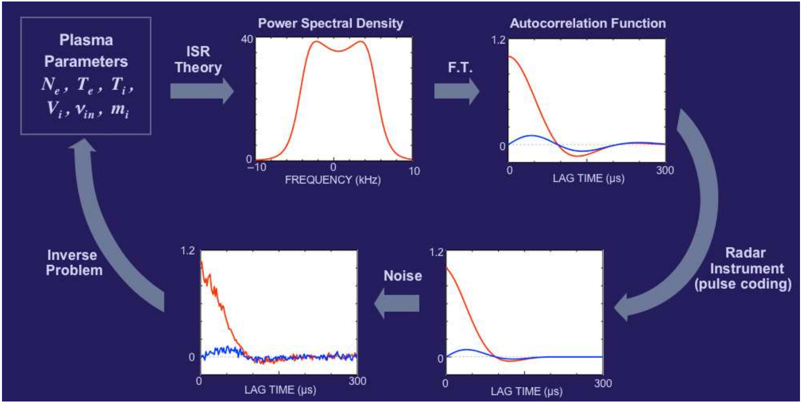


θ_{BS} - angle off boresight

A, B - constants



Fitting Spectra



Fitting Spectra

General Complicating Factors:

- Range smearing
- Lag smearing
- Pulse coding effects / "Self"-clutter
- Clutter (geophysical and not - e.g., mountains, irregularities, turbulence, non-Maxwellian)
- Signal strength / statistics
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Specific Based on Altitude:

- *F*-region/Topside - Light ion composition
- Bottomside - Molecular ion composition
- *E*-region - Collision frequency, Temperature
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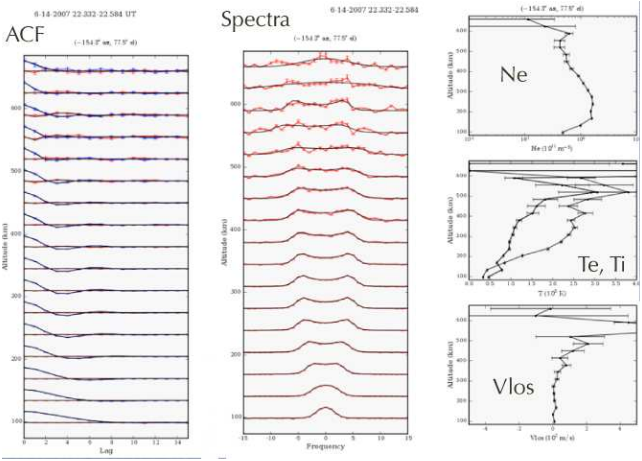
Approach:

- F-region - T_e, T_i, v_{los}, N_e
- Bottomside - Assume a composition profile
- E-region - $<\sim 105 km$, assume $T_e = T_i$
- D-region - Fit a Lorentzian (width, Doppler, N_e)

Fitting Spectra - Example



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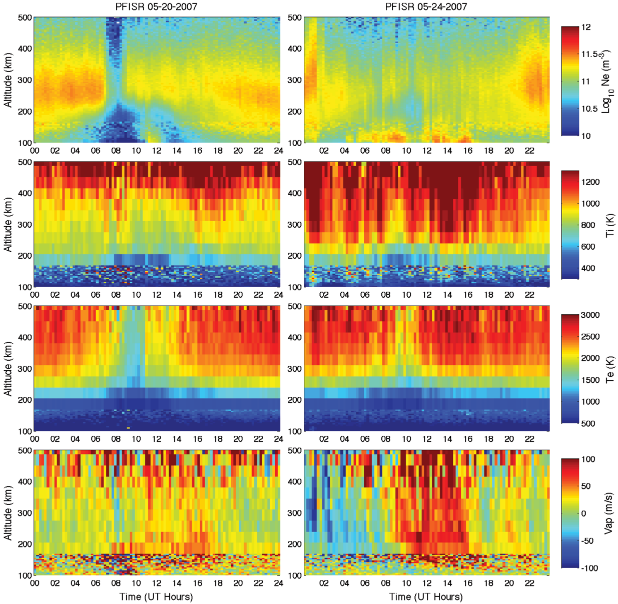

