Project 3: Convection

Katheryn Decker French

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Abstract

Convection is studied in several cases to test its efficiency as a mode of heat transport. Convection was modeled in a tank experiment in the lab, and cases of both dry and moist convection were studied in the atmosphere. A trend of instabilities leading to convection was seen in all cases, and convection was seen to be an efficient way to transport heat through a fluid to regain stability.

1 Introduction

Many important atmospheric phenomena are the result of convection. Convection functions to balance out instabilities in the atmosphere caused by solar heating of the ground. When cold, dense fluid is below hot, less dense fluid, the system is stable. If the bottom of this system is heated, convection will occur, and the newly warm fluid at the bottom will rise up, replacing a parcel of cooler fluid, which will sink down. By this method, the energy supplied by heating the bottom of this system is carried up through the system to a height at which the heat can be radiated away.

2 Convection in the Laboratory

Convection was modeled in the laboratory by setting up a stable system of dense fluid below less dense fluid, then introducing instabilities with a heating pad at the bottom of the tank. Two different methods were used to set up a density gradient; in the first, the tank was filled with half salt water below half fresh water, and in the second, the tank was filled with seven layers of increasingly warm fluid to the top.

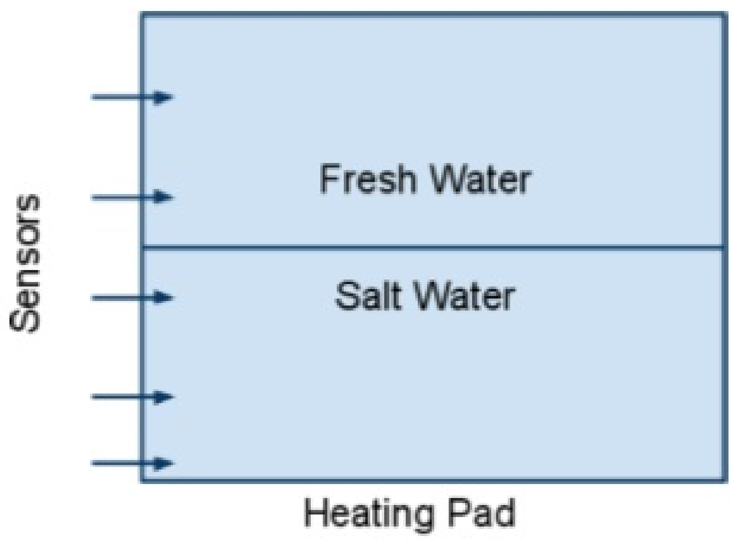


Figure 1: Diagram of experiment setup for two- layer tank experiment.

Two-Layer Experiment

In this experiment, a tank was set up with a heating pad on the bottom, and five temperature sensors places vertically along one corner of the tank. The bottom three sensors were within the area filled with salt water, and the top two sensors were in the area filled carefully with fresh water (Figure 1.) Once the heating pad was turned on, dye was put into the tank in order to observe the convection.

At first, convection only occurred in the bottom layer of salt water. Dyed plumes could be seen rising up from the bottom of the tank to the boundary between the two layers. This boundary gradually rose, until the mixing between the two layers had homogenized the tank, and plumes broke through the boundary, creating convection in the entire tank.

A graph of the temperatures at different locations can be seen in Figure 2. Initially the temperatures of the two layers were different, with the fresh water colder than the salt water, which introduced an unwanted variable into our experiment. The temperature profiles for each of the sensors is flat until t = 2 minutes on the figure. Convection can be seen in the bottom three sensors, as their temperatures increase, while the temperature difference between these bottom three sensors goes away. Once convection reached the fresh water boundary at around 17 minutes, some heat was transferred to the salt water through conduction. This effect was exaggerated by the initial temperature difference in

