Introduction to Radar Studies of the D-region and Mesosphere

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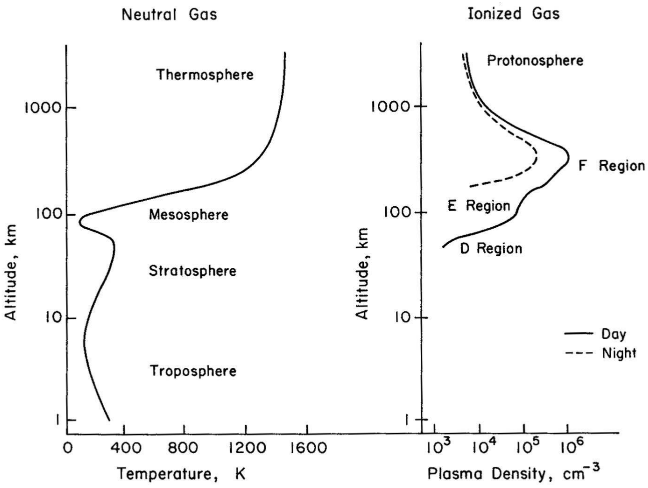
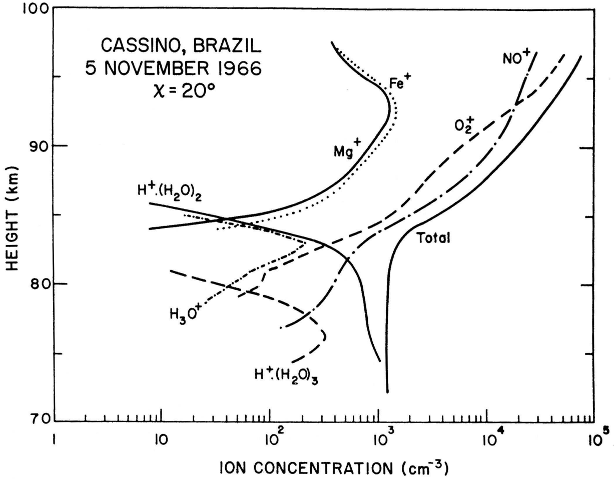
Layers of the Atmosphere



[Kelley, 2009]

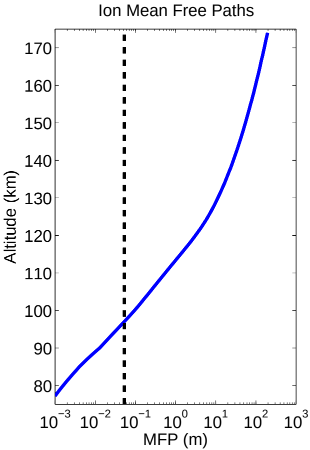
D-region Composition



[Narcisi *et al.*, *JATP*, 1972]

D-region Properties

- Weakly ionized
    - $N_n \approx 10^{20}\mathrm{m}^{-3}$
    - $N_e \approx 10^{9}\mathrm{m}^{-3}$
- Collisionally dominated
- Temperatures of all species are identical
- Ion velocity = Neutral velocity



Incoherent Scatter from Collisional Plasmas

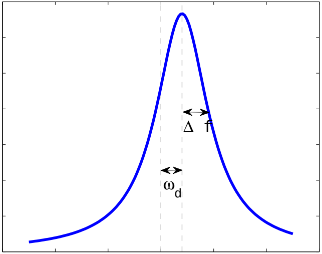
$$\text{If } MFP \ll \frac{\lambda_R}{4\pi}$$

$$\sigma(\omega_0 + \omega)d\omega = \frac{N_e r_e^2}{2\pi} \frac{2\pi\Delta f}{(2\pi\Delta f)^2 + (\omega - \omega_d)^2} d\omega$$

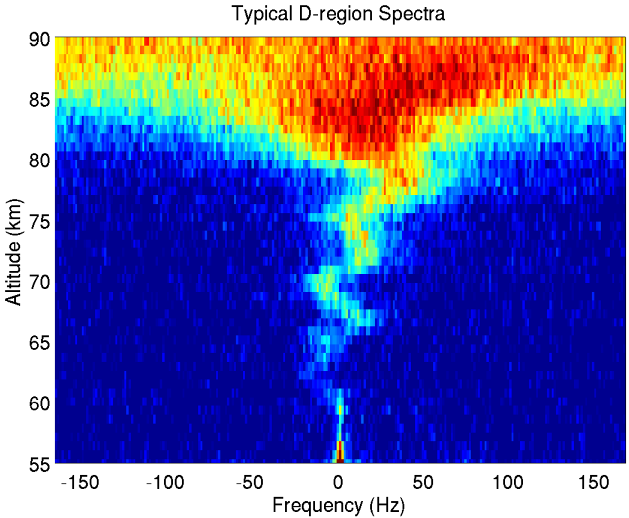
$$R(\tau) \propto \exp\left(-2\pi\Delta f\,|\tau|\right)$$

