LECTURE 2. IMPEDANCE MATCHING

2.1. Main principles
(conjugate matching, maximum delivered power)

2.2. Smith chart

2.3. Matching with lumped elements

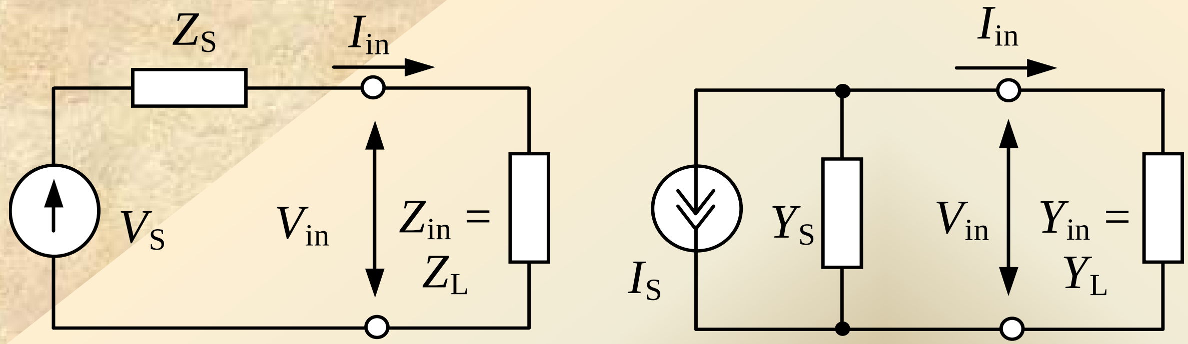
2.4. Matching with transmission lines

2.5. Determination of active device impedances

2.6. Types of transmission lines
(coaxial line, stripline,microstrip line,slotline, coplanar waveguide)

2.1. Main principles

Impedance matching is necessary to provide maximum delivery of RF power to load from source



$Z_S = R_S + jX_S$ - source impedance

$Z_L = R_L + jX_L$ - load impedance

$$P = \frac{1}{2} V_{in}^2 \,\mathrm{Re}\left(\frac{1}{Z_L}\right) = \frac{1}{2} V_S^2 \left|\frac{Z_L}{Z_S + Z_L}\right|^2 \mathrm{Re}\left(\frac{1}{Z_L}\right)$$

- power delivered to load

⇓

(substitution of real and imaginary parts of source and load impedances)

$$P = \frac{1}{2} V_S^2 \frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

- power delivered to load as function of circuit parameters

2.1. Main principles

For fixed source impedance Z_S, to maximize output power

$$\frac{\partial P}{\partial R_L} = 0 \qquad \frac{\partial P}{\partial X_L} = 0$$

$$P = \frac{1}{2} V_S^2 \frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

$$\begin{cases} R_S^2 - R_L^2 + (X_L + X_S)^2 = 0 \\ X_L (X_L + X_S) = 0. \end{cases}$$

$$\boxed{\begin{cases} R_S = R_L \\ X_L = -X_S \end{cases} \quad \text{or} \quad Z_L = Z_S^*}$$

- impedance conjugate matching conditions

$$\boxed{P = \frac{V_S^2}{8R_S}}$$

- maximum power delivered to load

$$\boxed{\begin{cases} G_S = G_L \\ B_L = -B_S \end{cases} \quad \text{or} \quad Y_L = Y_S^*}$$

- admittance conjugate matching conditions

$$\boxed{W_L = W_S^*}$$

- immitance conjugate matching conditions (Z or Y)

2.2. Smith chart

Smith chart represents relationships between load impedance Z and reflection coefficient Γ

$$\frac{Z}{Z_0} = \frac{1+\Gamma}{1-\Gamma}$$

with real and imaginary parts of

$$\frac{Z}{Z_0} = \frac{R}{Z_0} + j\frac{X}{Z_0} \qquad \Gamma = \Gamma_r + j\Gamma_i$$

$$\frac{R}{Z_0} + j\frac{X}{Z_0} = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i}$$

Equating real and imaginary parts:

$$\left(\Gamma_r - \frac{R}{R+Z_0}\right)^2 + \Gamma_i^2 = \left(\frac{Z_0}{R+Z_0}\right)^2$$

- constant-(R/Z_0) circles: family of circles centered at points $\Gamma_r = R/(R+Z_0)$ and $\Gamma_i = 0$ with radii of $Z_0/(R+Z_0)$

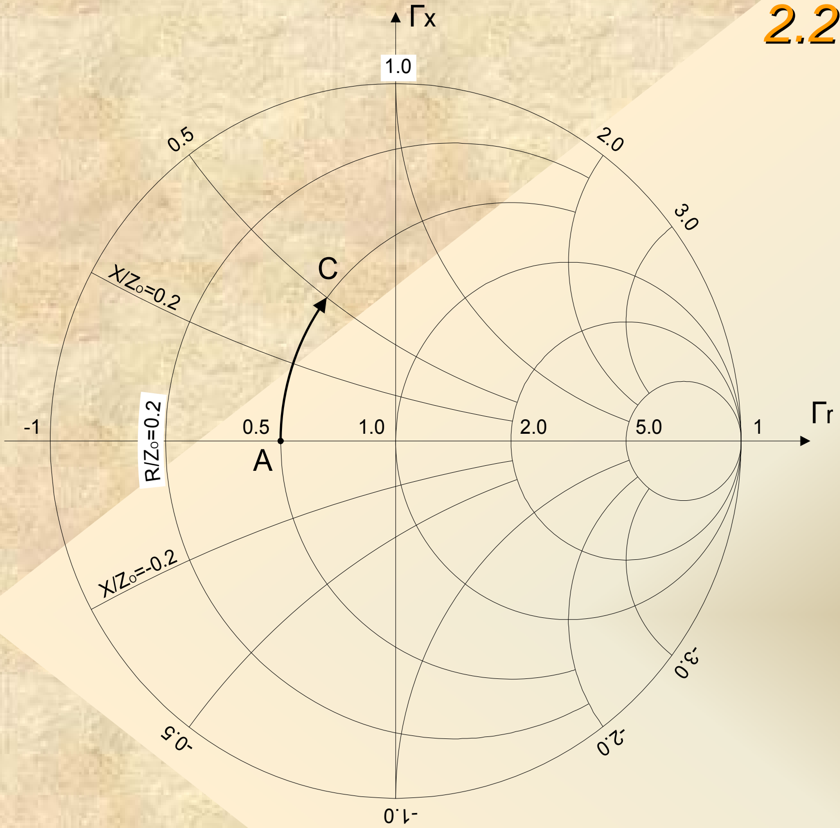
$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{Z_0}{X}\right)^2 = \left(\frac{Z_0}{X}\right)^2$$

- constant-(X/Z_0) circles: family of circles centered at points $\Gamma_r = 1$ and $\Gamma_i = Z_0/X$ with radii of Z_0/X

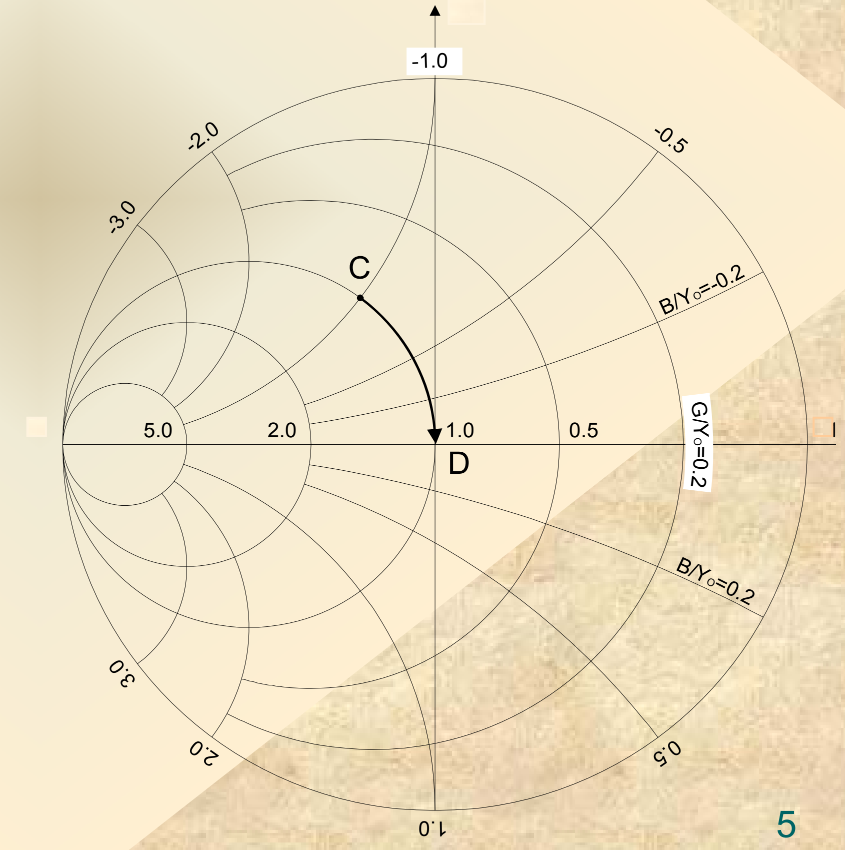
In admittance form:

$$\left(\Gamma_r + \frac{G}{G+Y_0}\right)^2 + \Gamma_i^2 = \left(\frac{Y_0}{G+Y_0}\right)^2 \qquad (\Gamma_r + 1)^2 + \left(\Gamma_i + \frac{Y_0}{B}\right)^2 = \left(\frac{Y_0}{B}\right)^2$$