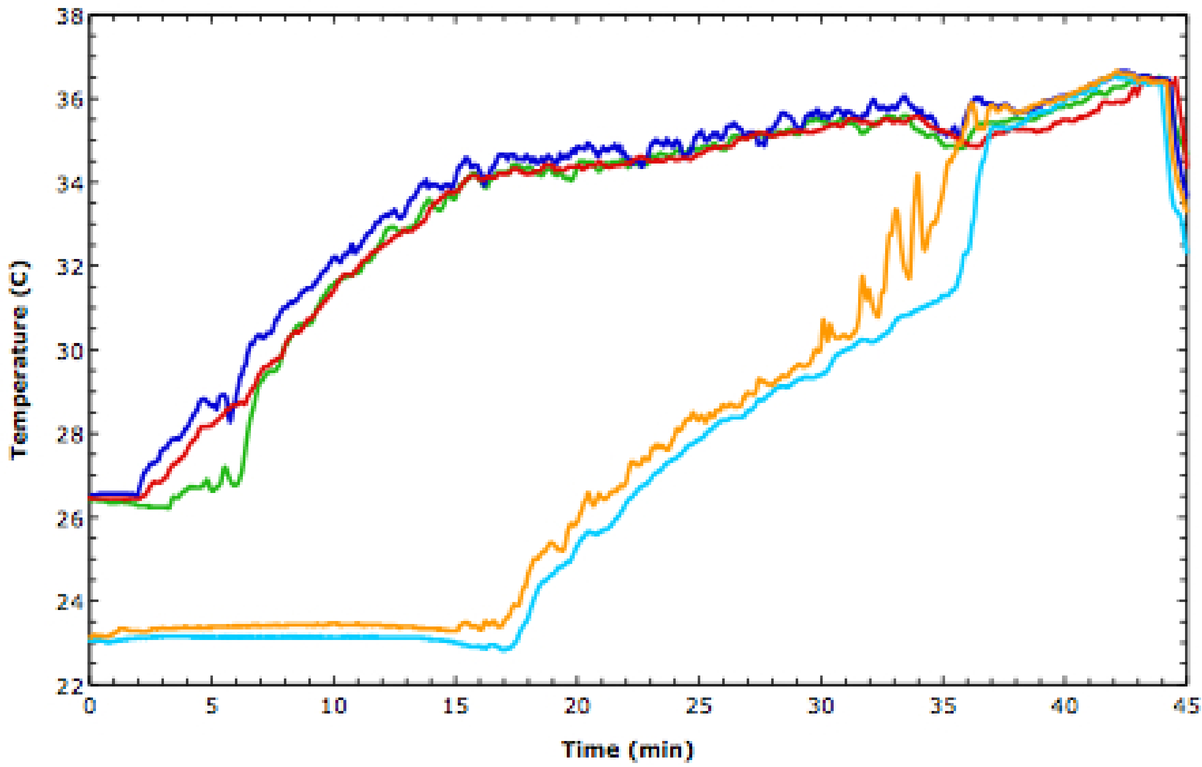


Figure 2: Temperature with respect to time at different sensors for the two-layer tank experiment. The bottom three sensors are represented by the blue, red, and green curves, and the two sensors initially above the salt water are represented by the orange and cyan curves.

the tank. At around 30 minutes, large plumes were seen rising from the convecting layer up to the top of the tank. These plumes can be seen as the large spikes in the orange plot, at the sensor second from the top. This was caused when mixing between the two layers had finally homogenized the tank. After 36 minutes, all five sensors are seen to have around the same temperature, demonstrating that convection is occurring throughout the tank.

Stratified Temperature Experiment

For this experiment, a tank was filled with seven layers of fluid (Figure 3) with cold water on the bottom and increasingly warm water to the top. The same procedures were followed as described in the previous section, with data measured by five temperature sensors at different vertical locations, and the bottom of the tank heated by a heating pad. Dye was dropped in to observe the convection, and a boundary was seen up to where convection was occurring, which rose with time.

To balance the heat coming into the system from the heating pad with the heat transfered through the tank via convection, the equation

$$\rho c_p \frac{dT}{dt} = \frac{H}{h} \tag{1}$$

should hold true. By assuming $\frac{dT}{dt}$ to be constant, a linear temperature distribution, one can solve for either h, the height of the boundary layer, or T the temperature at that height in terms of time t by the equation

$$h = \sqrt{\frac{2Ht}{\rho c_p \overline{T_z}}}. \tag{2}$$

A similar relation will be true for T, because the assumed linear temperature distribution means that $T = \overline{T_z} h$.

The temperatures recorded by the five sensors can be seen in Figure 4. Each sensor shows a roughly constant temperature, except for the top sensor, which was affected by cooling by air flow above it. Once convection reaches the level of a sensor, it follows the predicted square root dependence on time, as seen by the curve of best fit.

The vertical temperature gradient was plotted at several different times in Figure 6 to see how it was affected by convection. As time increases and convection occurs, a region of constant vertical temperature develops, and is pushed upward with time. By the end of the experiment, when the entire tank was convecting, the entire tank is seen to have a nearly constant temperature, accounting for cooling of the top by air flow.

