Balanced Vortex
AOS-801
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Introduction

The right combination of balanced forces can yield vortex phenomena such as hurricanes. This experiment is designed to observe those forces in the rotating tank and compare the obtained data to a past hurricane. The particle tracker software will be utilized in order to determine surface flow speeds which can be incorporated into the Rossby number equation. Actual and theoretical Rossby numbers will be computed for the tank experiment which will determine if the flow is geostrophic, cyclostrophic, or in gradient wind balance. The same will be done for a past hurricane using scatterometer data to estimate wind speeds. The three Rossby numbers will be compared and a conclusion that Rossby numbers much greater than one, indicating cyclostrophic flow, near the vortex will be shown.

Motivation

Hurricanes intrigue us from a synoptic standpoint as well as a shear fascination standpoint. Since neither of us have the opportunity to research hurricanes, an experiment like this was a great way for us to further explore and understand another area of weather. We wanted to be able to connect the fluid dynamics we have learned this year to something that really sparked our interest and curiosity.

Theory

As this is the "balanced vortex" experiment, the balances at play in the horizontal and vertical are of primary importance in understanding flow behavior. In the vertical, hydrostatic balance is used to describe the system (1). This assumption is maintained in the atmosphere except in the case of extremely strong small scale disturbances. It is also a good approximation within the tank for the same reason, as the pressure is simply the weight of the water directly above a particular location. It is worth noting that the pressure is considered zero above the surface of the water because the pressure from the atmosphere is sufficiently small and does not vary over the surface of the tank.

$$\frac{\partial p}{\partial z} = -\rho g \quad (1)$$

In the horizontal, we use the inviscid momentum equations in the natural coordinate system (2). The assumption that the flow is inviscid is not accurate in all cases, but for this simplified tank experiment friction will have little effect away from the edges. In the atmosphere, analysis will be performed at a distance sufficiently far from the surface to minimize frictional effects.

$$\frac{V^2}{R} + fV = -\frac{\partial \Phi}{\partial n} \quad (2)$$

In this equation, *V* is azimuthal velocity, *R* is the radius from the center of the vortex, *f* is the Coriolis parameter, and the right-hand side of the equation is the change in geopotential in natural coordinates. Through use of the hydrostatic balance equation, we are able to convert the pressure gradient force into a term based upon the departure from the reference height of the water (3).

$$\frac{v_\theta^{\ 2}}{r} = \frac{g\partial \eta}{\partial r} - 2\Omega v_\theta \quad (3)$$

Where v_θ is the azimuthal speed, *r* is the radius, *g* is gravity, $\frac{\partial \eta}{\partial r}$ is the departure from the reference height of the water with respect to radius, and $-2\Omega v_\theta$ is the Coriolis acceleration.

Now that a general equation has been established to determine the forces affecting the flow, we must use a method to determine which forces are dominant. The Rossby number (4) is particularly useful for this as it is the ratio of acceleration effects to Coriolis effects where v_θ is the azimuthal speed, *f* is the Coriolis parameter, and *r* is the radius.

$$Ro = \frac{v_\theta}{fr} \quad (4)$$

From this number we can determine 3 balances: geostrophic, cyclostrophic, and gradient wind. In geostrophic flow, the curvature term is much smaller than the other terms and as a result the flow is balanced between Coriolis forces and pressure gradient forces. This is represented by a Rossby number much less than one. Cyclostrophic flow is the balance between curvature effects and pressure gradient forces when the horizontal scale is small enough to be neglected. It is represented by Rossby numbers much larger than one. The gradient wind balance occurs when all terms are of similar magnitude and is represented by Rossby numbers close to unity. The Rossby number is also useful because a theoretical Rossby number can be derived through application of conservation of angular momentum beginning in the non-rotating framework (5).

$$v_\theta r = \Omega r_1^{\ 2} \quad (5)$$

Next, we convert this to the rotating framework by subtracting the rotation rate multiplied by the radius, and solve for the azimuthal velocity. We then put this velocity into the Rossby number (4) which results in a theoretical Rossby number (6) where r_1 is the radius of the tank and *r* is the radius from the center to the particle. This number has been compared to the data gathered in both the wet lab and dry lab sections.

$$Ro_{theory} = \frac{1}{2}\left(\frac{r_1^{\ 2}}{r^2} - 1\right) \quad (6)$$

Wet Lab

Experiment:

The experiment was performed several times at varying rotation rates. Initially, a non-rotating case, a 10 rpm case, and a 15 rpm case were observed. Red dye was added during the experiment which made it too difficult to obtain data through the particle tracker software. Therefore, two more cases at 10 rpm and 20 rpm were conducted without the addition of red dye in order to obtain velocity data.