Vector Velocities - Preliminaries

LOS Velocity measurement can be represented as:

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where the radar **k** vector in geographic coordinates is:

$$\mathbf{k} = \begin{bmatrix} k_e \\ k_n \\ k_z \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} R^{-1}$$

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If we can neglect Earth curvature ("high enough" elevation angles),

$$\mathbf{k} = \begin{bmatrix} k_e \\ k_n \\ k_z \end{bmatrix} = \begin{bmatrix} \cos\theta\sin\phi \\ \cos\theta\cos\phi \\ \sin\theta \end{bmatrix}$$

where θ, ϕ are elevation and azimuth angles, respectively.

Vector Velocities - Preliminaries

For a local geomagnetic coordinate system we can use the rotation matrix,

$$R_{geo \rightarrow gmag} = \begin{bmatrix} \cos\delta & -\sin\delta & 0 \\ \sin I \sin\delta & \cos\delta \sin I & \cos I \\ -\cos I \sin\delta & -\cos I \cos\delta & \sin I \end{bmatrix}$$

where δ ($\sim 22^\circ$) and I ($\sim 77.5^\circ$) are the declination and dip angles, respectively.

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where δ ($\sim 22^\circ$) and I ($\sim 77.5^\circ$) are the declination and dip angles, respectively. Then,

$$\mathbf{k} = \begin{bmatrix} k_{pe} \\ k_{pn} \\ k_{ap} \end{bmatrix} = \begin{bmatrix} k_e \cos\delta - k_n \sin\delta \\ k_z \cos I + \sin I (k_n \cos\delta + k_e \sin\delta) \\ k_z \sin I - \cos I (k_n \cos\delta + k_e \sin\delta) \end{bmatrix}.$$

Vector Velocities - Two Point

Two LOS velocity measurements can be written as,

$$\begin{bmatrix} v_{los}^1 \\ v_{los}^2 \end{bmatrix} = \begin{bmatrix} k_{pe}^1 & k_{pn}^1 & k_{ap}^1 \\ k_{pe}^2 & k_{pn}^2 & k_{ap}^2 \end{bmatrix} \begin{bmatrix} v_{pe} \\ v_{pn} \\ v_{ap} \end{bmatrix}$$

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Can be solved for v_{pn} and v_{pe} assuming $v_{ap} \approx 0$,

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Vector Velocities - Two Point

Two LOS velocity measurements can be written as,

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Implies that you need look directions with different **k** vectors.

Vector Velocities - Generalization

Multiple measurements can be written as,

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or

$$\mathbf{v}_{los} = A\mathbf{v}_i + \mathbf{e}_{los}$$

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Treat $\mathbf{v}_i$ as a Gaussian random variable (Bayesian), use linear theory to derive a least-squares estimator. $\mathbf{v}_i$ zero mean, Σ_v (*a priori*). Measurements zero mean, covariance Σ_e. Solution,

$$\hat{\mathbf{v}}_i = \Sigma_v A^T (A \Sigma_v A^T + \Sigma_e)^{-1} \mathbf{v}_{los}$$

Error covariance,

$$\Sigma_{\hat{v}} = \Sigma_v - \Sigma_v A^T (A \Sigma_v A^T + \Sigma_e)^{-1} A \Sigma_v = (A^T \Sigma_e^{-1} A + \Sigma_v^{-1})^{-1}$$