2.2. Smith chart

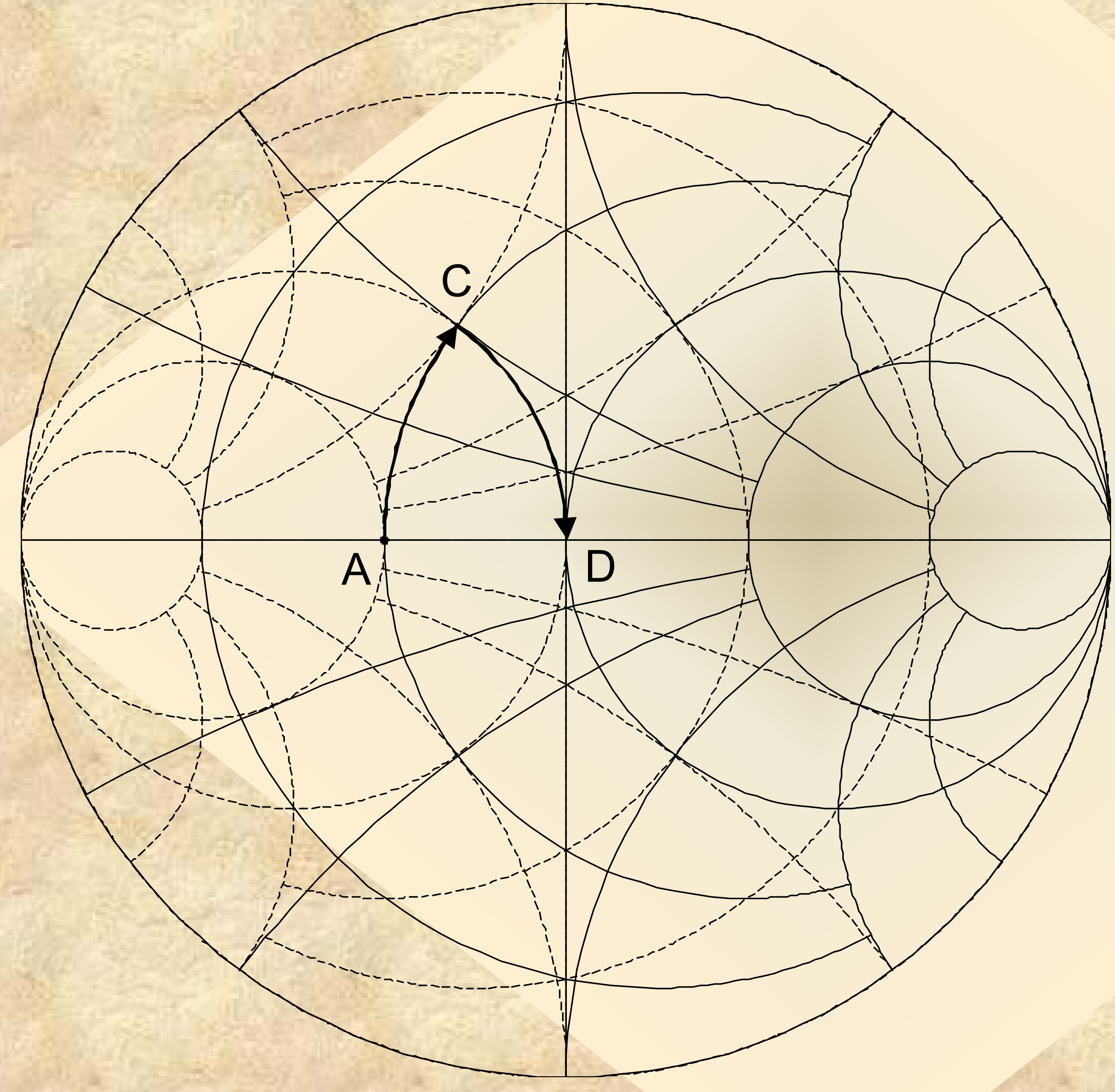


At Z Smith chart, curve from point A to pint C indicates impedance transformation from resistance 25 Ohm to inductive impedance (25 +j25) Ohm



At Y Smith chart, curve from point C to point D indicates admittance transformation from inductive admittance (20 - j20) mS to conductance 20 mS (50 Ohm)

2.2. Smith chart



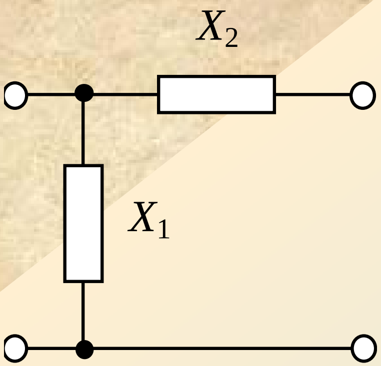
At combined Z-Y Smith chart:

Z Smith chart provides transformation from point A to point C

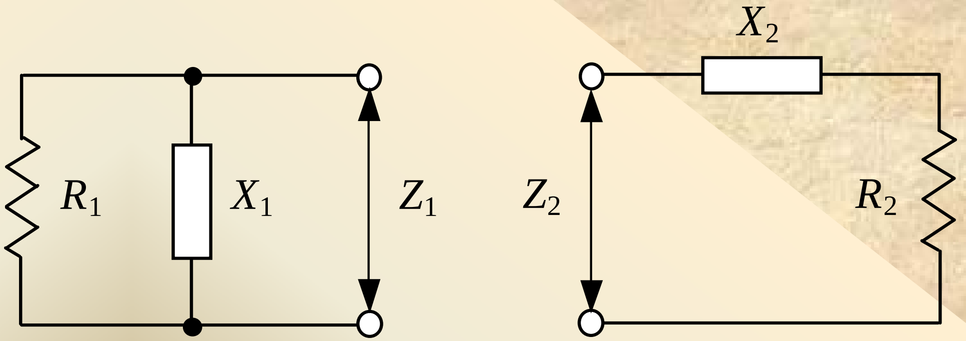
Y Smith chart provides transformation from point C to point D

2.3. Matching with lumped elements

L-transformer



Impedance parallel and series circuits



Equivalence when $Z_1 = Z_2$:

$$R_2 + jX_2 = \frac{R_1 X_1^2}{R_1^2 + X_1^2} + j\frac{R_1^2 X_1}{R_1^2 + X_1^2}$$

$$\boxed{R_1 = R_2\left(1 + Q^2\right) \qquad X_1 = X_2\left(1 + Q^{-2}\right)}$$

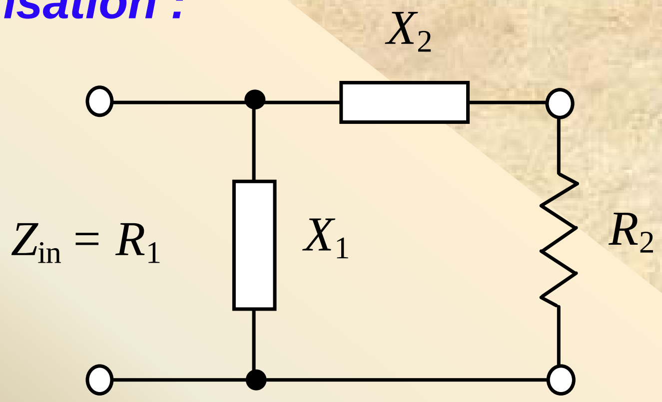
where Q = $R_1/|X_1| = |X_2|/R_2$

- quality factor equal for series and parallel circuits

2.3. Matching with lumped elements

For conjugate matching with reactance compensation :

$$R_1 = R_2\left(1 + Q^2\right)$$
$$X_1 = -X_2\left(1 + Q^{-2}\right)$$



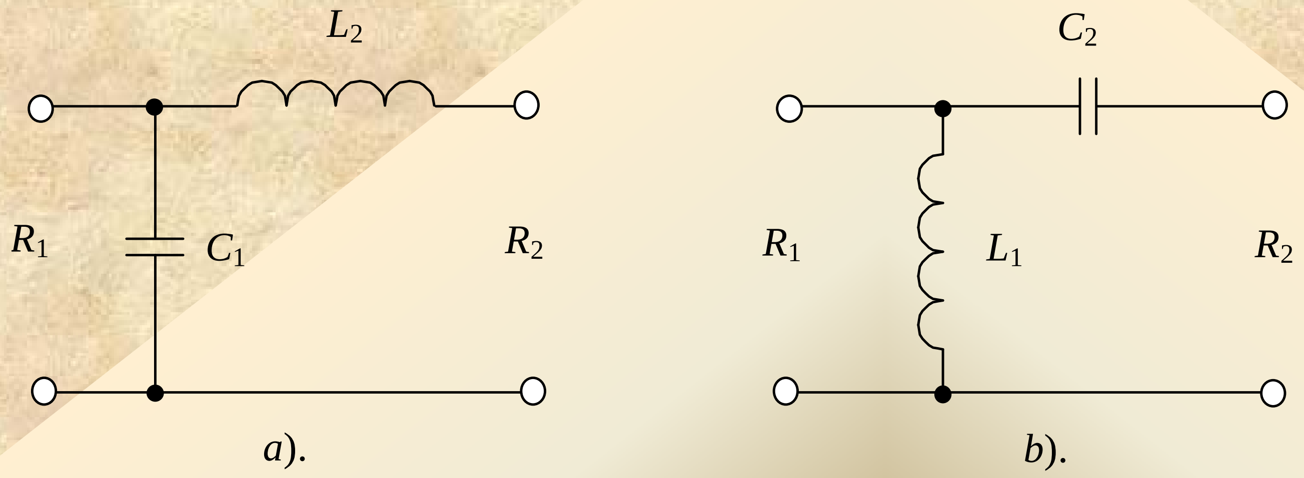
Input impedance Z_{in} will be resistive and equal to R_1 when :