The setup of the experiment was the same for all of the cases and relatively simple to prepare. Three small beakers (~100 mL each) were inverted and placed evenly around the center of the tank. Then a cylindrical, plastic bucket (30 cm in diameter by 50 cm high) was placed on top of the beakers. A hole approximately 1 cm was drilled in the center of the bucket and a stopper plugged the hole. The bucket was filled three quarters full of water and blue dye was added for visualization.

Once steady-state equilibrium was reached, black paper dots were added to the surface and a large metal rod was used to quickly push the stopped out of the bucket. This created a low pressure gradient over the hole and thus the water began to flow radially inward. Through conservation of angular momentum, a vortex formed at the hole and the paper dots swirled down through the hole. Red dye was added during the first few experiments so that the swirling motion could be visualized.

Data and Calculations:

The data from the particle tracker is originally formatted as the number of pixels away from the top right corner of the recorded image. As such the first alteration to the data was to move the origin point to the center of the hole in the tank and convert from pixels to centimeters. Figures 1 and 2 depict particle tracks, in the 10 rpm and 20 rpm cases respectively, after such conversions. The tracks are relatively smooth as they circle the drain with decreasing radius, though they decrease in smoothness as they approach the center. This decreasing smoothness is due to an increase in velocity. In Figure 2, this lack of smoothness indicates a major source of error in calculating the Rossby numbers, specifically the particle may have traveled much further than calculated if it has circled the drain multiple times. There is also a lack of data near the center. This is due to the particle tracker being unable to keep up with the particle, and also due to a decreased contrast between the black dots and the dark drain hole.

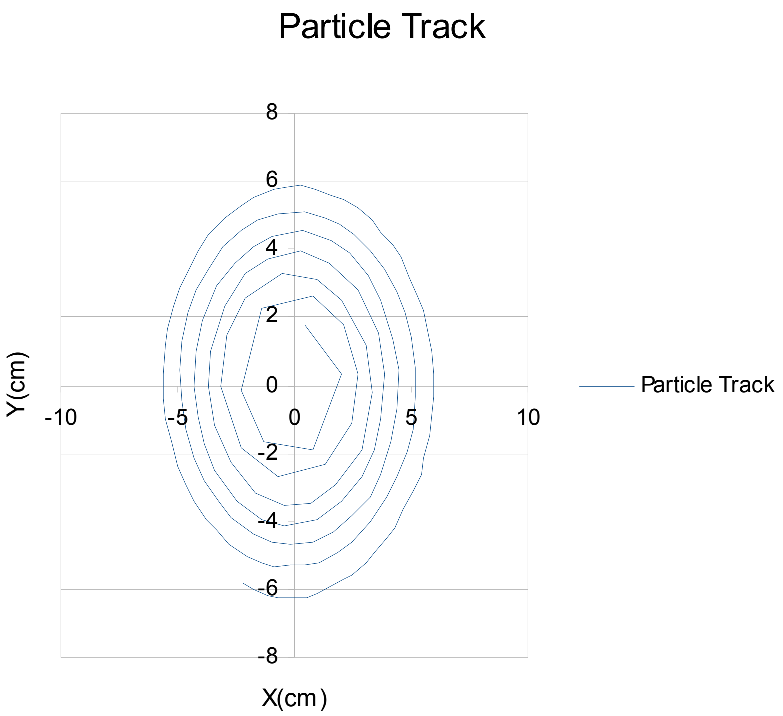

Figure 1: 10 rpm particle track plot.