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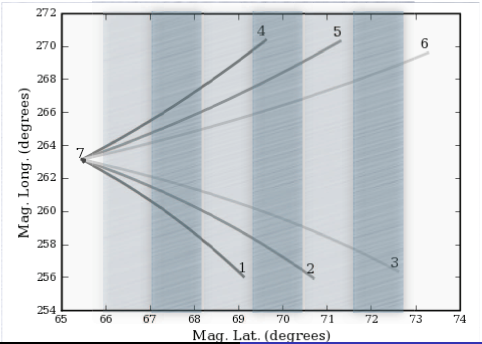
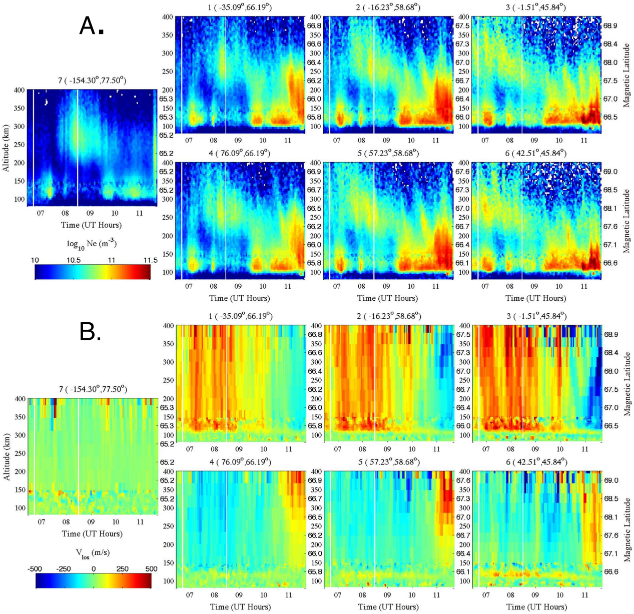


Electric Fields - Example

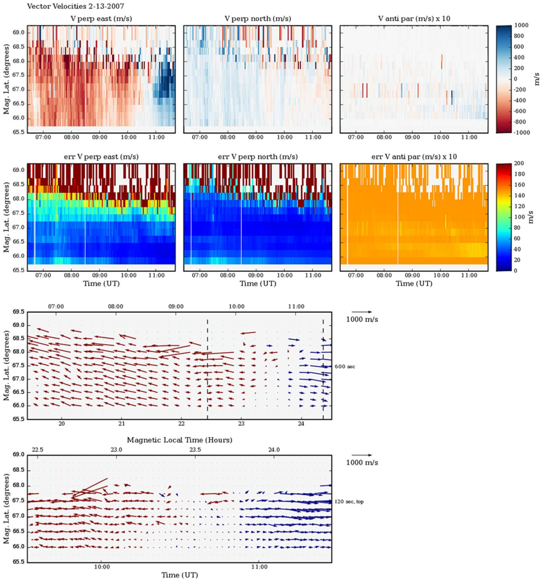


Electron Density

LOS Velocities

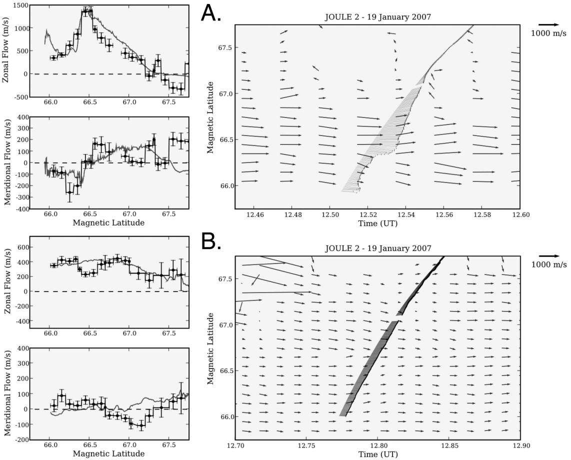
Electric Fields - Example

Resolved Vectors



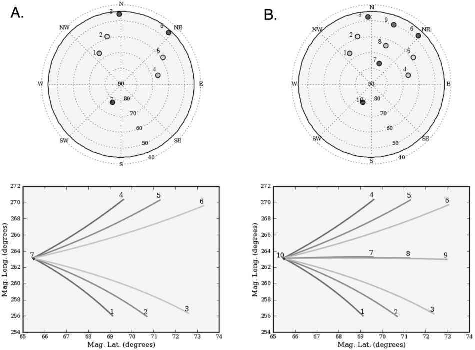
Electric Fields - Example

Comparison to rocket-measured E-fields.