$$\Delta f = \frac{16\pi k_B T}{\lambda_R^2 m_i \nu_{in}}$$

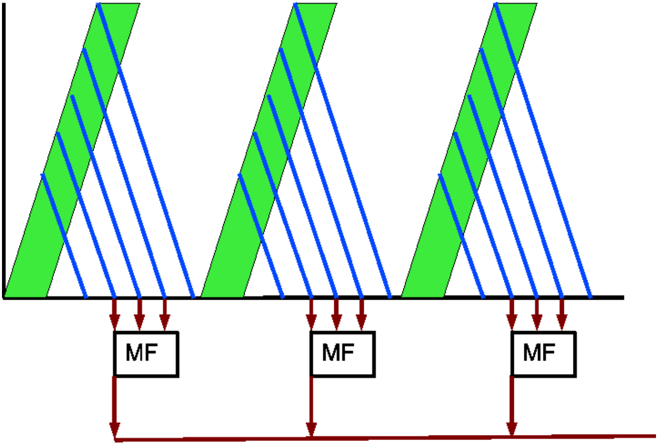
[Dougherty and Farley, *JGR*, 1963]



Example Spectrum

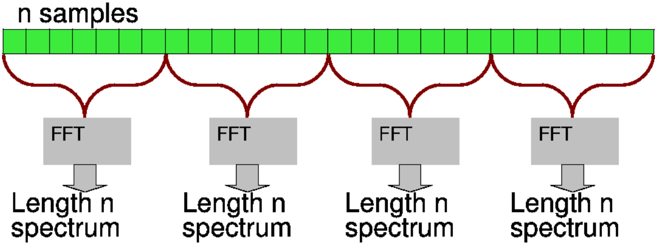


Canonical Underspread Experiment



The matched filter outputs form a time series for each altitude sampled once per IPP

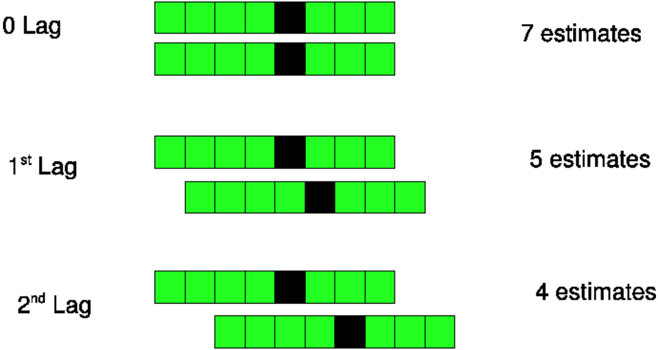
Pulse to Pulse Spectra



- Nyquist Frequency $= \frac{1}{2}\frac{1}{IPP}$
- Frequency Resolution $= \frac{1}{nIPP}$

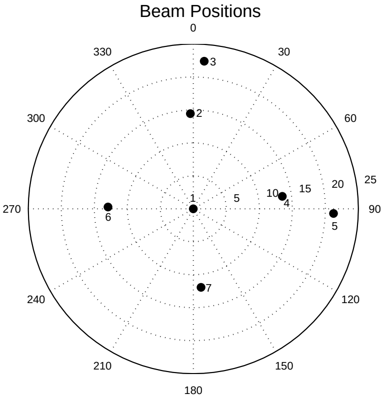
Pulse to Pulse Correlation

Direct computation of the ACF provides a good way to deal with missing pulses

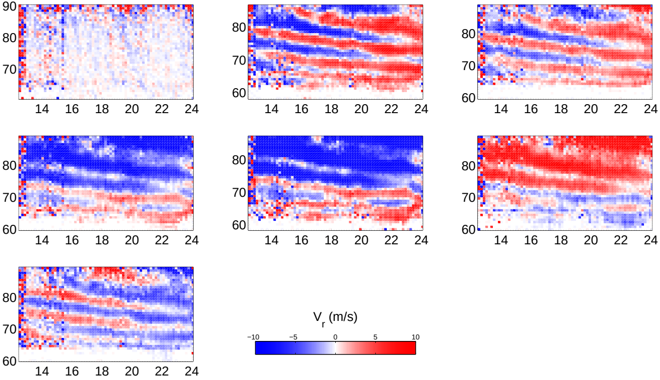


Typical PFISR D-region modes

- Transmit Barker codes or similar binary phase codes
- IPP= 2-4 ms (250-125 Hz Nyquist)
- 128 consecutive pulses in each beam position
- 1-3 Hz spectral resolution



Radial Velocities



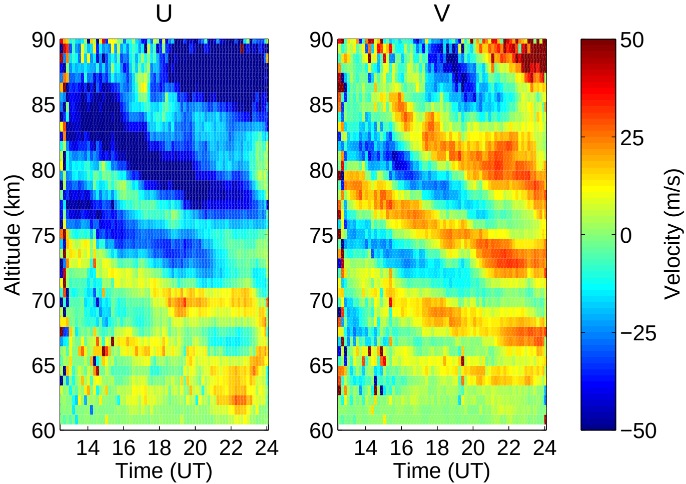
Vector Velocities

$$\begin{aligned} \theta_i &= \text{Elevation} \\ \phi_i &= \text{Azimuth} \\ V_{r,i} &= \text{Radial Velocity} \end{aligned}$$