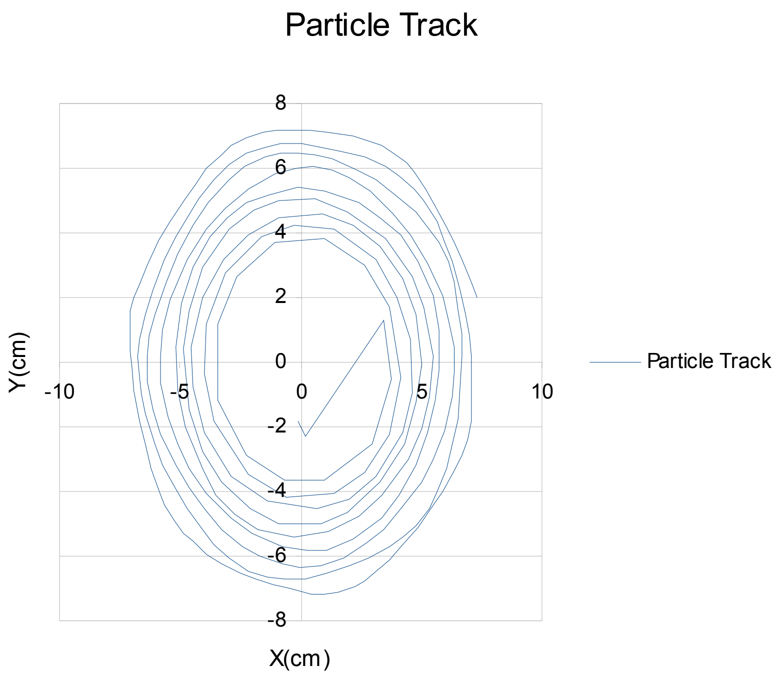

Figure 2: 20 rpm particle track data.

After this data has been converted into azimuthal velocity, it can be used in the Rossby number (4) and is depicted along with the theoretical Rossby number in Figures 3 and 4. This calculation introduces another source of error regarding the radius of the tank. More specifically, as the water drains from the tank the surface gets further from the camera. As such it takes up a smaller portion of the screen and in turn the apparent radius of the tank shrinks. This has been taken into consideration and adjusted for by changing the radius by approximately .006 centimeters per millisecond in the 10 rpm case and .004 centimeters per millisecond in the 20 rpm case. These numbers were obtained through careful examination of the video. As seen in Figures 3 and 4, the experimental values correlate well with the theoretical values. The 20 rpm case correlates slightly better than the 10 rpm case. From the figures, it can be determined that the balance of forces is geostrophic near the outer wall because the Rossby number asymptotically approaches 1. Near the center, the Rossby number is much larger than 1 which indicates cyclostrophic balance. Therefore, between the wall and the center, there must be a region of gradient wind balance.

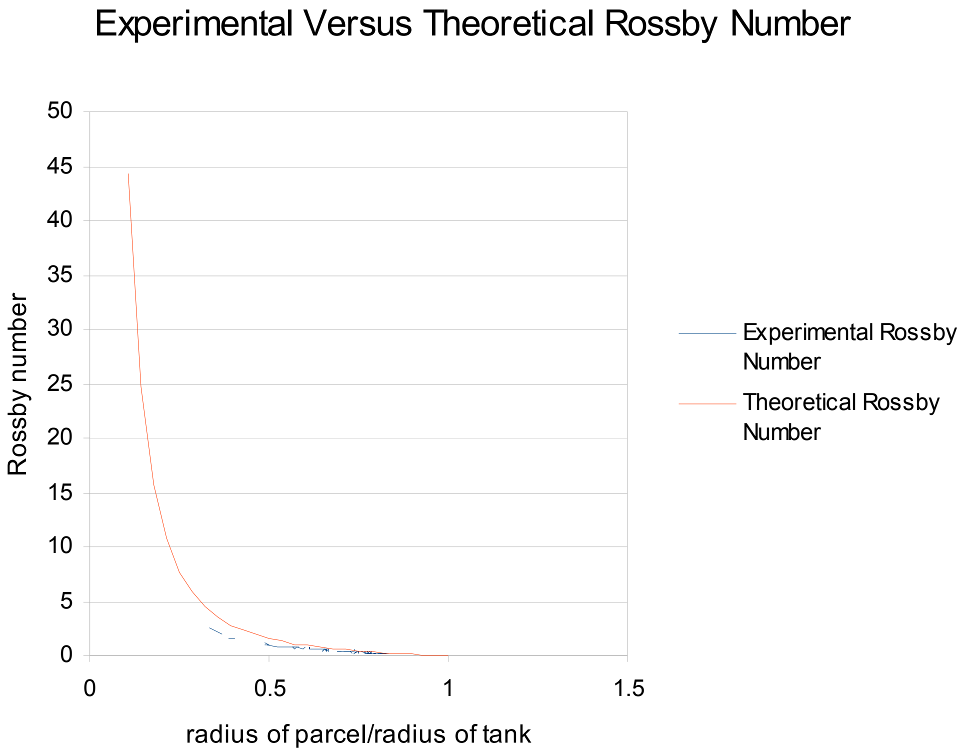

Figure 3: Theoretical vs. Experimental Rossby numbers for 10 rpm.