Experiment Planning

The approach also allows for an efficient means of experiment planning, since the output covariance of the measurements is independent of the actual measurements.



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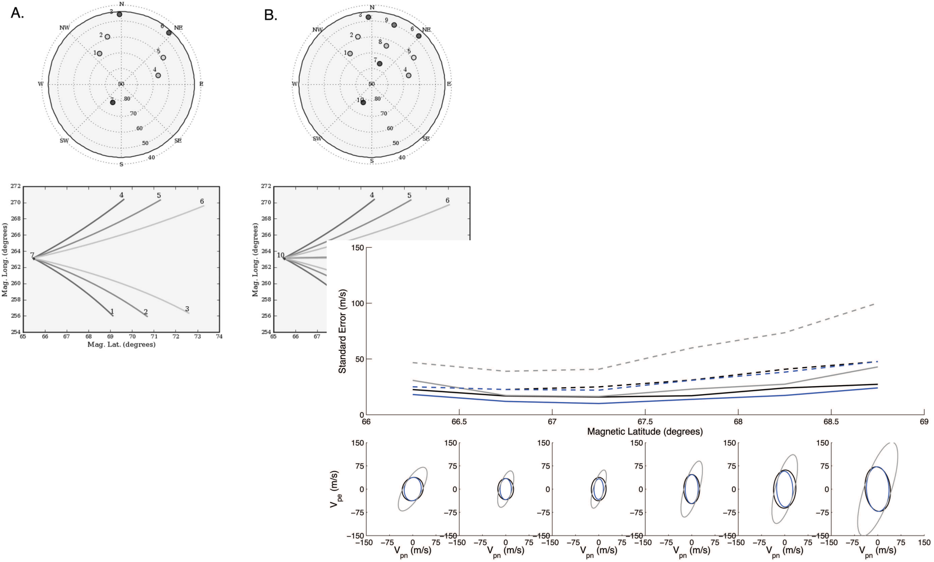

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Defining the matrix C as,

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$$\mathbf{v}_i = b_i C \mathbf{E} + C \mathbf{u}$$

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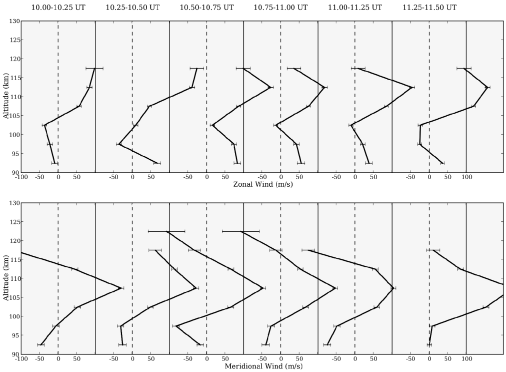
This allows for direct constraint of both the vertical wind and the parallel electric field, both of which we expect to be small.

$$\Sigma_v^{gmag} = J_{geo \rightarrow gmag} \Sigma_v^{geo} J^T_{geo \rightarrow gmag}$$

E-Region Winds - Example



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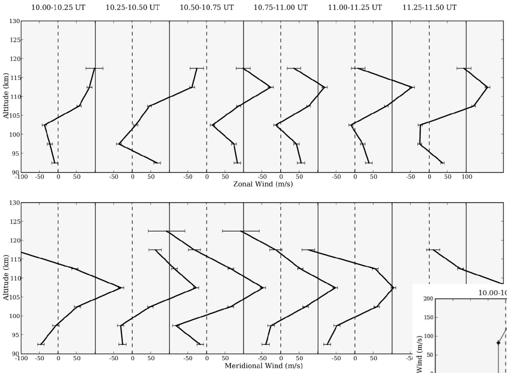