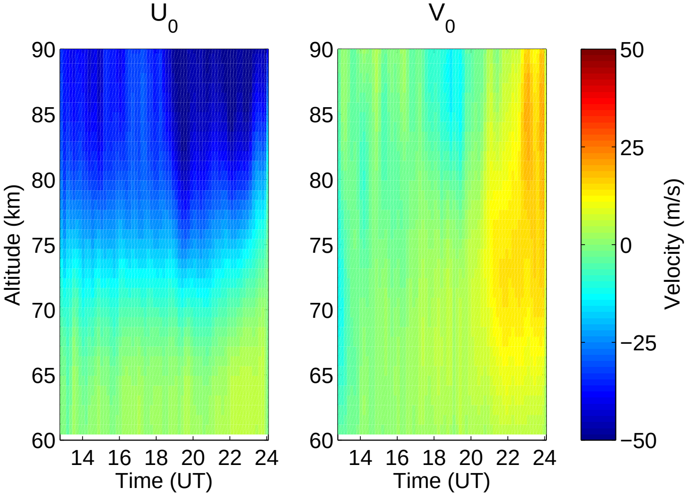
$$\begin{pmatrix} V_{r,1} \\ \vdots \\ V_{r,7} \end{pmatrix} = \begin{pmatrix} \cos(\theta_1)\sin(\phi_1) & \cos(\theta_1)\sin(\phi_1) & \sin(\theta_1) \\ \vdots & \vdots & \vdots \\ \cos(\theta_7)\sin(\phi_7) & \cos(\theta_7)\sin(\phi_7) & \sin(\theta_7) \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$$\begin{aligned} \mathbf{V}_r &= \mathbf{D}\mathbf{U} \\ \mathbf{U} &= (\mathbf{D}^{\mathrm{T}}\mathbf{D})^{-1}\mathbf{D}^{\mathrm{T}}\mathbf{V_r} \end{aligned}$$

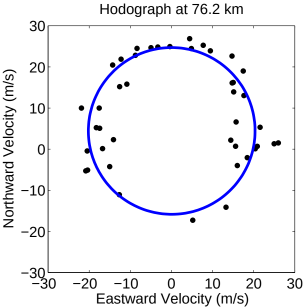
Vector Velocities

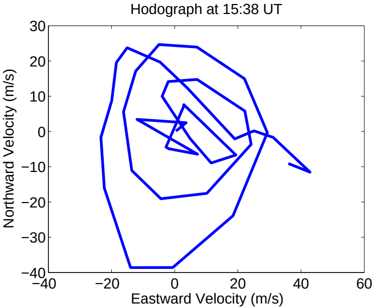


Background Winds



Hodographs


Hodograph at 76.2 km
Northward Velocity (m/s)
Eastward Velocity (m/s)
30
20
10
0
−10
−20
−30
−30
−20
−10
0
10
20
30


Hodograph at 15:38 UT
Northward Velocity (m/s)
Eastward Velocity (m/s)
30
20
10
0
−10
−20
−30
−40
−40
−20
0
20
40
60

Spectral Widths

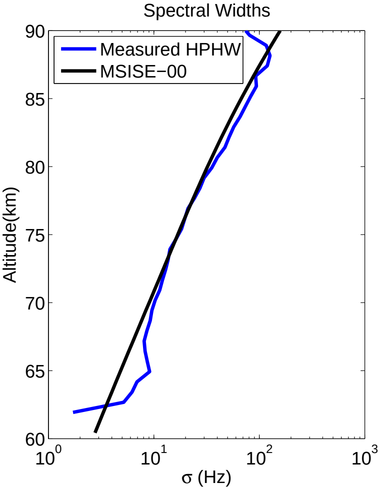
$$\sigma = \frac{16\pi k_B T}{\lambda_R^2 m_i \nu_{in}}$$

$$T = \mathrm{Temperature}$$

$$m_i = \mathrm{Ion\ Mass}$$

$$\nu_{in} = \mathrm{Ion-neutral\ collision\ frequency}$$

$$\nu_{in} \propto \mathrm{Density}$$



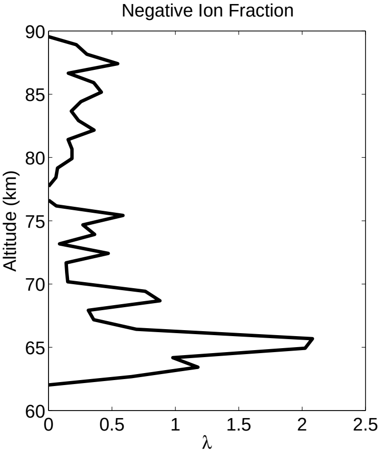
Negative Ions

- Negative ions enhance electron diffusion.
- The broadening above 70 km is probably due to different sources (e.g. turbulence).