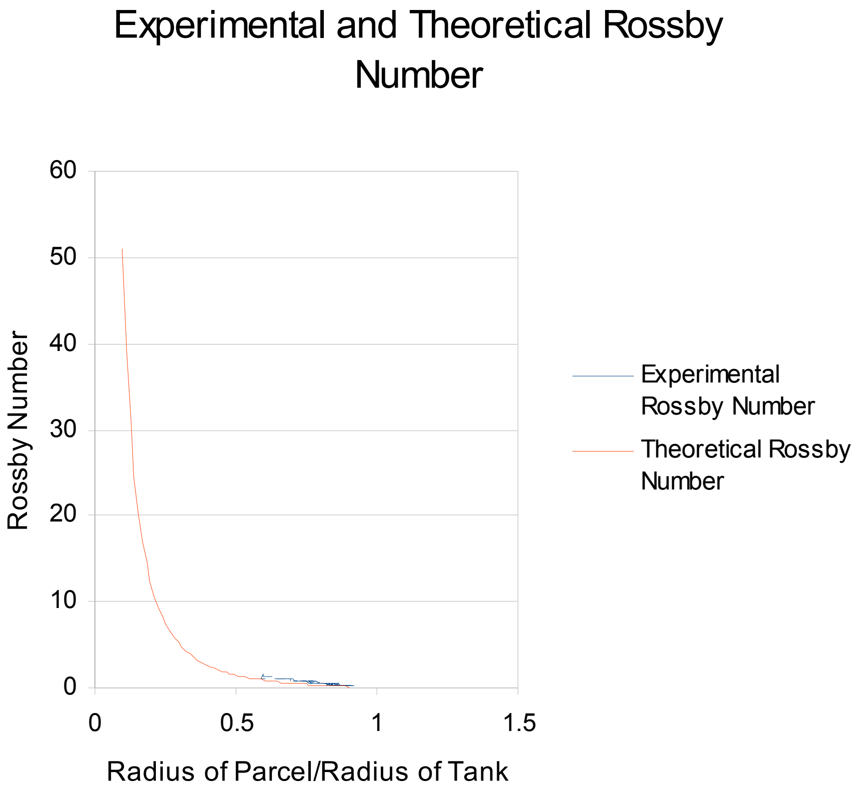

Figure 4: Theoretical vs. Experimental Rossby numbers for 20 rpm.

Dry Lab

In this section of the lab, scatterometer data was used to calculate Rossby numbers for Hurricane Ivan, and compare those results to the results from the tank experiment. This can be done because in a hurricane, air flows radially inward toward a low pressure center. Due to conservation of angular momentum, the air flow increases rapidly as it approaches the center (or vortex). This is exactly what happened with the tank experiment where water flowed inward toward the low pressure center and increased speed.

The scatterometer data was obtained through the "Weather in a Tank" website (www-paoc.mit.edu/12307/mass wind/scatterometer_instructions.htm) which had access to the data from Remote Sensing Systems. After the data was downloaded for Ivan, it was run through a MATLAB program provided through the website in order to generate wind speed data shown in Figure 5. Hurricane Ivan was a category 4 storm on the Saffir-Simpson scale at the time this data was collected on September 14, 2004. It made landfall around Orange Beach, AL on September 16, 2004 as a category 3.