$$\begin{cases} |X_1| = R_1 / Q \\ |X_2| = R_2 Q \\ Q = \sqrt{R_1 / R_2 - 1}. \end{cases}$$

where $Q = R_1/|X_1| = |X_2|/R_2$
- quality factor equal for series and parallel circuits

2.3. Matching with lumped elements

Two L-type matching circuits



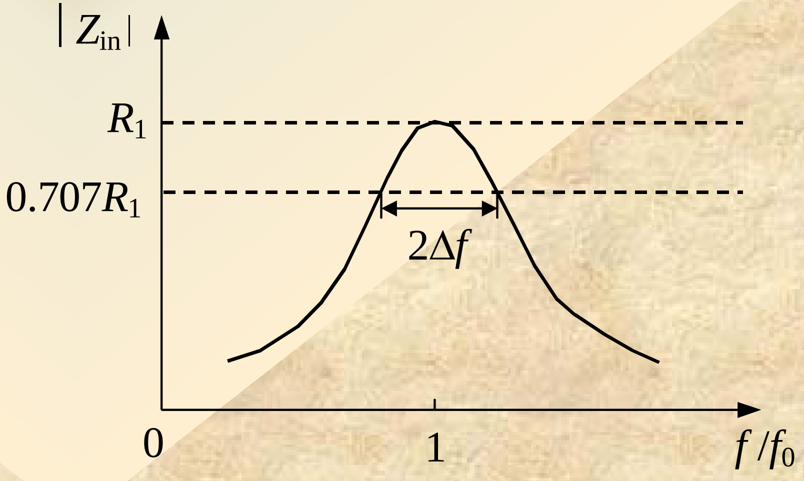
Resistance R_1 connected to parallel reactive element must be greater than resistance R_2 connected to series reactive element

$$\left.\begin{aligned}\omega C_1 &= Q/R_1\\ \omega L_2 &= Q R_2\end{aligned}\right\}\quad Q=\sqrt{\frac{R_1}{R_2}-1}\quad \left\{\begin{aligned}\omega L_1 &= R_1/Q\\ \omega C_2 &= 1/(Q R_2)\end{aligned}\right.$$

Bandwidth properties

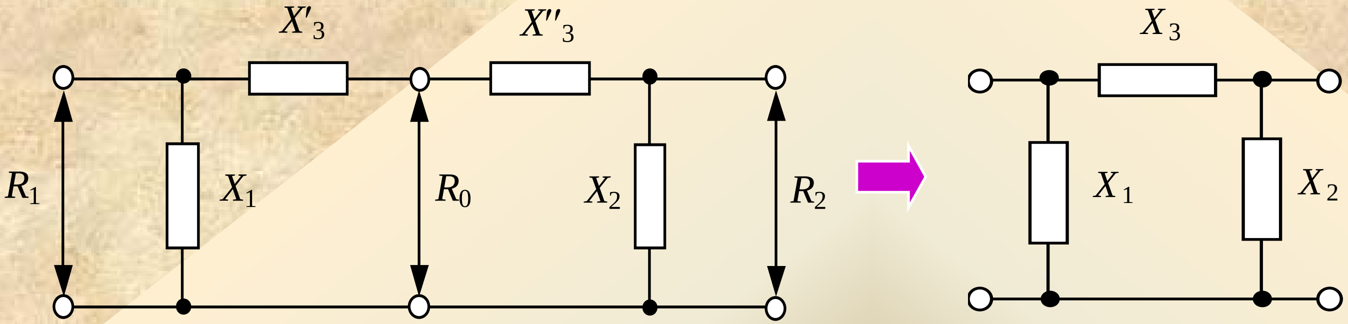
$$\begin{cases} Q \cong f_0/2\Delta f_0 \\ F_{\mathrm{n}} \cong Q^2\left(n^2-1\right)\end{cases}$$

where F_n - out-of-band suppression factor
n - harmonic number

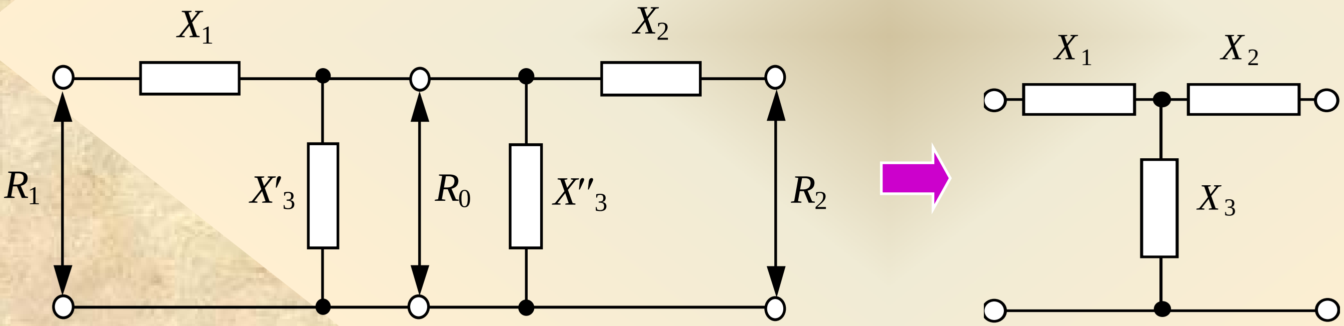


2.3. Matching with lumped elements

Connection of two L-transformers



π - transformer

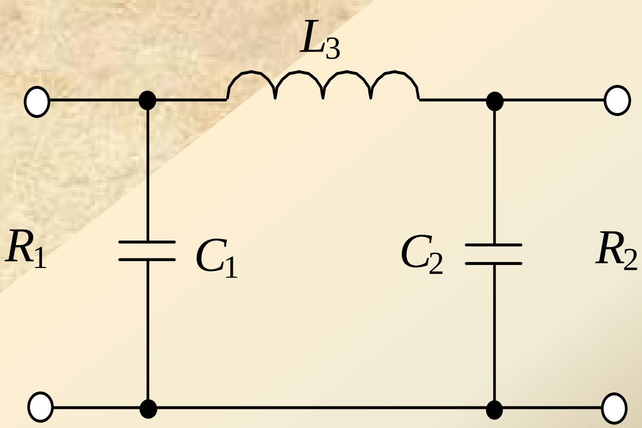


T- transformer

- ***for each L-transformer, resistances R_1 and R_2 are transformed to some intermediate resistance R_0 with value of $R_0 < (R_1, R_2)$***
- ***for same resistances R_1 and R_2, T- and π-transformers have better filtering properties, but narrower bandwidth compared with single L-transformer***

2.3. Matching with lumped elements

π -type matching circuits



$$\omega C_1 = Q_1 / R_1 \qquad \omega C_2 = Q_2 / R_2$$

$$\omega L_3 = R_1 \left(Q_1 + Q_2 \right) / \left(1 + Q_1^2 \right)$$

$$Q_2 = \sqrt{\frac{R_2}{R_1}\left(1 + Q_1^2\right) - 1} \qquad Q_1^2 > \frac{R_1}{R_2} - 1$$

- ***widely used as output matching circuit to provide Class B operation with sinusoidal collector voltage***

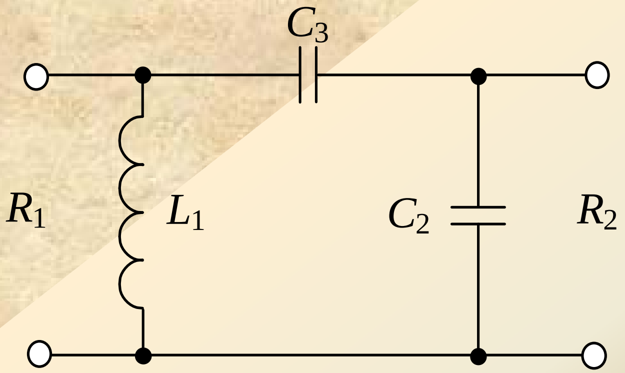
- ***useful for interstage matching when active device input and output capacitances can be easily incorporated inside matching circuit***