By taking the logarithm of both sides of the temperature version of Equation 2, there should be a linear relation between $log(T)$ and $log(t)$. This can be seen in Figure 5. As expected, there is a linear relation between $log(T)$ and $log(t)$ for the temperature sensors inside of the convecting layer.

To calculate the efficiency of heat transport by the convection, the equation

$$H = \rho c_p \overline{\omega \delta T} \tag{3}$$

which predicts the heat transfer due to convection based on the time averaged temperature fluctuations and the time scale of the convecting plumes. To calculate the time scale of the convecting plumes, the equation

$$\omega^2 \simeq \frac{2}{3} agh\Delta T \tag{4}$$

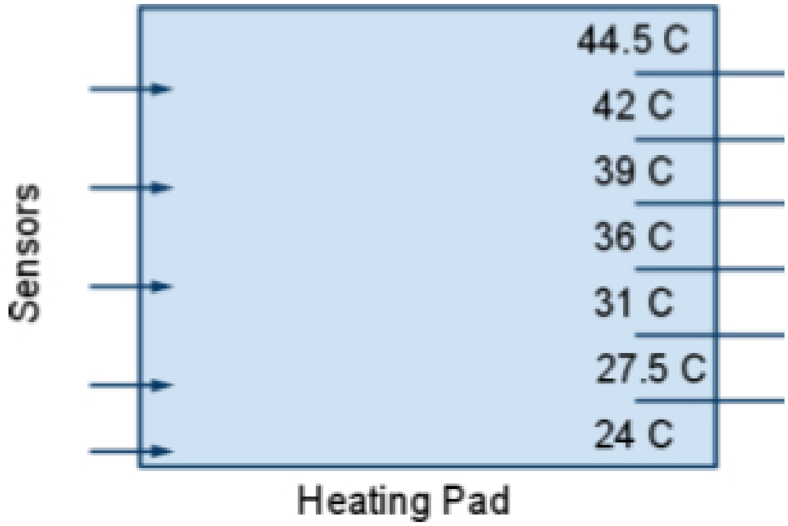


Figure 3: Diagram of experiment setup for stratified tank experiment.

was used, where h was the height of the convecting layer, and ΔT was the temperature fluctuation. The temperature fluctuations were estimated to be between 0.1 degrees C and 0.4 degrees C. The resulting values for ω were 0.25 and 0.5 cm/s, which was consistent with the observed plume speeds. These values were put into the equation for H, for an estimated value of between 2600 and 21000 W/m^2. This is a large range, but consistent with the heat supplied by the heating pad, which was 7750 W/m^2. The consistency here shows that convection in the tank is an efficient way to transport the heat from the heating pad.

3 Dry Convection in the Atmosphere

The stratified tank experiment should be an analogue to dry convection in the atmosphere, but a new temperature variable must be introduced. Because the dominating effect on temperature in the atmosphere is pressure, temperature decreases with altitude. The potential temperature, θ, is defined as