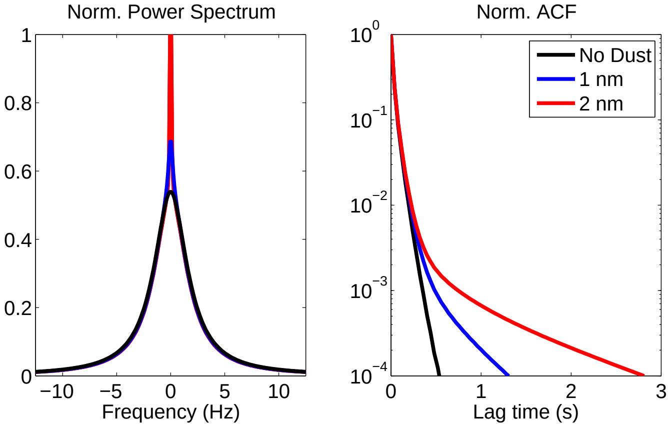
$$\sigma = \frac{16\pi k_B T}{\lambda_R^2 m_i \nu_{in}}(1+\lambda)$$

$$\lambda = \frac{N_-}{N_e}$$

[Matthews, *JGR*, 1978]

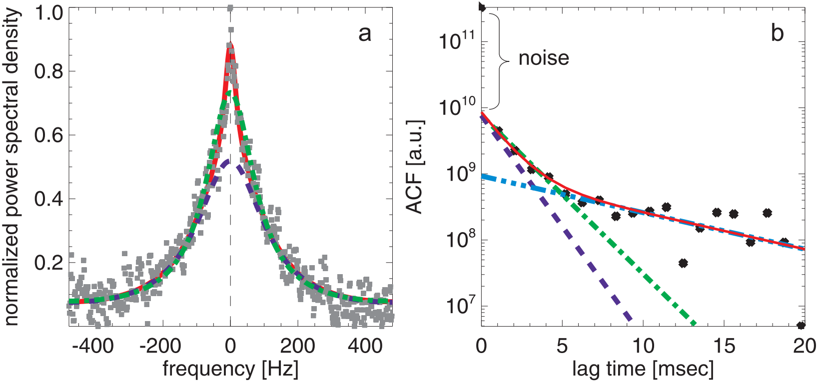


Dust Effects on ISR Spectra



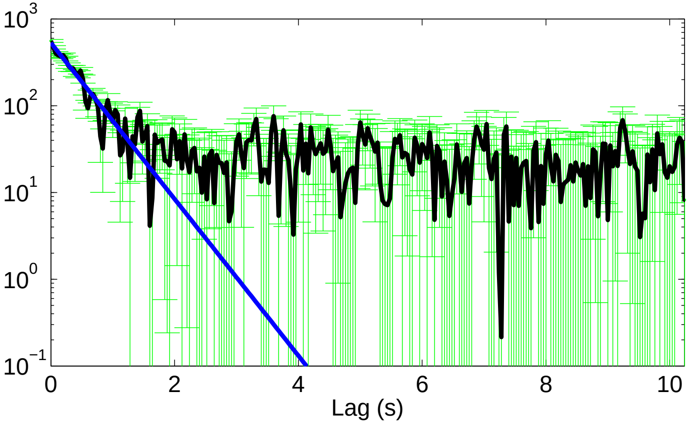
[Cho *et al.*, *JGR*, 1998]

Arecibo Measurements of Meteoric Smoke

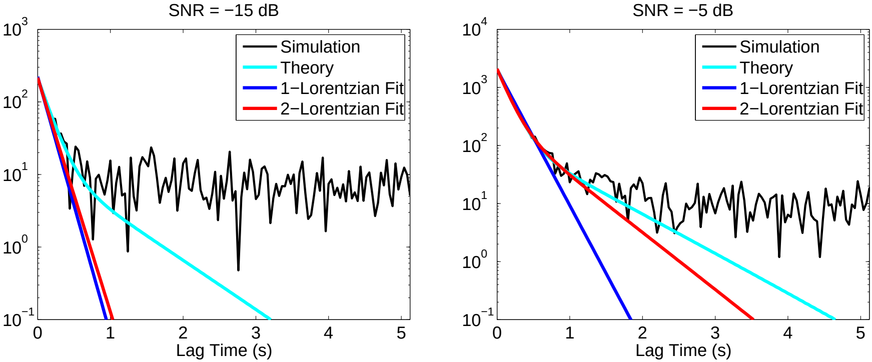


[Strelnikova *et al.*, *GRL*, 2007]

My Efforts at Jicamarca



Simulated Autocorrelation Functions



Why can Arecibo Detect Dust?