- ***provides significant level of harmonic suppression***

- ***with additional series LC-filter, can be directly applied to realize Class E mode with shunt capacitance***

2.3. Matching with lumped elements

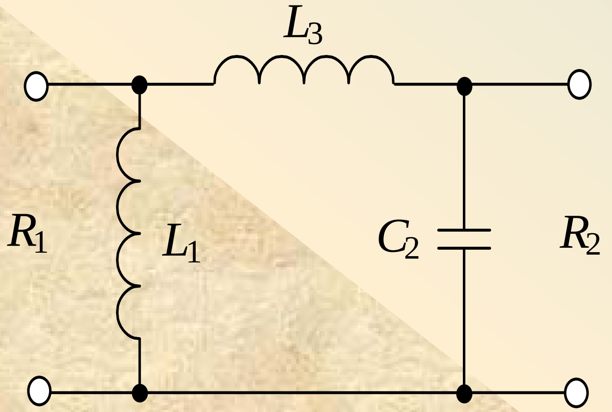
π -type matching circuits



$$\omega L_1 = R_1 / Q_1 \qquad \omega C_2 = Q_2 / R_2$$

$$\omega C_3 = \left(1 + Q_2^2\right) / \left[R_2\left(Q_1 - Q_2\right)\right]$$

$$Q_2 = \sqrt{\frac{R_2}{R_1}\left(1 + Q_1^2\right) - 1} \qquad Q_1^2 > \frac{R_1}{R_2} - 1$$



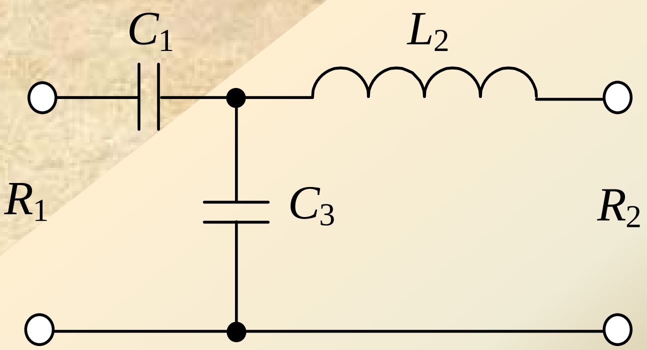
$$\omega L_1 = R_1 / Q_1 \qquad \omega C_2 = Q_2 / R_2$$

$$\omega L_3 = R_2\left(Q_2 - Q_1\right) / \left(1 + Q_2^2\right),$$

$$Q_1 = \sqrt{\frac{R_1}{R_2}\left(1 + Q_2^2\right) - 1} \qquad Q_2^2 > \frac{R_2}{R_1} - 1$$

2.3. Matching with lumped elements

T-type matching circuits



$$\omega C_1 = 1/(R_1 Q_1) \qquad \omega L_2 = Q_2 R_2$$

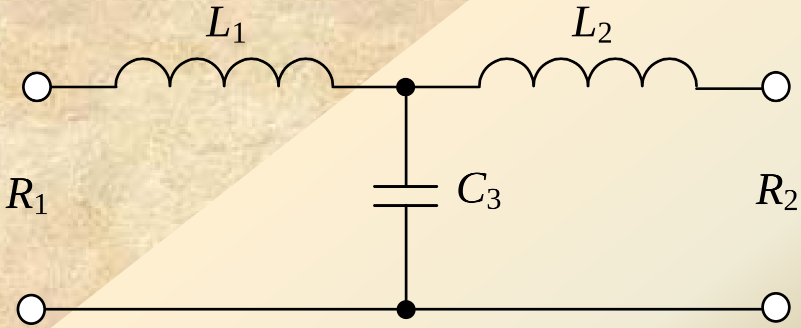
$$\omega C_3 = (Q_2 - Q_1)/\left[R_2\left(1 + Q_2^2\right)\right]$$

$$Q_1 = \sqrt{\frac{R_2}{R_1}\left(1 + Q_2^2\right) - 1} \qquad Q_2^2 > \frac{R_1}{R_2} - 1$$

- ***widely used as input, interstage and output matching circuits in high power amplifiers***
- ***can incorporate active device lead and bondwire inductances within matching circuit***
- ***provides significant level of harmonic suppression***
- ***can be directly applied to realize Class F mode providing high impedances at harmonics***

2.3. Matching with lumped elements

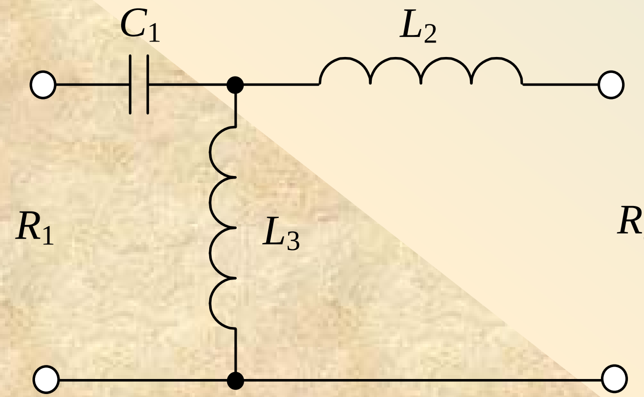
T-type matching circuits



$$\omega L_1 = Q_1 R_1 \qquad \omega L_2 = Q_2 R_2$$

$$\omega C_3 = (Q_1 + Q_2)/\left[R_2\left(1 + Q_2^2\right)\right]$$

$$Q_1 = \sqrt{\frac{R_2}{R_1}\left(1 + Q_2^2\right) - 1} \qquad Q_2^2 > \frac{R_1}{R_2} - 1$$



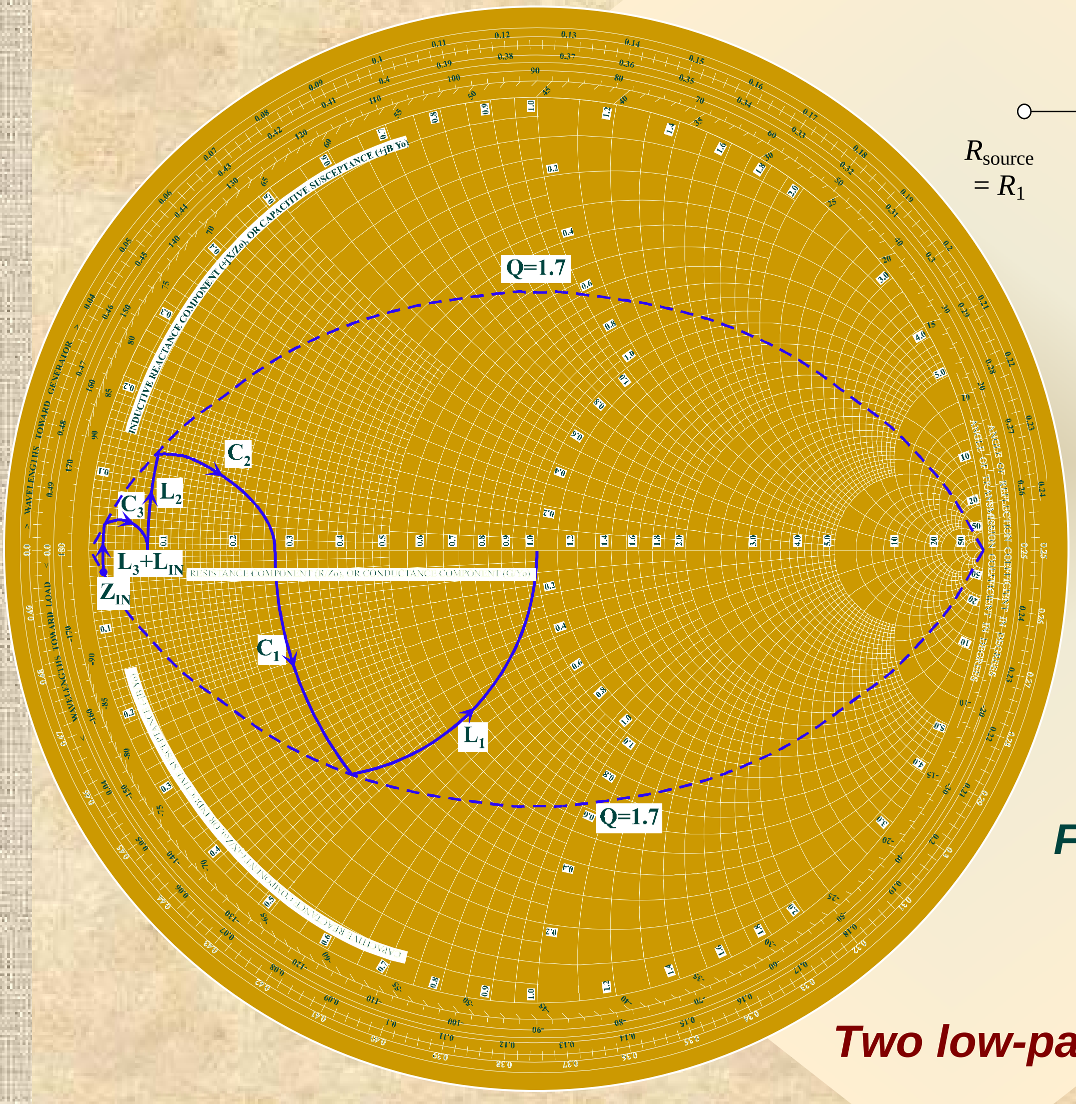
$$\omega C_1 = 1/\left(R_1 Q_1\right) \qquad \omega L_2 = Q_2 R_2$$