$$\theta = T(\frac{P_o}{P})^{\frac{R}{c_p}} \tag{5}$$

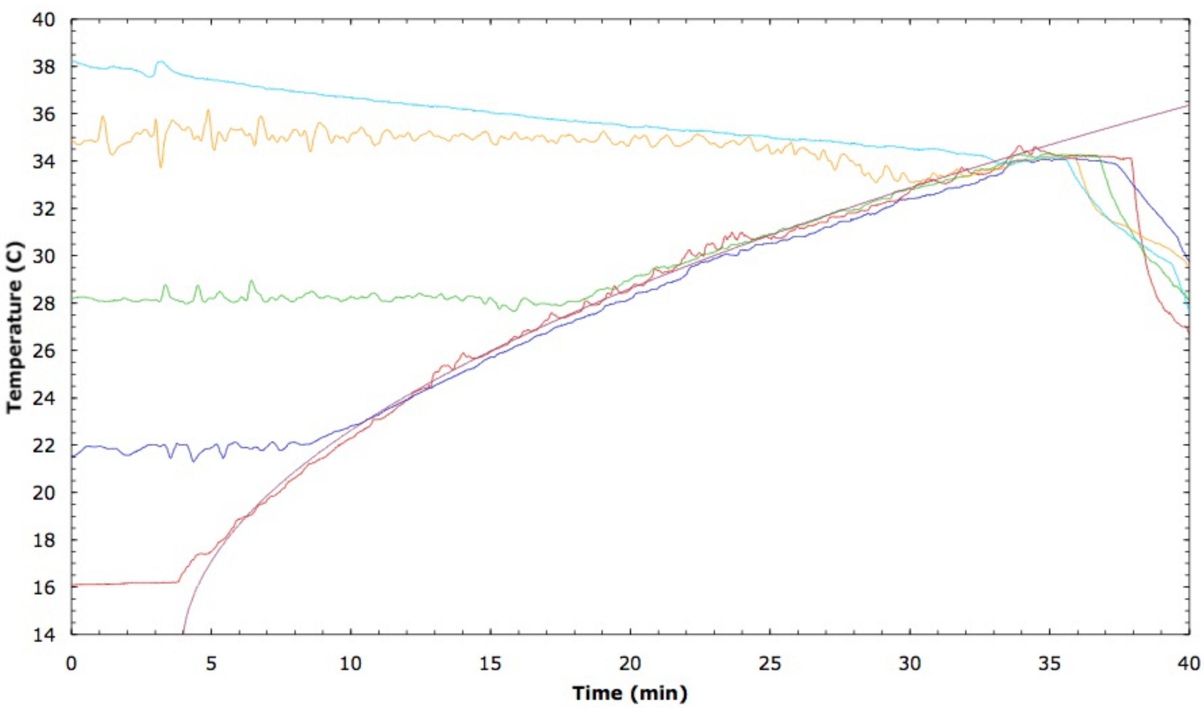


Figure 4: Temperature with respect to time at different sensors for the stratified tank experiment. The sensors are represented from bottom to top by red, blue, green, orange, and cyan. A line of best fit for the square root dependence is shown in brown.

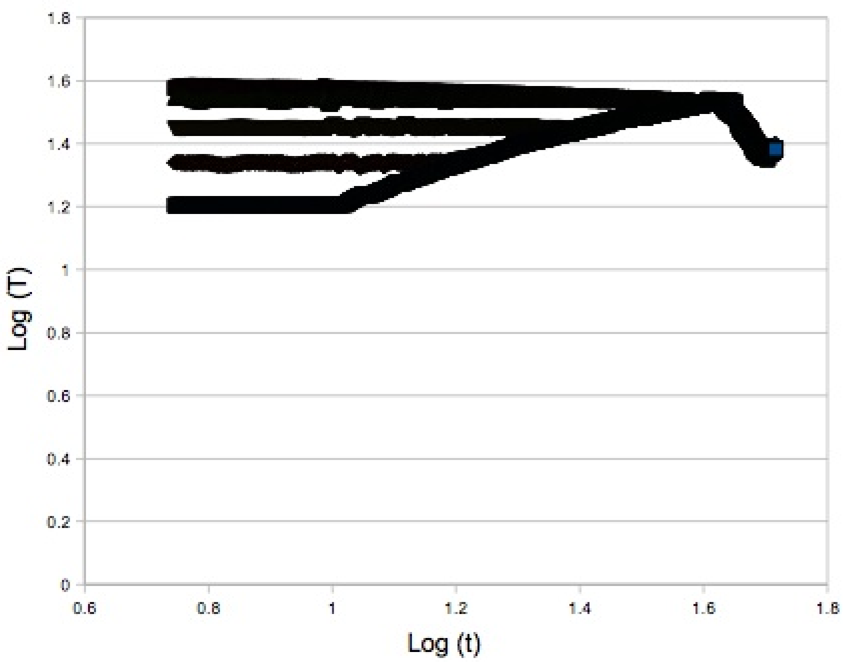


Figure 5: Log-log plot of the results of the five temperature sensors. The plots are flat until the convecting layer reaches the temperature sensor, and the plot becomes linear.