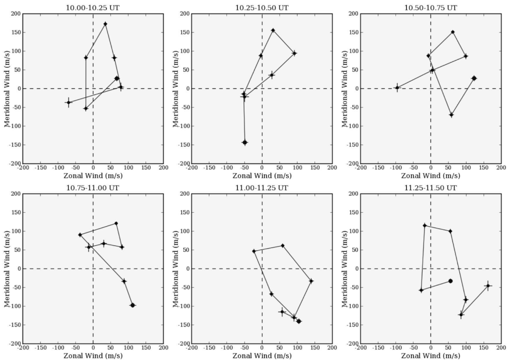

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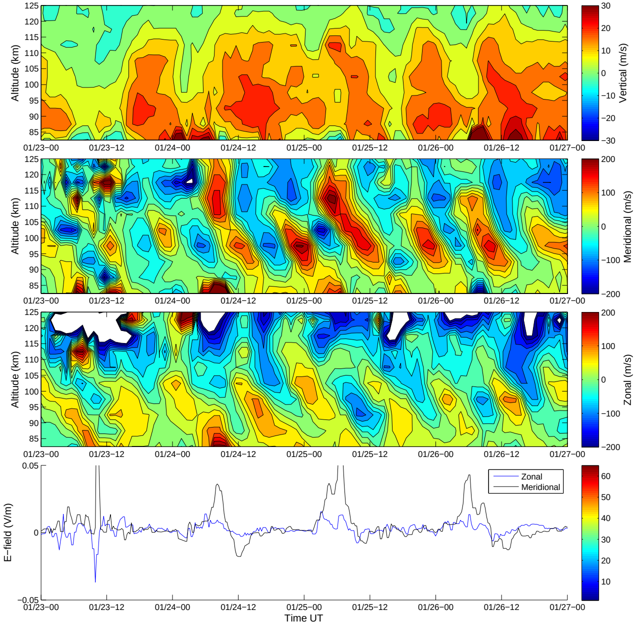

Collision Frequency

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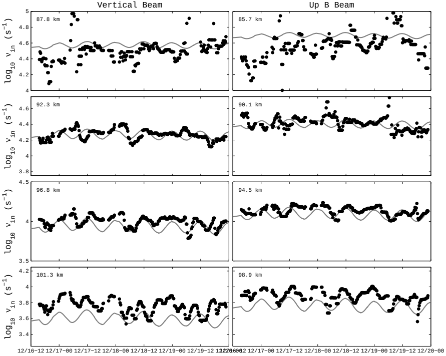
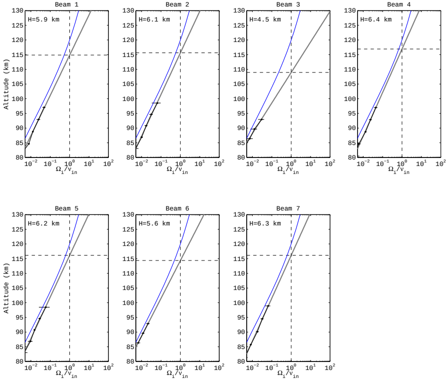
1. Direct fits at lower altitudes (spectral width $\sim\propto T_n/\nu_{in}$)
2. Examination of variation of LOS velocity with altitude

Collision Frequency - Method 1

Semi-diurnal variation over several days.



Altitude profile and extrapolation.