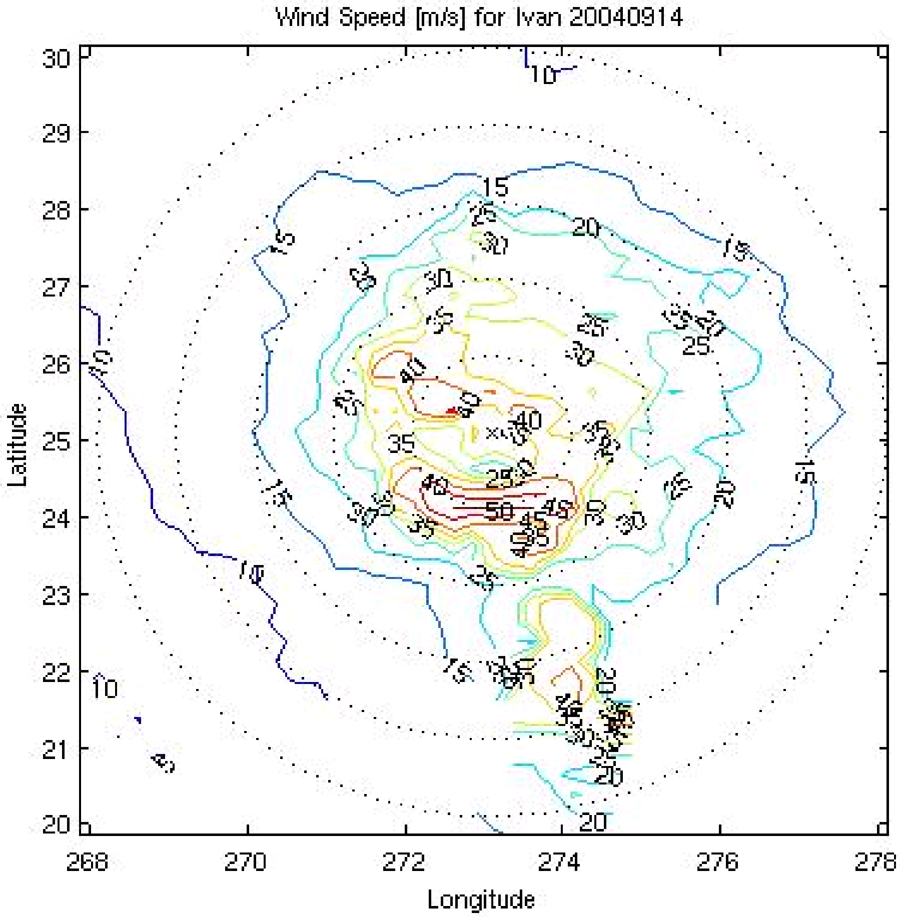

Figure 5: Wind speed for Hurrican Ivan on September 14, 2004
plotted with respect to longitude and latitude.

A scatterometer measures backscatter radiation from microwave pulses and is a useful tool in order to measure ocean surface wind speeds as in Figure_. The dotted circles in the figure are approximately 100 km apart and the winds are about parallel to the dotted circles. We calculated the Rossby number by roughly estimating the average wind speed at a given circle via the standard formula (4). Figure 6 denotes the various radii used and the black line indicates the latitude of the center of the storm. Table 1 shows the data obtained from the figure.