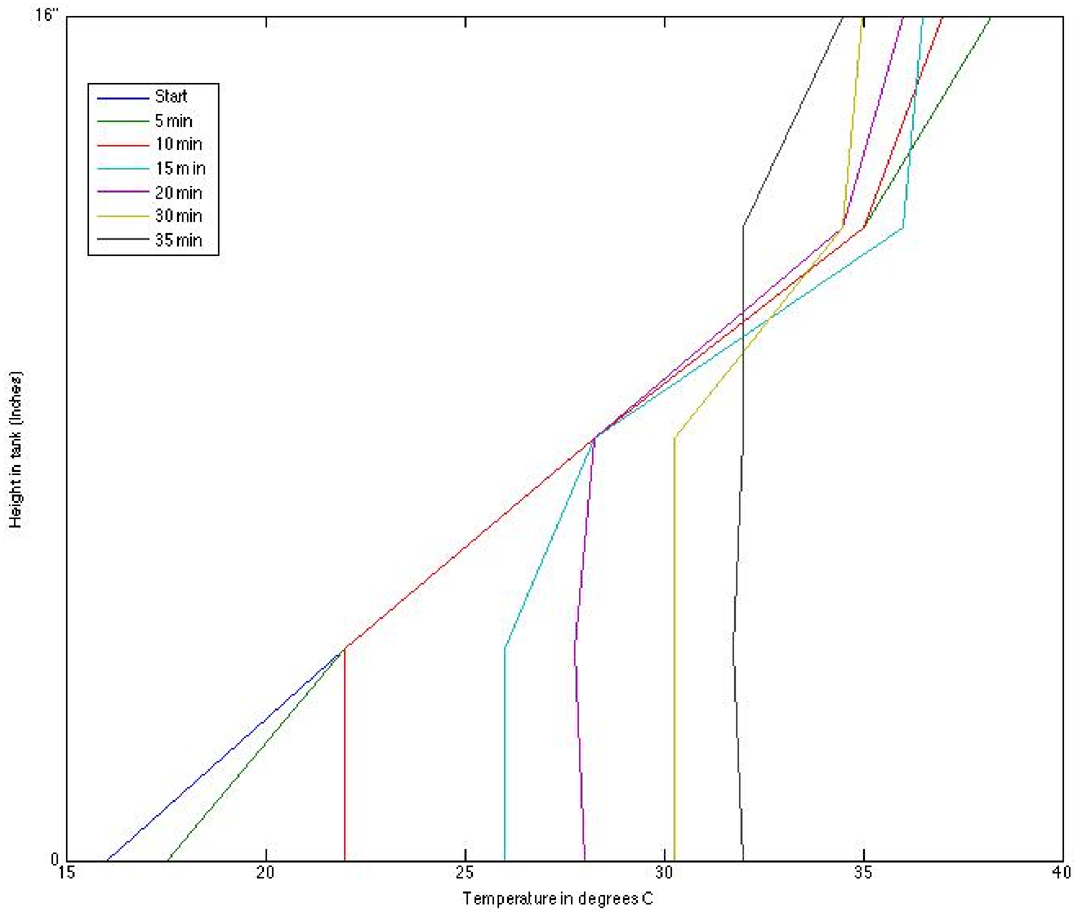


Figure 6: Vertical temperature profiles of stratified tank experiment at five different times throughout the experiment.

which is the equivalent of the temperature the parcel of air would have if brought to some reference pressure P_o adiabatically. Potential temperature follows the same stability trend as temperature, where if $\frac{\partial \theta}{\partial z} > 0$, the air is stable, but if $\frac{\partial \theta}{\partial z} < 0$, there is instability.

Dry Convection Example in Yuma, AZ

Dry convection was analyzed over the desert in Yuma, AZ. A dataset was used with temperature profiles taken several times over the course of one day, as well as measurements of the solar heat. The potential temperature profiles throughout the day can be seen in Figure 7. In the first two readings, the atmosphere is stable, with cool air below warmer air. By the third reading, there is a region of instability close to the ground, where the sun has begin heating the surface. The fourth reading shows a region where potential temperature is constant, which shows that convection is occurring there. This region of constant potential temperature grows over the next two readings, showing that the convecting layer is growing. This resembles the tank experiment, where the convecting layer grew as heat was continually supplied to the bottom.

To analyze the efficiency of convection, the internal energy of the atmosphere in the convecting layer was estimated, and compared to the solar flux (Figure 8) through the equation

$$H\Delta t = \frac{c_p}{g} \Sigma_{i=1}^{n} \Delta T_i \delta p_i. \tag{6}$$

Using the case of the two hours between the last two temperature readings, the right side of the above equation is equal to 2,559,000 Ws/m^2. This time period corresponds to 12:30 pm to 2:30 pm local time, which is the time when the solar flux was the greatest. The left side of the above equation is equal to 3,420,000 Ws/m^2. These numbers are in good agreement, assuming some loss of heat, or the imprecise assumption of the albedo equalling 0.5. This shows that convection is an efficient method of transferring solar heat away from the ground and up through the atmosphere.