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Under strong convection (electric field) conditions, neglect winds

$$v_z' \sim b_i (1+\kappa_i^2)^{-1} \left[\kappa_i E_{\perp e} + E_{\perp n} \right] \cos I$$

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If $\kappa_i(z) = \kappa_0 e^{(z - z_0)/H}$, vertical ion velocity will maximize at

$$z_{\max\ v_z'} = z_0 + H \ln \kappa_0^{-1} + H \ln \left[\frac{\cos\alpha \pm 1}{\sin\alpha}\right]$$

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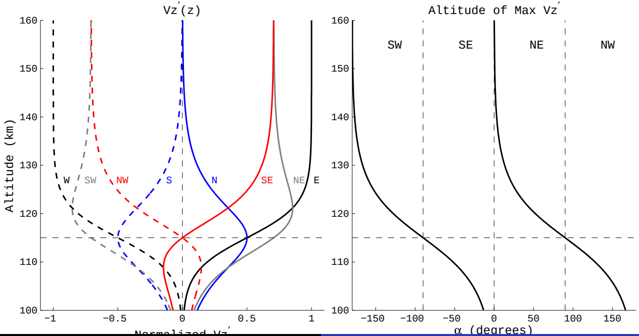
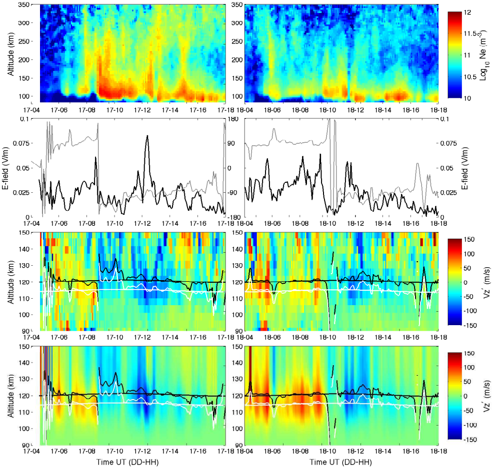
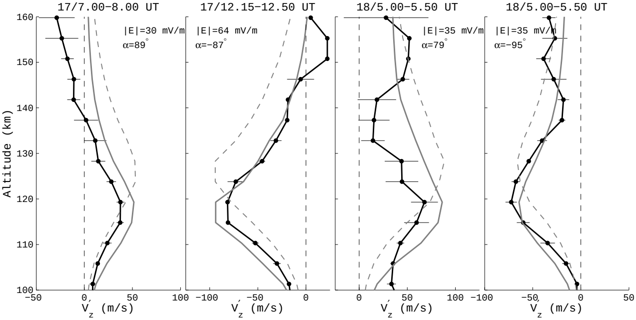


Collision Frequency - Method 2

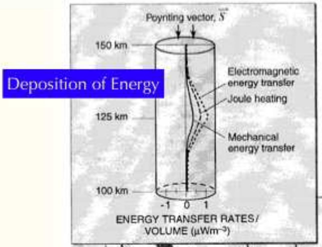


Collision Frequency - Method 2

Profiles of v_z' during high convection conditions.
Dashed - with MSIS; Solid - scaled by a factor of 2.



Conductivities / Currents / Joule Heating Rates

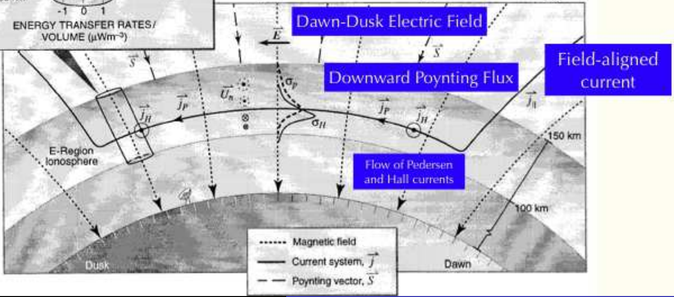


- Supplied electromagnetic energy is converted to heat (Joule heating) or momentum (Lorentz forcing)

$$q_j = \mathbf{j} \cdot \mathbf{E}' = \mathbf{j} \cdot (\mathbf{E} + \mathbf{U} \times \mathbf{B}) = \sigma_P |\mathbf{E} + \mathbf{U} \times \mathbf{B}|^2$$

$$q_m = \mathbf{U} \cdot (\mathbf{j} \times \mathbf{B})$$

- q_m negative - neutral gas is a sink (load)
- q_m positive - neutral gas is a source (generator)