$$\omega L_3 = R_2\left(1 + Q_2^2\right)/(Q_1 - Q_2),$$

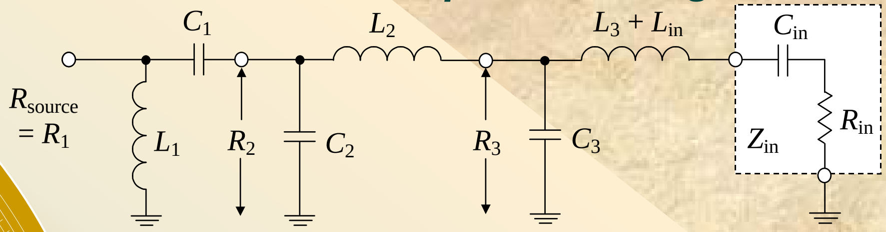
$$Q_2 = \sqrt{\frac{R_1}{R_2}\left(1 + Q_1^2\right) - 1} \qquad Q_1^2 > \frac{R_2}{R_1} - 1$$

2.3. Matching with lumped elements

Matching design example



132-174 MHz 150 W MOSFET power amplifier: three-section input matching



$$Z_{\text{in}} = (0.9 - j1.2)\,\Omega$$

$$f_{\text{c}} = \sqrt{132 \cdot 174} = 152 \text{ MHz}$$

Q = 152/(174 - 132) = 3.6

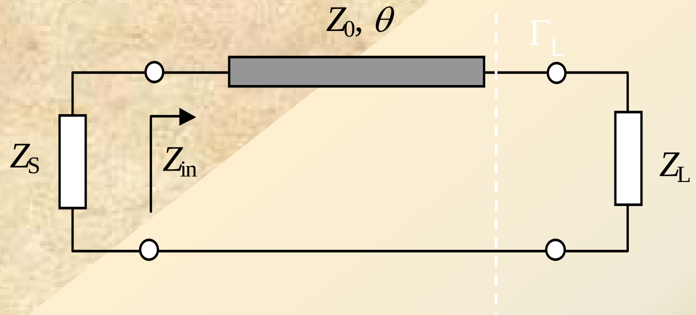
$$\boxed{\frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_{\text{in}}}}$$

For R_{in} = 0.9 Ohm and R_1 = 50 Ohm:
R_3 = 3.5 Ohm, R_2 = 13 Ohm
Q = 1.7

Two low-pass and one high-pass L-sections

2.4. Matching with transmission lines

Transmission-line transformer



Impedance at input of loaded transmission line:

$$\frac{Z_{in}}{Z_0} = \frac{1 + \Gamma_L \exp(-2j\theta)}{1 - \Gamma_L \exp(-2j\theta)}$$

Input impedance for loaded transmission line with electrical length of θ, normalized to its characteristic impedance Z_0, can be found by rotating this impedance point clockwise by 2θ around Smith chart center point with radius $|\Gamma_L|$

$$\frac{Z_L}{Z_0} = \frac{1 + \Gamma_L}{1 - \Gamma_L} \Rightarrow Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\theta}{Z_0 + jZ_L \tan\theta}$$

$$\Downarrow$$

For conjugate matching with reactance compensation when $Z_S = Z_{in}^*$:

$$\boxed{\begin{gathered} Z_0 = \sqrt{\frac{R_S\left(R_L^2 + X_L^2\right) - R_L\left(R_S^2 + X_S^2\right)}{R_L - R_S}} \\ \theta = \tan^{-1}\left(Z_0 \frac{R_S - R_L}{R_S X_L - X_S R_L}\right) \end{gathered}}$$

For quarter-wave transmission line with $\theta = 90°$:

$$Z_{in} = Z_0^2 / Z_L$$

2.4. Matching with transmission lines

For pure resistive source impedance $Z_S = R_S$:

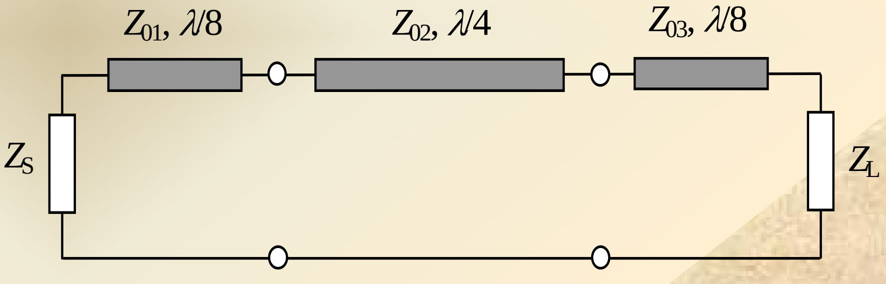
$$X_L Z_0 \left(1 - \tan^2\theta\right) + \left(Z_0^2 - X_L^2 - R_L^2\right)\tan\theta = 0$$

For electrical length $\theta = 45°$

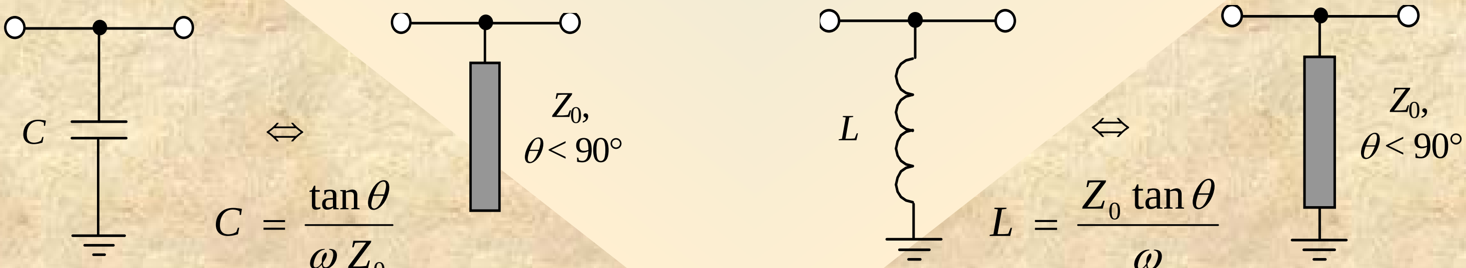
$$Z_0 = |Z_L| = \sqrt{R_L^2 + X_L^2} \qquad R_S = R_L \frac{Z_0}{Z_0 - X_L}$$

Any load impedance can be transformed into real source impedance using λ /8-transformer whose impedance is equal to magnitude of load impedance

To match any source impedance Z_S and load impedance Z_L, matching circuit can be designed with two λ /8-transformers and one λ /4-transformer



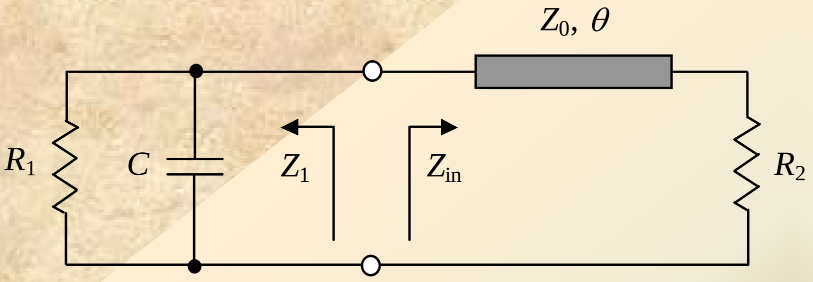
Lumped and transmission line single-frequency equivalence



$$C = \frac{\tan\theta}{\omega Z_0} \qquad L = \frac{Z_0 \tan\theta}{\omega}$$

2.4. Matching with transmission lines

L-type transformer



Conjugate matching:

$$R_{in} - jX_{in} = \frac{R_1 X_1^2}{R_1^2 + X_1^2} + j\,\frac{R_1^2 X_1}{R_1^2 + X_1^2}$$

$$\boxed{\begin{aligned} R_1 &= R_{in}\left(1 + Q^2\right) \\ X_1 &= -X_{in}\left(1 + Q^{-2}\right) \end{aligned}}$$