4 Moist Convection in the Atmosphere

While dry convection can be described by potential temperature, convection in moist air is further complicated by the latent heat of the water vapor. As a moist parcel of air rises, it will cool, and water vapor will condense out of the parcel, increasing the energy of the parcel so it can rise further. A new quantity, the equivalent potential temperature, θ_e, can be defined, depending on the humidity. To simplify measurements, one can assume the air is saturated with moisture, and define the saturated equivalent potential temperature

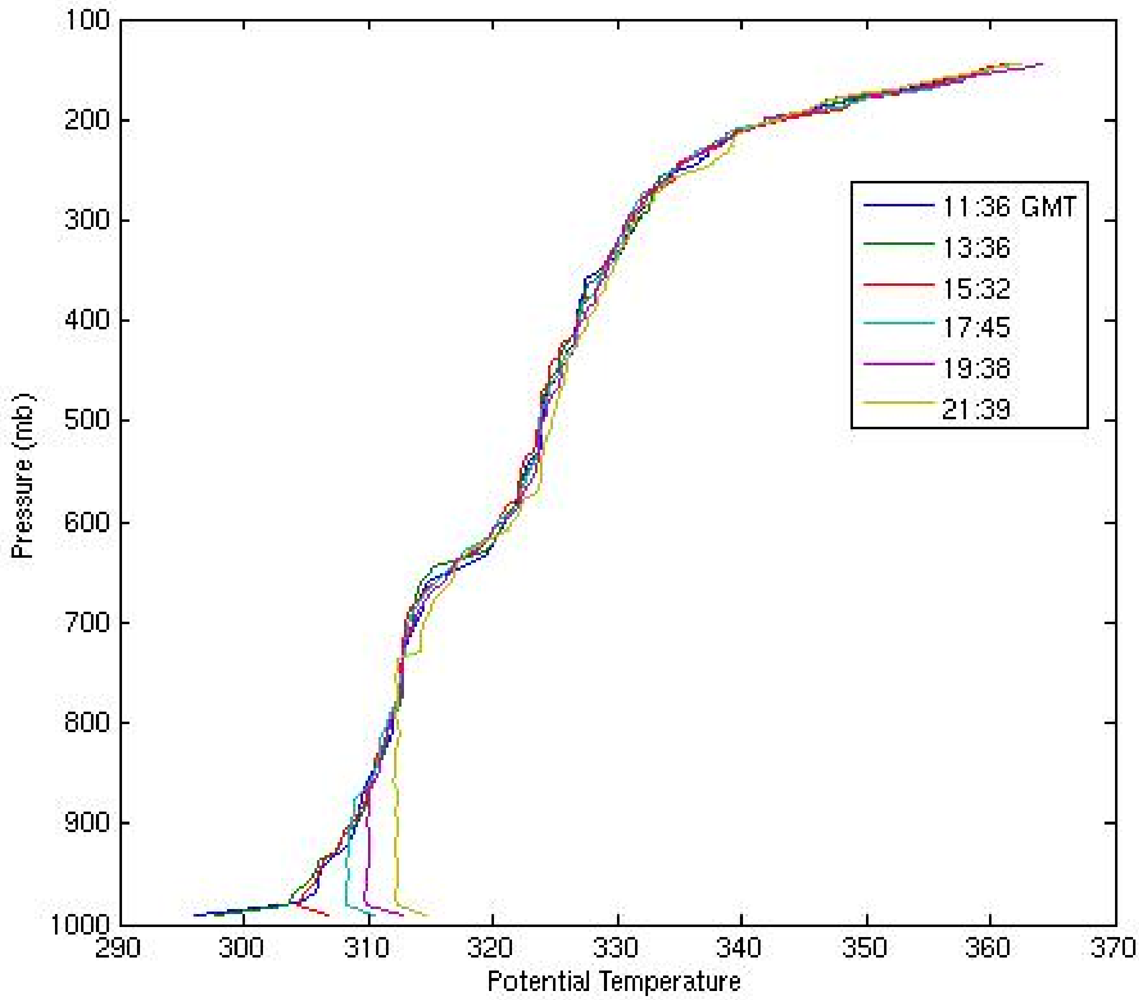
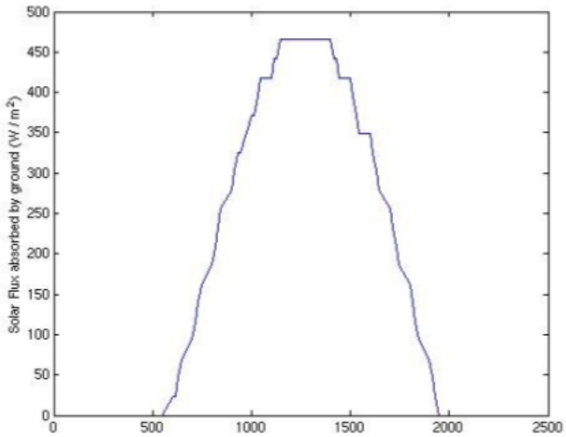


Figure 7: Potential Temperature against Pressure for different times for a day in Yuma, AZ. 11:36 GMT corresponds to 04:36 am in Yuma, AZ, and 21:39 GMT corresponds to 02:39 pm.