Thayer [2000]

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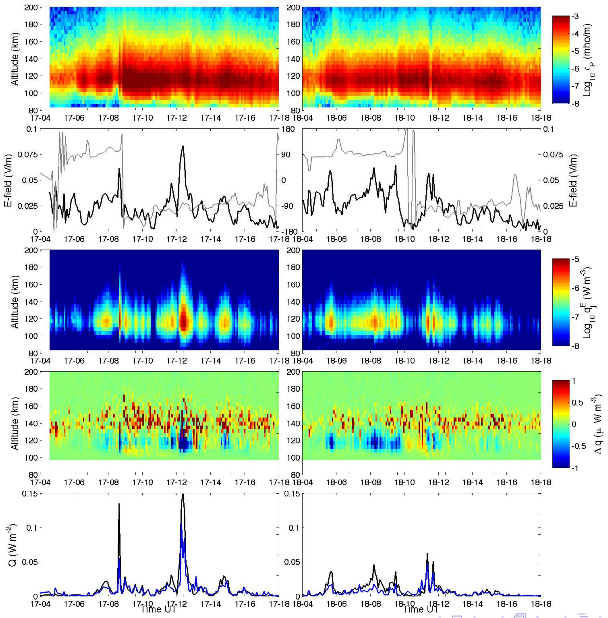
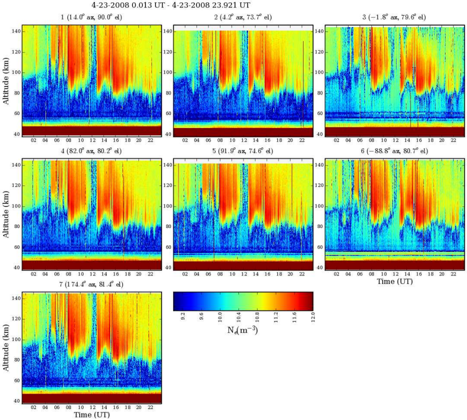


D-Region Parameters - Raw Power and Spectra



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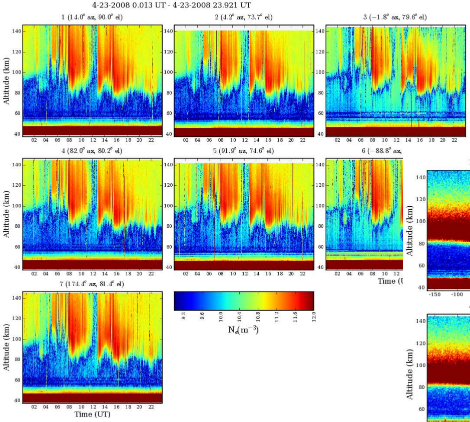
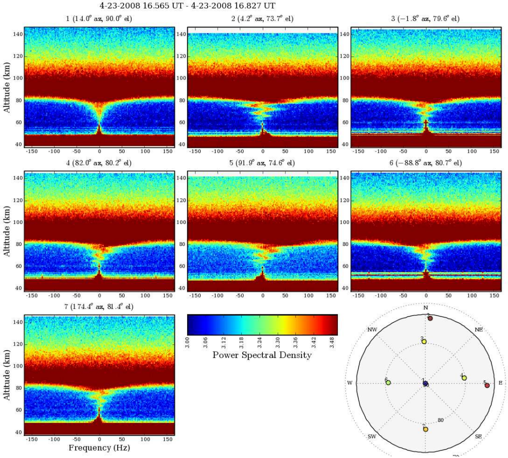
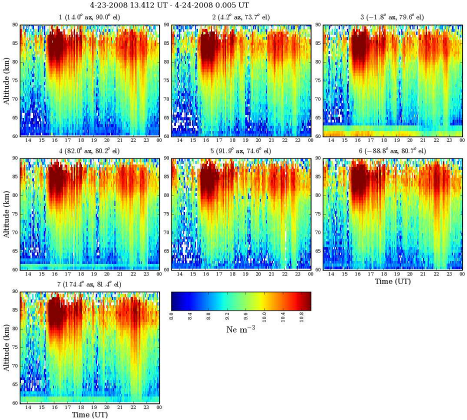