Arecibo

- $SNR = -15\mathrm{dB}$
- IPP $=$ 1.04 ms
- 256 point spectra
- 0.266 s per spectrum
- 13521 spectra per hour of integration

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Jicamarca

- $SNR = -15$ to -10dB
- IPP $=$ 40 ms
- 256 point spectra
- 10.24 s per spectrum
- 4 channels
- 1406 spectra per hour of integration

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Jicamarca

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To match the statistical accuracy which Strelnikova et al. (2007) achieves with 1 h 40 min of integration at Arecibo, we would need to integrate for 16 hours!!

Statistical Model of Turbulence

Imagine a passive scalar tracer, ξ, of a turbulent flow (i.e. potential temperature, electron density, index of refraction) is a random process.

This process could be described by an autocorrelation function

$$R_\xi(\mathbf{r}) = \langle \xi(\mathbf{x}) \xi(\mathbf{x} - \mathbf{r}) \rangle$$

or by a 3-D power spectrum

$$\Phi_\xi(k_x, k_y, k_z) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_\xi(\mathbf{r}) e^{(-i r_x k_x - i r_y k_y - i r_z k_z)} \, dr_x \, dr_y \, dr_z$$

1-D Power Spectra

Often a 1-D version of the power spectrum is used instead. There are two different definitions:
The sum of $\Phi_\xi(\mathbf{k})$ over spherical shells of constant k:

$$E_\xi(k) = 4\pi k^2 \Phi_\xi(k)$$

or the fluctuations that would be seen by a probe traveling in a straight line:

$$F_\xi(k_x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Phi_\xi(k_x, k_y, k_z)\, dk_y\, dk_z$$

Kolmogorov Theory

- Inertial subrange

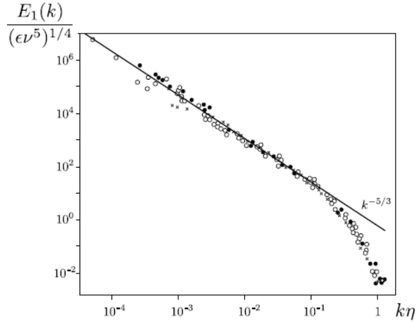
$$E(k) \propto k^{-\frac{5}{3}}$$

- Viscous subrange

$$E(k) \propto k^{-7}$$

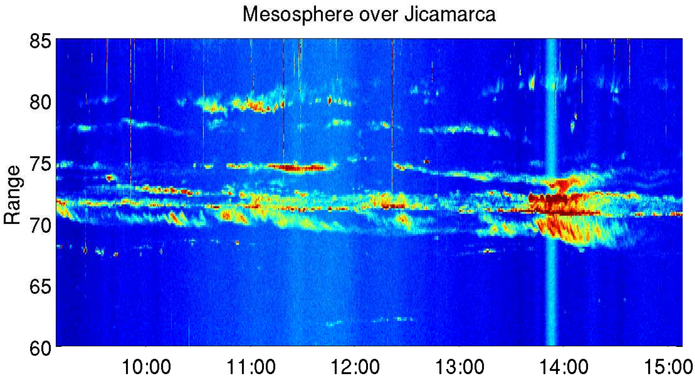
- Transition point is the Kolmogorov microscale

$$\eta_K = \left(\frac{\nu^3}{\epsilon}\right)^{\frac{1}{4}}$$



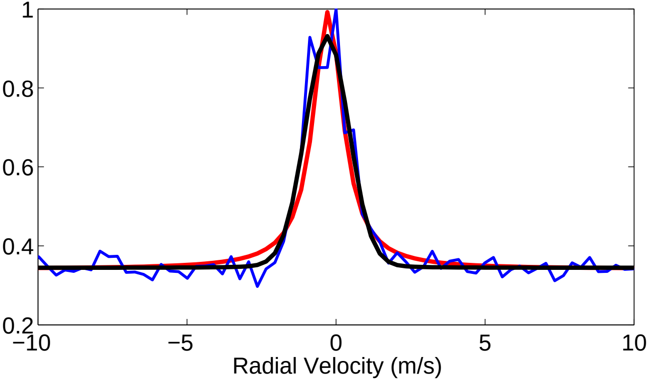
Coherent Scatter from Turbulence

$$\eta(k_B) = \frac{\pi^2}{2} k_B^4 \Phi_n(k_B) \qquad k_B = \frac{4\pi}{\lambda_R}$$

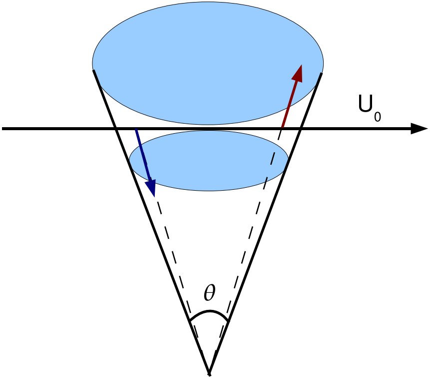


Radar Spectra and Energy Dissipation Rates

$$\epsilon = 0.47 \frac{(\lambda_R/2)^2 \Delta f^2 N}{2 \ln 2}$$



Beam Broadening



$$\Delta f'^2 = \Delta f^2 + \Delta f_B^2$$

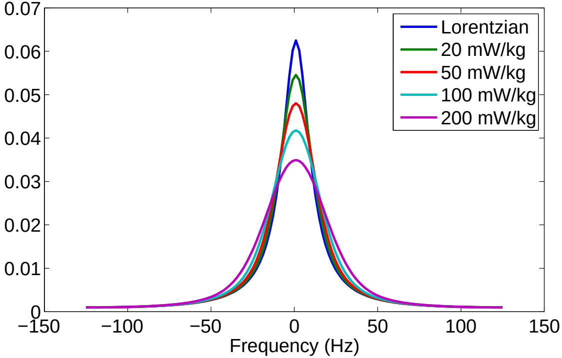
$$\Delta f_B \approx \frac{2}{\lambda_R} \theta U_0$$