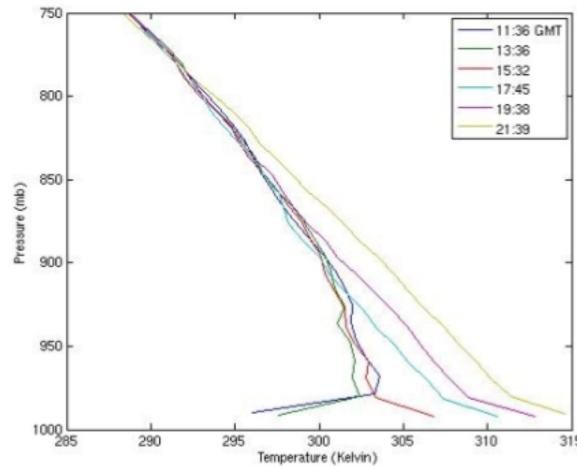


Figure 8: Solar Flux and Temperature in Yuma, AZ. The figure on the left shows the solar flux with respect to time from midnight to midnight the next day. The figure on the right shows the temperature profiles of the lower troposphere taken at six times throughout the day.

as

$$\theta_e^* = \theta exp(\frac{Ldq*}{c_p T}) \tag{7}$$

where θ is the potential temperature as described earlier, L is the latent heat, $dq*$ is the change in the saturation specific humidity, c_p is the heat capacity, and T is the temperature.

Moist Convection Example in Charleston, SC

A storm passing through Charleston, SC on April 13th, 2009, was analyzed to study moist convection. Radar images of the storm at 17z and 3z, before and after the storm passed through, can be seen in Figures 9 and 10.

The saturated equivalent temperature was plotted for four times throughout the day, and is shown in Figure 11. The first three profiles show instabilities, where θ_e^* decreases with increasing pressure. Radar images show that the storm passed over the area around the time of the third profile, and by the time of the fourth profile, the atmosphere is seen to be neutral, as there is no negative slope to θ_e^*.

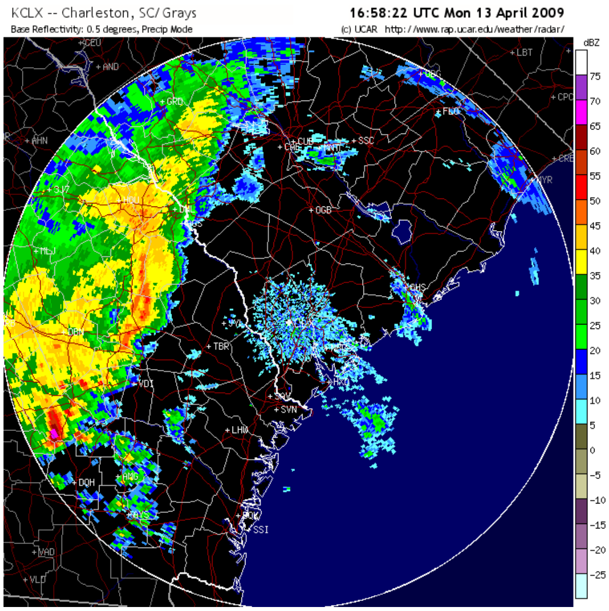


Figure 9: Radar image of the Charleston, SC area at 17z. Charleston is indicated by the white cross. A storm can be seen traveling towards Charleston. (UCAR)