where $X_1 = -1/\omega C$

Real and imaginary parts of

$$Z_{in} = Z_0\,\frac{R_2 + jZ_0\tan\theta}{Z_0 + jR_2\tan\theta}$$

Matching for any ratio of R_1/R_2

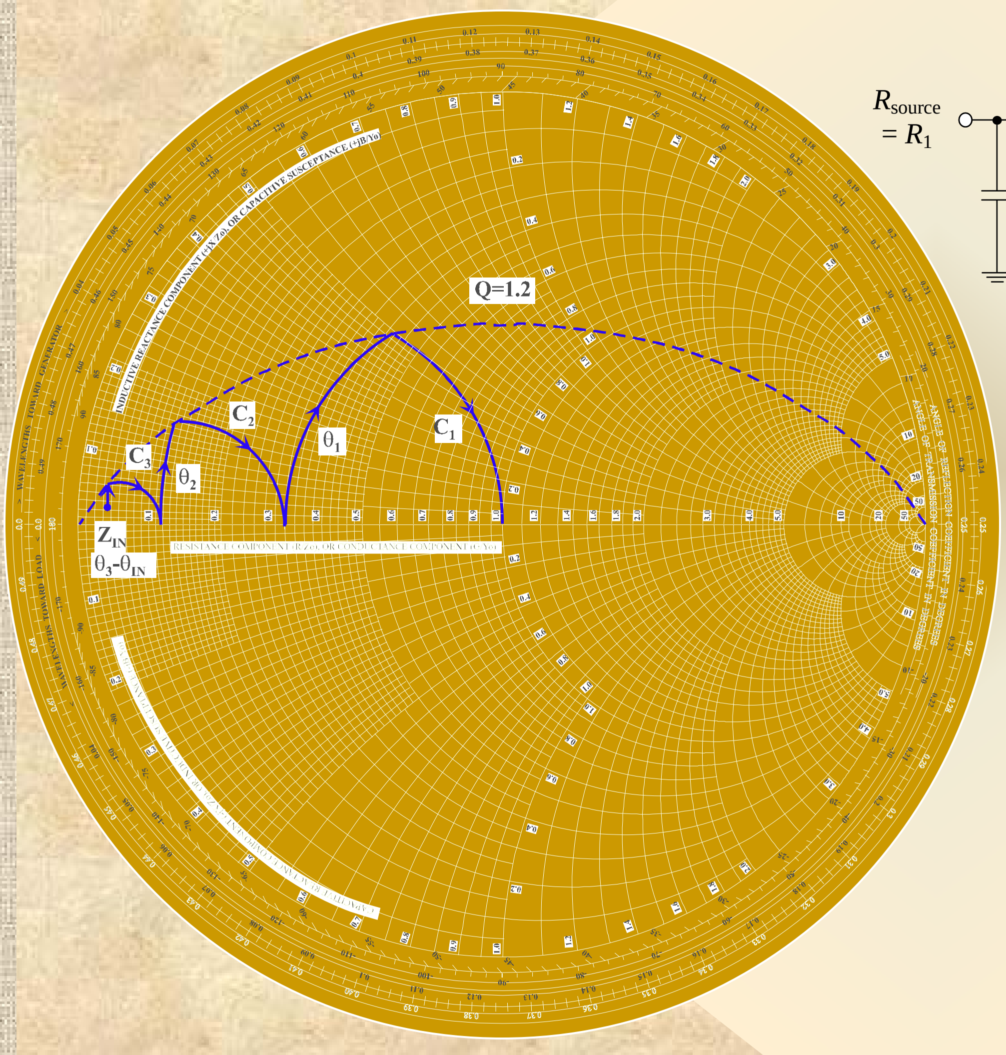
$$\boxed{\begin{aligned} R_{in} &= Z_0^2 R_2\,\frac{1 + \tan^2\theta}{Z_0^2 + \left(R_2\tan\theta\right)^2} \\ X_{in} &= Z_0\tan\theta\,\frac{Z_0^2 - R_2^2}{Z_0^2 + \left(R_2\tan\theta\right)^2} \end{aligned}}$$

$$\boxed{\begin{aligned} C &= Q/\omega R_1 \\ \frac{R_1}{R_2} &= \frac{1 + \left(\dfrac{Z_0}{R_2} - \dfrac{R_2}{Z_0}\right)^2 \sin^2\theta\,\cos^2\theta}{\cos^2\theta + \left(R_2/Z_0\right)^2 \sin^2\theta} \end{aligned}}$$

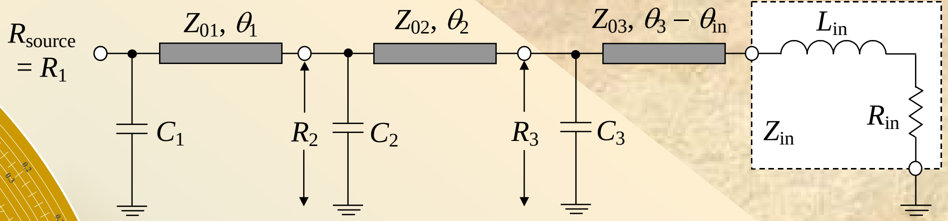
Second implicit equation : numerical or graphical solution

2.4. Matching with transmission lines

Matching design example



470-860 MHz 150 W LDMOSFET power amplifier: three-section input matching



$$Z_{in} = (1.7 + j1.3)\,\Omega$$

$$f_c = \sqrt{470 \cdot 860} = 635\ \text{MHz}$$

Q = 635/(860 - 470) = 1.63

$$\boxed{\frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_{in}}}$$

Q = 1.2

For R_{in} = 1.7 Ohm and R_1 = 50 Ohm: R_3 = 5.25 Ohm, R_2 = 16.2 Ohm

For Z_{01}= Z_{02} = Z_{03} = 50 Ohm ⇒ θ_1 = 30°, θ_2 = 7.5°, θ_3 = 2.4°

For $\theta_1 = \theta_2 = \theta_3$ = 30° ⇒ Z_{01} = 50 Ohm, Z_{02} = 15.7 Ohm, Z_{03} = 5.1 Ohm

2.5. Determination of active device impedances

Analytical evaluation

Output resistance in Class B : $$R_{\mathrm{out}}^{(\mathrm{B})} = \frac{\left(V_{\mathrm{cc}} - V_{\mathrm{sat}}\right)^2}{2P_{\mathrm{out}}}$$

where V_{sat} is defined from load line analysis

Output capacitance :

$$C_{\mathrm{out}} = C_{\mathrm{c}}$$ ***- bipolar device***

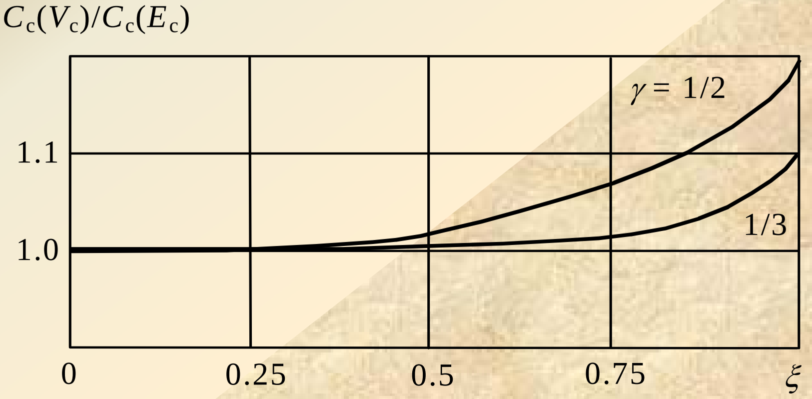
$$C_{\mathrm{out}} = C_{\mathrm{ds}} + C_{\mathrm{gd}}$$ ***- FET device***

Large-signal collector capacitance

$$C_{\mathrm{c}} = C_{\mathrm{co}} \Big/ \left(1 + \frac{v_{\mathrm{c}}}{\varphi}\right)^{\gamma}$$ ***- junction capacitance***

$$v_{\mathrm{c}} = E_{\mathrm{c}} + V_{\mathrm{c}}\sin\omega t \quad \Rightarrow \quad i_{\mathrm{c}} = C_{\mathrm{c}}(v_{\mathrm{c}})\frac{dv_{\mathrm{c}}}{dt}$$

$$C_{\mathrm{c1}} = \frac{I_{\mathrm{c1}}}{\omega V_{\mathrm{c}}} = \frac{C_{\mathrm{c}}(E_{\mathrm{c}})}{\pi}\int_0^{2\pi}\frac{\cos^2\omega t}{(1+\xi\sin\omega t)^{\gamma}}\,d(\omega t)$$