[Hocking, *Radio Sci.*, 1985]
See also:
[Sloss and Atlas, *J. Atmos. Sci.*, 1968]
[Nastrom, *Ann. Geophys.*, 1997]

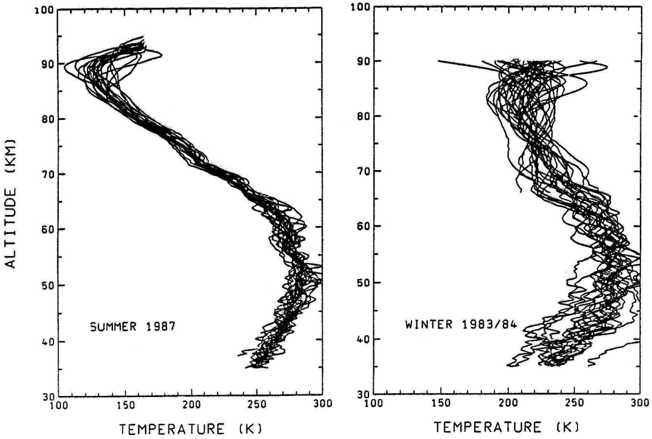
Turbulence Effects on ISR spectra

The ISR spectrum is the convolution of a Lorentzian and Gaussian

$$V(\omega) = L(\omega) * G(\omega)$$



Mesospheric Temperature Profiles

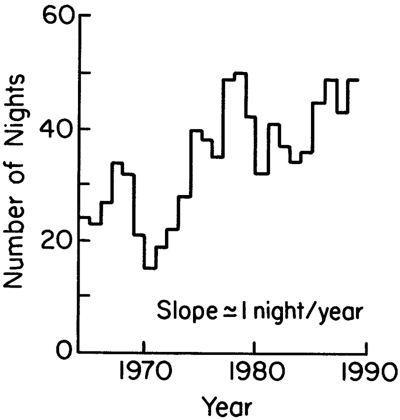


[Lübken and von Zahn, *JGR*, 1991]

Noctilucent Clouds

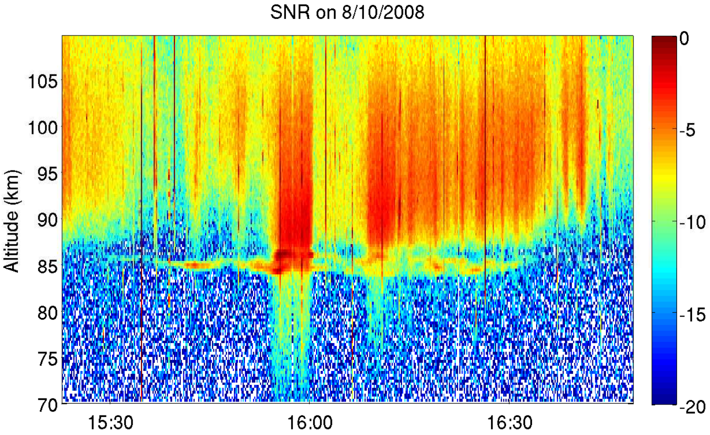
NLCs and Global Change

- First NLC observation was in 1885, after the eruption of Krakatoa in 1883.
- NLCs were observed over England in 1908 after a comet struck Siberia.
- NLC occurrences have been steadily increasing.
- Correlated with increased atmospheric $\mathrm{CO_2}$ and $\mathrm{CH_4}$.
- The Space Shuttle also releases a large amount of water.

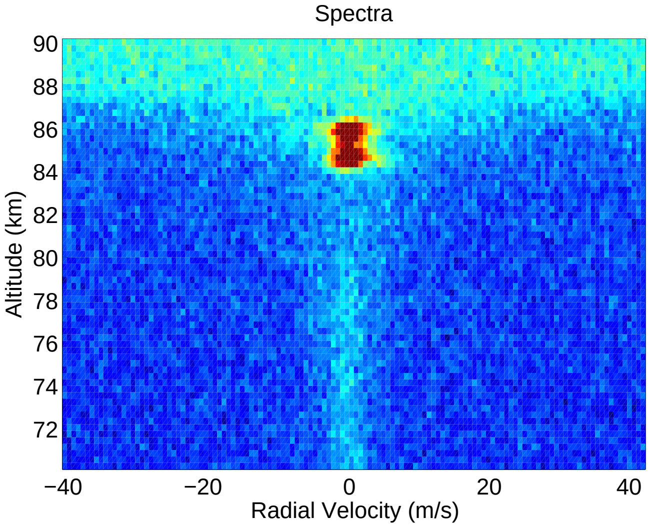


[Gadsden, *JATP*, 1990]