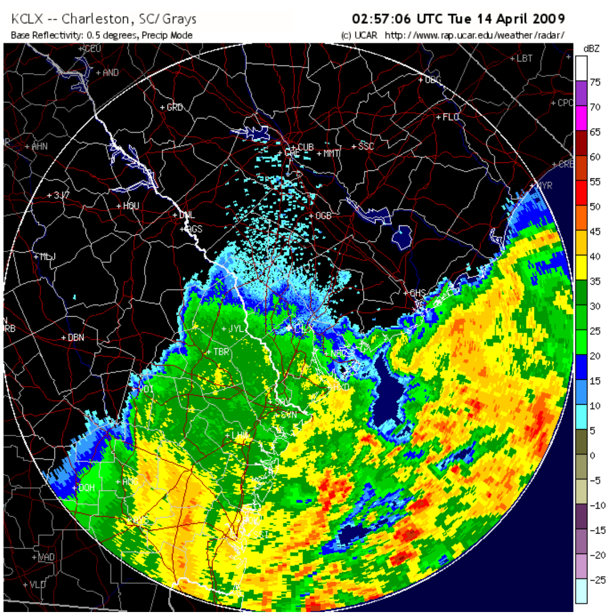


Figure 10: Radar image of the Charleston, SC area at 3z. Charleston is indicated by the white cross. A storm can be seen to have just passed through Charleston. (UCAR)

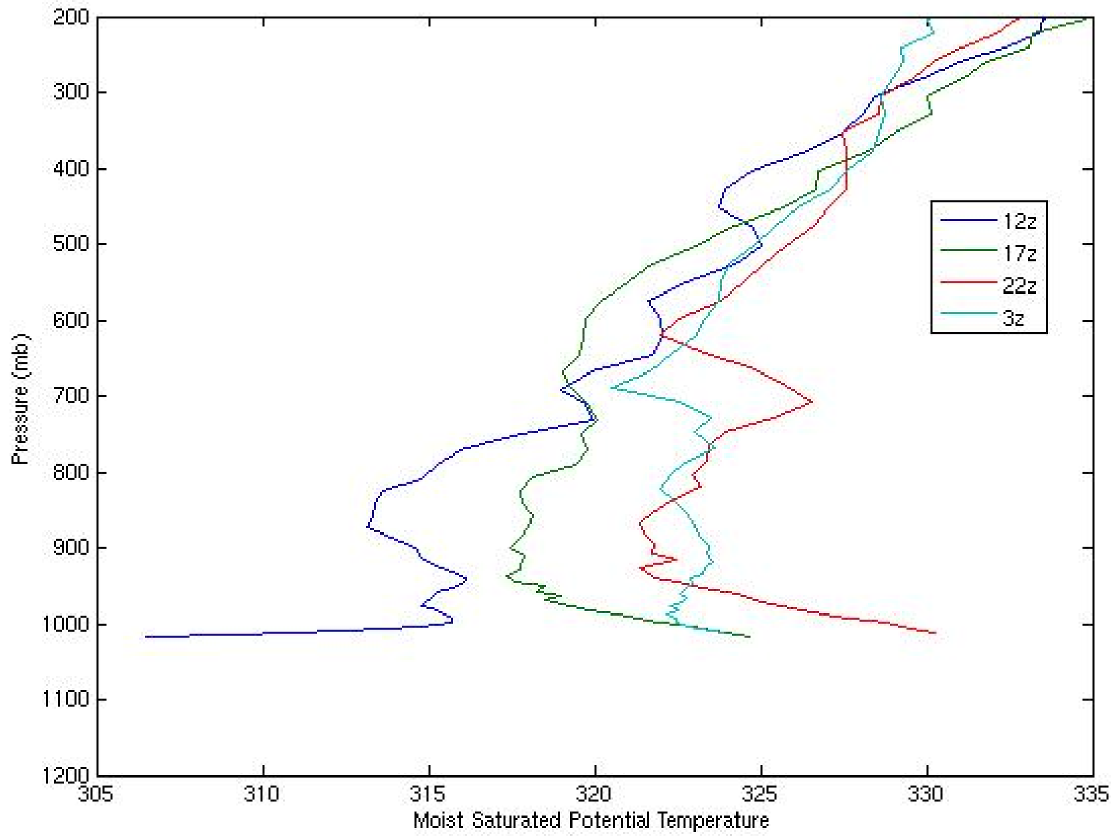


Figure 11: Saturated Equivalent Potential Temperature (θ_e^*) against Pressure for a day in Charleston, SC.

5 Conclusions: Comparison of Lab and Atmospheric Results

Convection in both dry and moist air can be well modeled by a tank experiment in which solar heating of the ground is replaced by a heating pad. Instabilities can be seen to lead to convection in fluids, assuming the correct temperature variable is used. By defining different conserved quantities, the same trends of convection removing instabilities can be seen in water, dry convection, and moist convection. Convection was seen to efficiently transport the heat supplied to the system in both the stratified tank experiment and the dry convection case study.

References

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