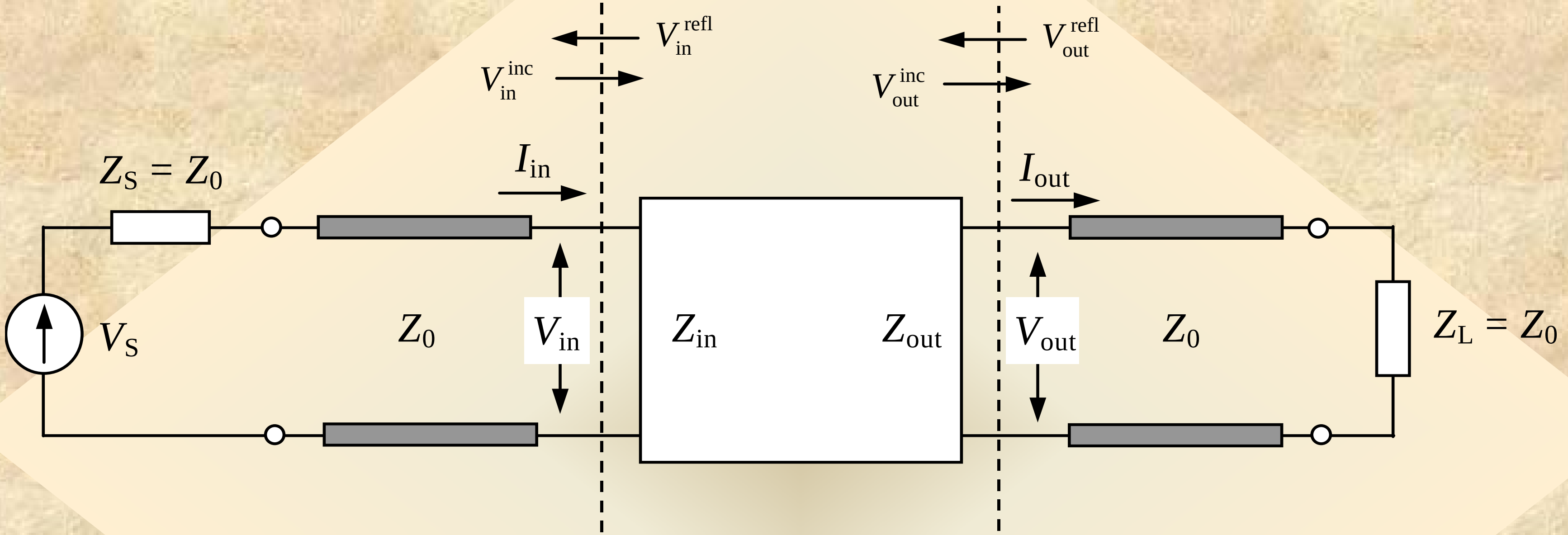


where $$\xi = V_{\mathrm{c}}/(E_{\mathrm{c}} + \varphi)$$

where

2.5. Determination of active device impedances

S-parameter measurements



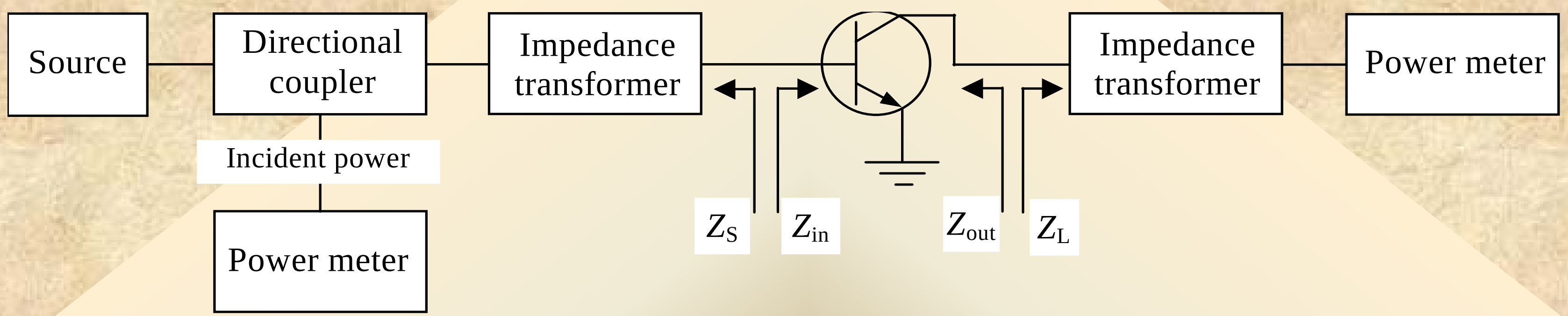
$$Z_{in} = \frac{V_{in}}{I_{in}} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} \quad \textit{where} \quad \Gamma_{in} = \frac{V_{in}^{refl}}{V_{in}^{inc}}$$

$$Z_{out} = \frac{V_{out}}{I_{out}} = Z_0 \frac{1 + \Gamma_{out}}{1 - \Gamma_{out}} \quad \textit{where} \quad \Gamma_{out} = \frac{V_{out}^{refl}}{V_{out}^{inc}}$$

To define Z_{out}, source with nominal power is placed instead of load, and load becomes source

2.5. Determination of active device impedances

Power measurements

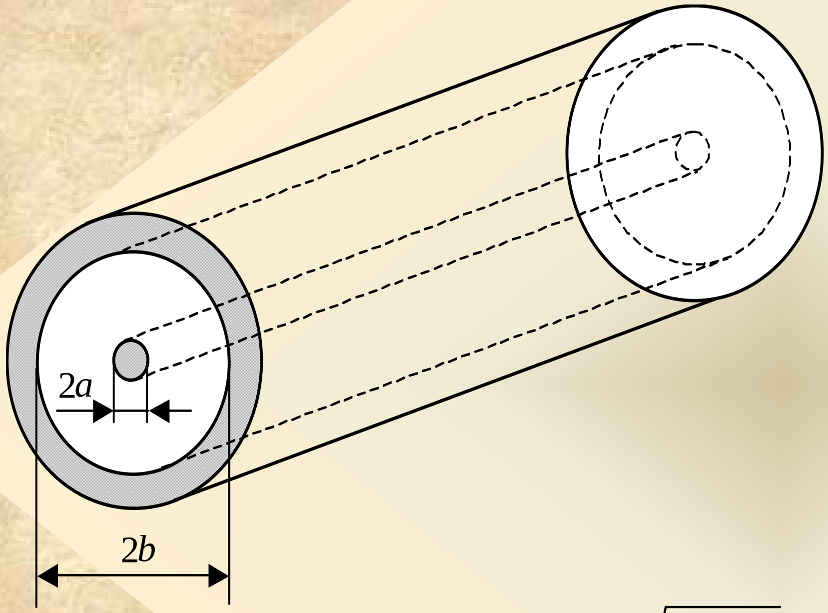


- ***tune input impedance transformer to maximize incident power, I.e., power delivery from source to active device***
- ***tune output impedance transformer to maximize output power delivered to load***
- ***measure transformer impedances seen from the active device input and output, I.e., Z_S and Z_L***
- ***calculate input and output active device impedances according to***

$$\boxed{Z_{in} = Z_S^* \qquad Z_{out} = Z_L^*}$$

2.6. Types of transmission lines

Coaxial line



Main wave type for coaxial line - transverse electromagnetic TEM wave

$$Z_0 = \frac{\eta}{2\pi}\ln\left(\frac{b}{a}\right)$$

- characteristic impedance

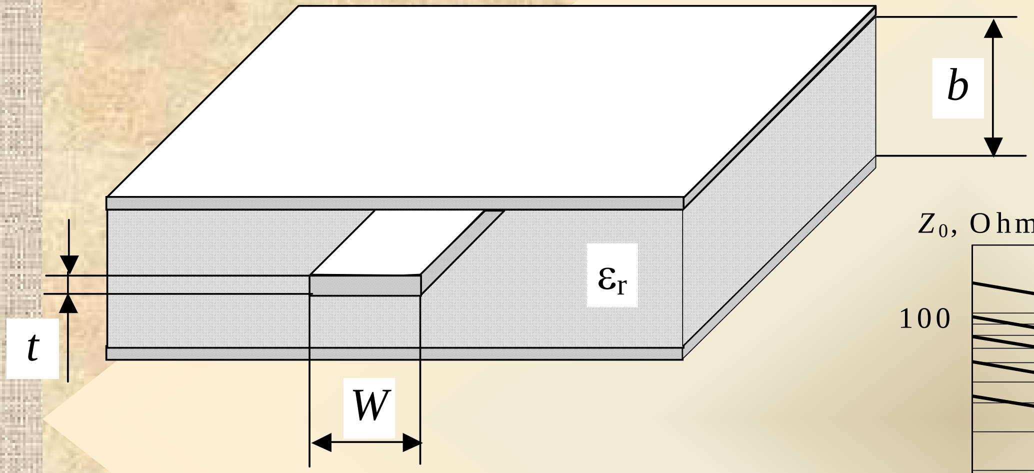
where $\eta = \sqrt{\mu / \varepsilon}$ **- wave impedance of lossless line equal to intrinsic medium impedance**

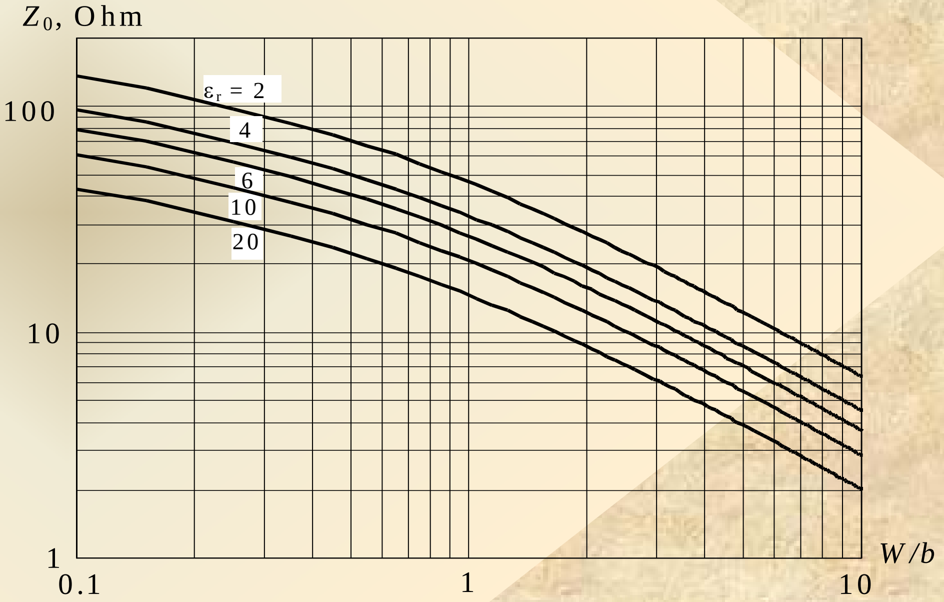
- **widely used for hybrid high power applications: combiners, dividers, transformers**