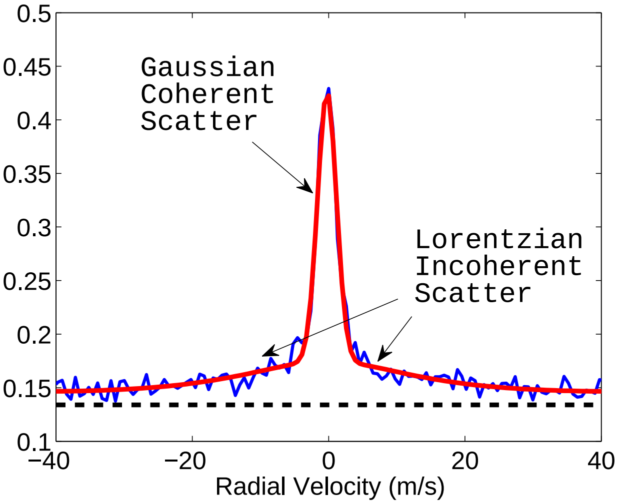
Polar Mesospheric Summer Echoes



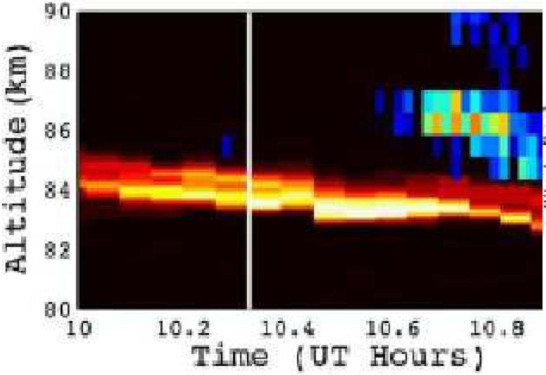
PMSE Spectra



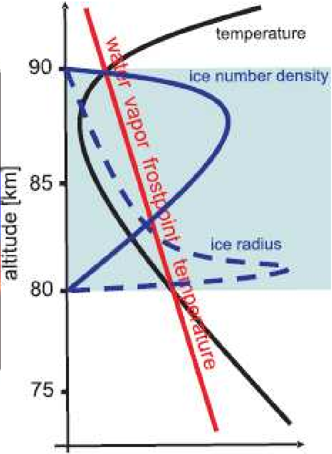
Spectral Superposition



PMSE and NLCs

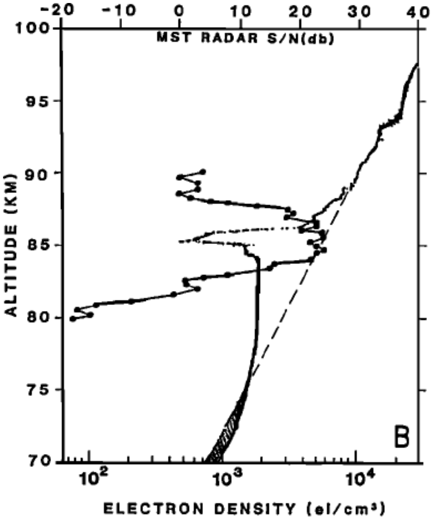


[Taylor *et al.*, *JASTP*, 2009]



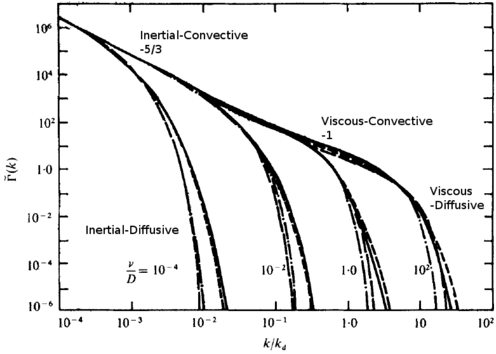
[Rapp and Thomas, *JASTP*, 2006]

Electron Density Biteouts



[Ulwick *et al.*, *JGR*, 1988]

Batchelor's Theory and The Schmidt Number



[Hill, *J. Fluid Mech.*, 1978]

Schmidt number: $Sc = \frac{\nu}{D}$
When $Sc > 1$, above η_K:

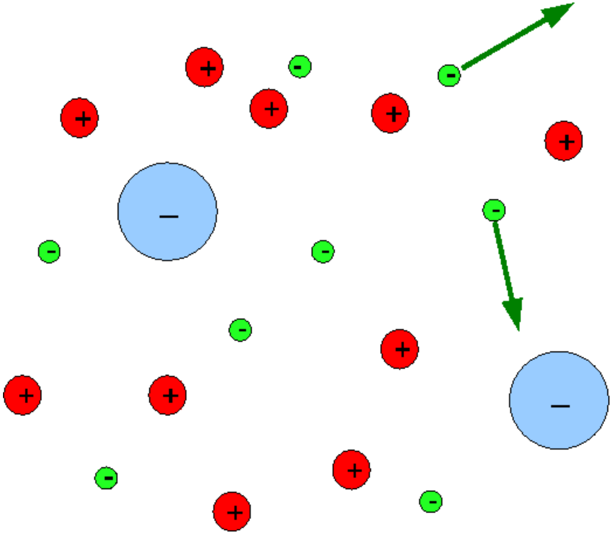
$$E(k) \propto k^{-1} \exp\left(\frac{-2Dk^2\sqrt{\nu}}{\sqrt{\epsilon}}\right)$$