D-Region Parameters - N_e and Spectral Widths



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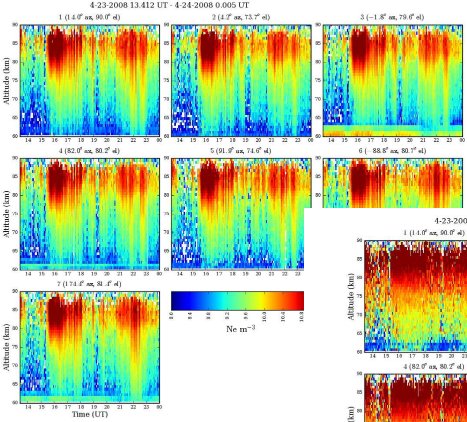
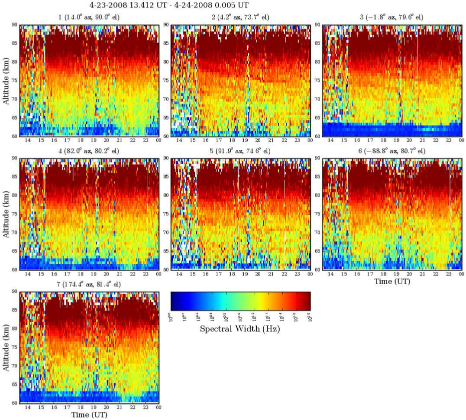
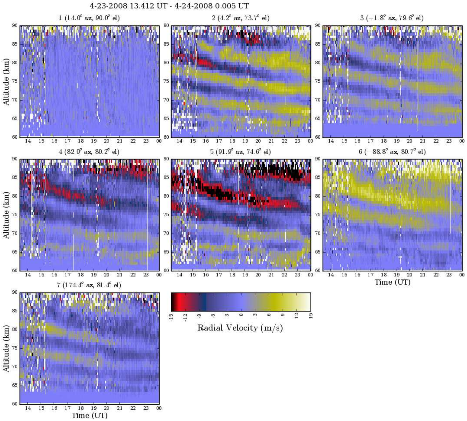




D-Region Parameters - Velocities and Winds



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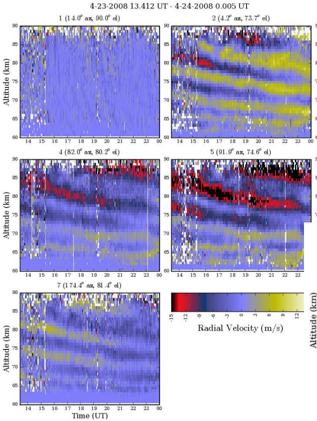

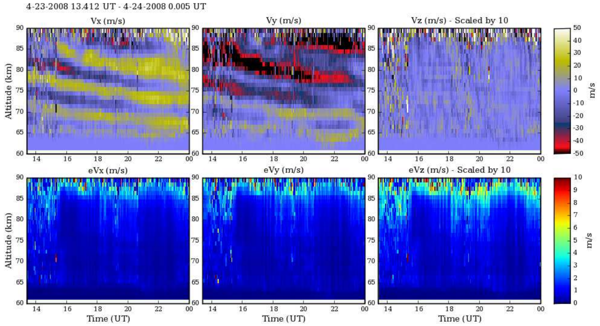

Future

1. Move towards full profile techniques
2. Take advantage of space and time information
3. Standardize approaches
4. Molecular ion composition, height-resolved plasma lines, topside parameters, etc.
5. Make these products available to interested users
6. Extend our arsenal of products (e.g., *D*-region momentum fluxes, higher altitude winds, etc.)