2.6. Types of transmission lines

Stripline

Main wave type for stripline - transverse electromagnetic TEM wave





$$Z_0 = \frac{30\pi}{\sqrt{\varepsilon_r}} \frac{b}{W_e + 0.441b}$$

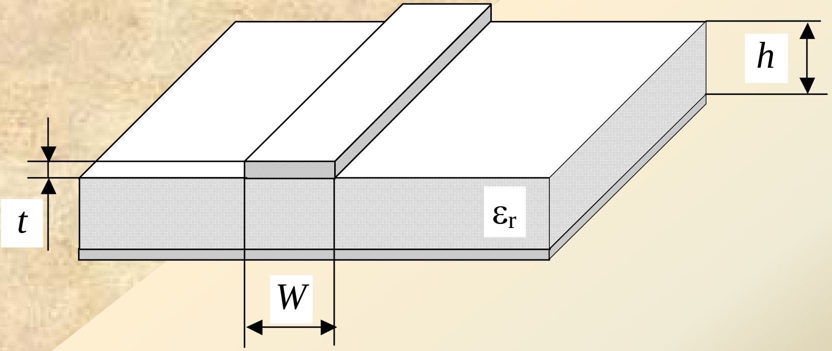
- characteristic impedance

$$\frac{W_e}{b} = \frac{W}{b} - \begin{cases} 0 & \text{for } \frac{W}{b} > 0.35b \\ \left(0.35 - \frac{W}{b}\right)^2 & \text{for } \frac{W}{b} \le 0.35b. \end{cases}$$

- ***provides lower characteristic impedance***

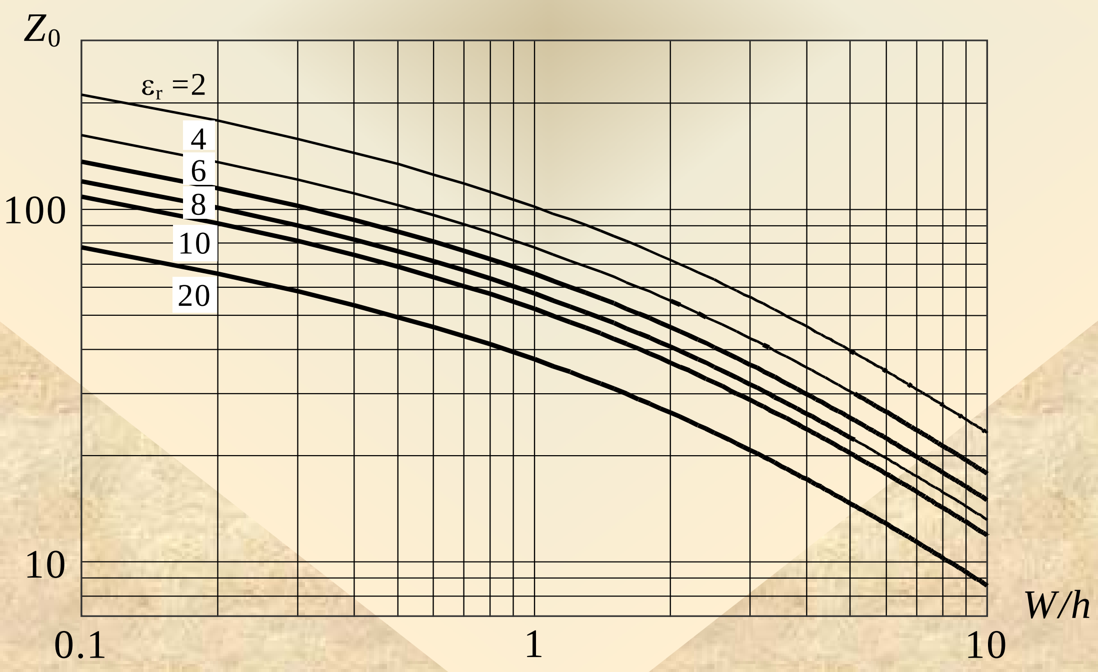
2.6. Types of transmission lines

Microstrip line



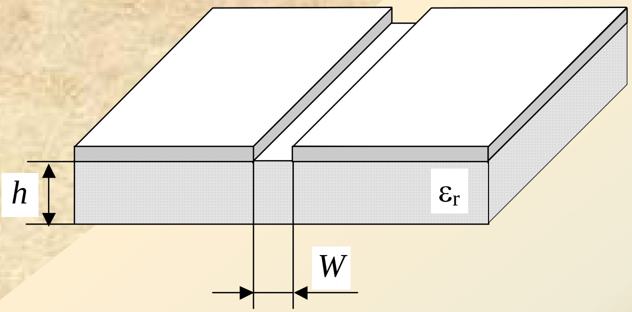
$$Z_0 = \frac{120\pi}{\sqrt{\varepsilon_r}}\,\frac{h}{W}\,\frac{1}{1 + 1.735\varepsilon_r^{-0.0724}\left(W/h\right)^{-0.836}}$$

- characteristic impedance

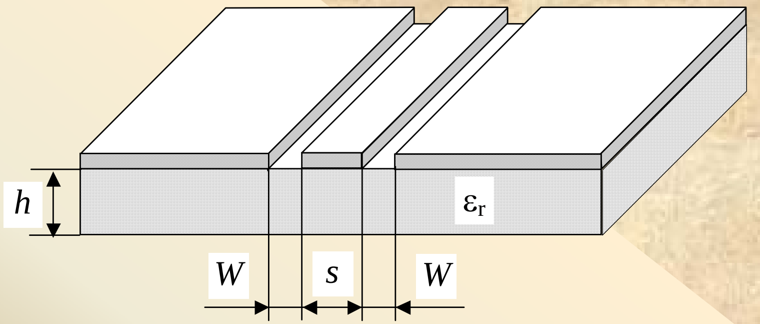


2.6. Types of transmission lines

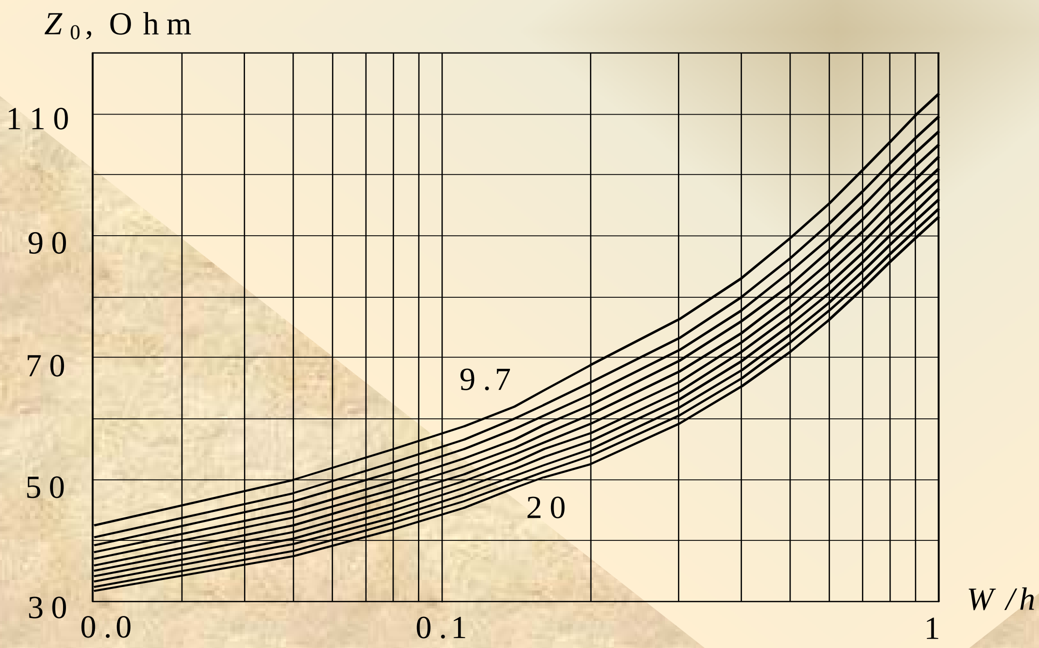
Slotline



Coplanar waveguide



Characteristic impedance



- *provide higher characteristic impedance*

- *widely used for hybrid and monolithic integrated circuits*