Viscous-convective to viscous-diffusive transition is at the Batchelor scale:

$$\eta_B = \left(\frac{D^2\nu}{\epsilon}\right)^{\frac{1}{4}} = \left(\frac{\nu^3}{Sc^2\epsilon}\right)^{\frac{1}{4}}$$

[Batchelor, *J. Fluid Mech.*, 1959]

Ambipolar Diffusion



[Hill, *JGR*, 1978]
[Rapp and Lübken, *JGR*, 2003]

PMSE reflectivity

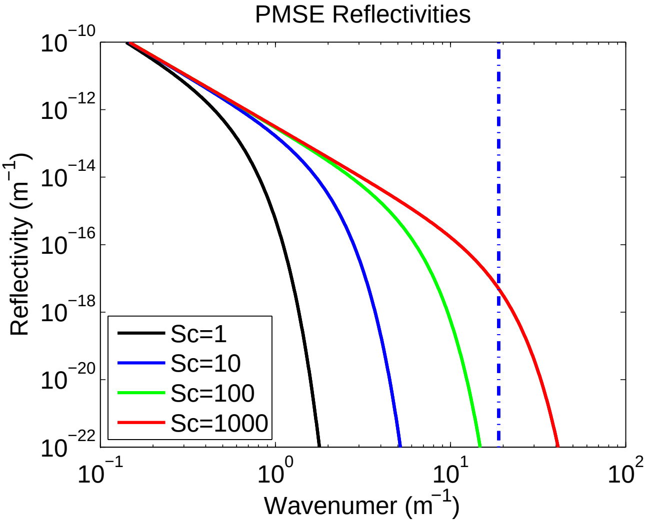
$$\eta(k) = \frac{\pi^2}{2} k^4 \Phi_n(k)$$

$$= 8\pi^3 \frac{f_\alpha q Ri}{Pr^t} \frac{\sqrt{\epsilon\nu}}{N^2} \tilde{M_e}^2 r_e^2 k^{-3} \exp\left(-\frac{2\eta_K^2}{Sc} k^2\right)$$

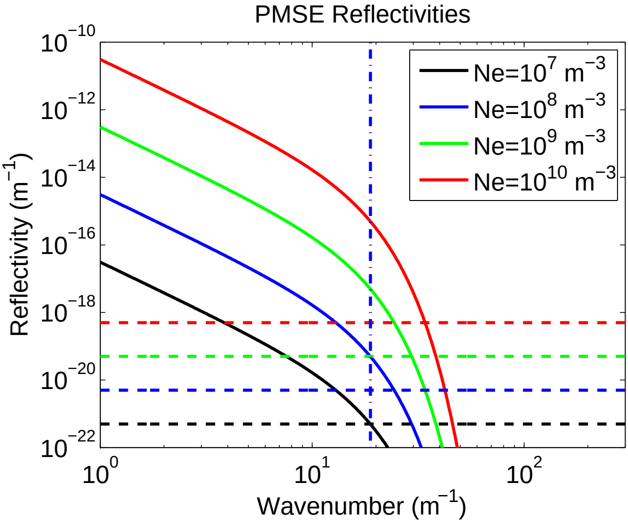
$$\tilde{M_e} = N_e \left(\frac{N^2}{g} - \frac{1}{N_e}\frac{dN_e}{dz} - \frac{1}{H}\right)$$

The reflectivity is a function of Sc, ϵ, N_e, and $\frac{dN_e}{dz}$.
[Rapp *et al.*, *JASTP*, 2008]

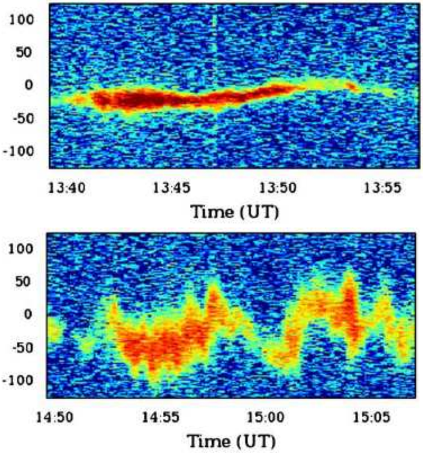
PMSE reflectivity



PMSE reflectivity



Fossil Turbulence?



[Nicolls *et al.*, *JASTP*, 2009]

Some Open Questions in PMSE research

- How high do the Schmidt numbers need to be?
- Are such large Schmidt numbers reasonable?
- How long can "fossilized" structures live?
- How much charge does the ice carry?
- How does the Aurora affect PMSE?
- How do electric fields affect PMSE?