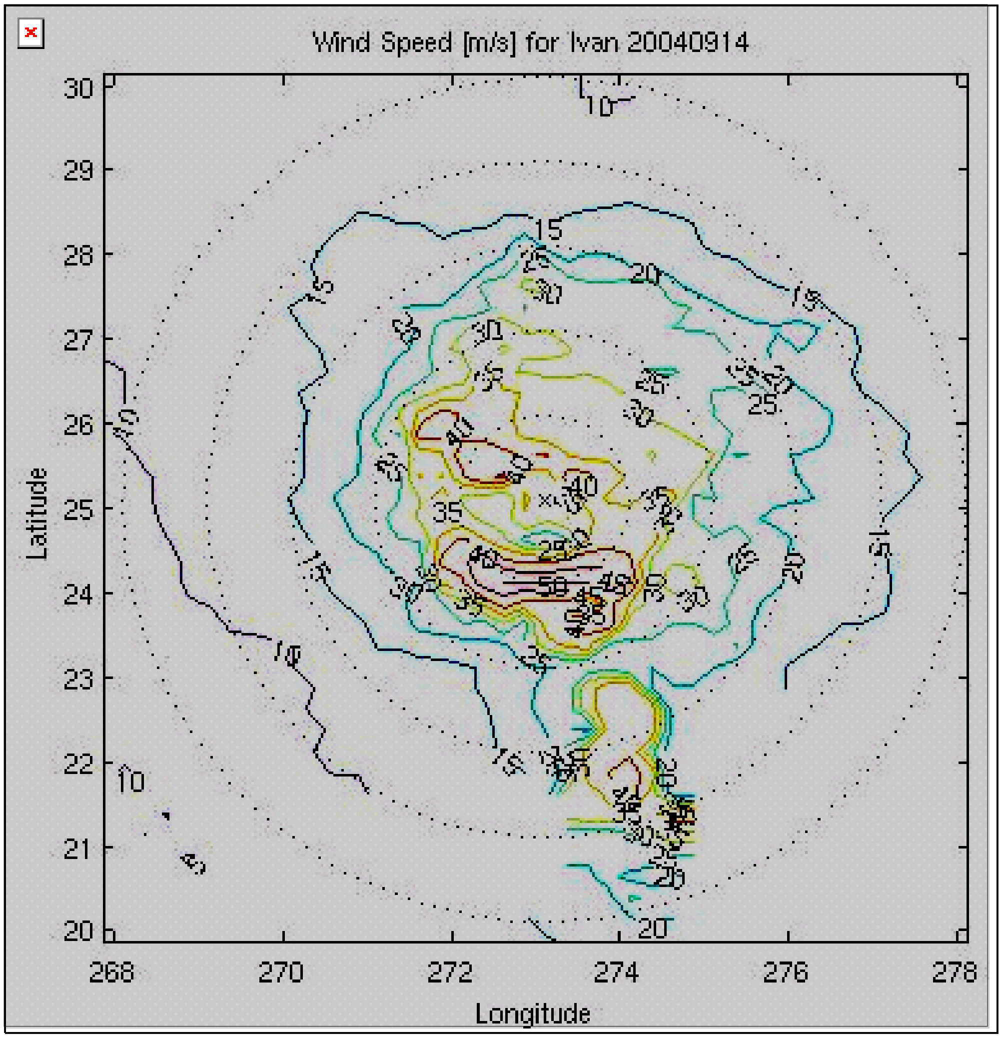

Figure 6: Same as Figure 5 but with radii and latitude highlighted.

Color	*Distance from Center (radius)*	*Average Wind Speed*
Pink	300 km	20 m/s
Burgandy	200 km	25 m/s
Purple	100 km	35 m/s
Blue	50 km	40 m/s
Latitude ~ 25.2°		

Table 1: Estimated *v*, *r*, and φ.

The calculated Rossby numbers for Hurricane Ivan at the four selected distances are shown below.

$$Ro_{50km} = \frac{40\,{}^{m}\!/\!{}_{s}}{2\Omega\sin(25.2°)(50km)} = 12.88$$

$$Ro_{100km} = \frac{35\,{}^{m}\!/\!{}_{s}}{2\Omega\sin(25.2°)(100km)} = 5.64$$

$$Ro_{200km} = \frac{25\,{}^{m}\!/\!{}_{s}}{2\Omega\sin(25.2°)(200km)} = 2.01$$

$$Ro_{300km} = \frac{20\,{}^{m}\!/\!{}_{s}}{2\Omega\sin(25.2°)(300km)} = 1.07$$

From these calculations, we can deduce that the balance of forces close to the vortex (center) is predominantly cyclostropic since the Rossby numbers are much greater than 1. This means that the pressure gradient and centrifugal forces dominate because the horizontal scale is so small that the Coriolis force can be neglected. The further away from the vortex, the more geostrophic the flow becomes.

Conclusion

The theoretical and calculated Rossby numbers from the tank experiment and the calculated Rossby numbers from the hurricane data all correlate well. In each case, the Rossby numbers become much greater than 1 near the vortex which indicates the flow is cyclostrophic. Errors were introduced into the wet lab due to the particle tracker being unable to track the paper dots close to the vortex center. This is because the dots were moving too quickly and there was a reduced color contrast at the vortex. Errors were also introduced in the dry lab due to the rough estimation of average wind speed from the scatterometer data. The size of the vortex is directly proportional to the rotation rate. Thus, the higher the rotation rate the larger the vortex as indicated in the wet lab. The Rossby numbers for the dry lab are significantly greater in magnitude than the Rossby numbers for the wet lab. This is primarily due to the fact that data can be obtained for the vortex (eye wall) of a hurricane whereas it cannot be obtained in the tank for reasons mentioned previously.

References

“Balanced Vortex: Introduction.” *Weather in a Tank*. EAPS. 27 May 2009 <http://www-paoc.mit.edu/labguide/balmo.html>.

Holton, James R. *An Introduction to Dynamical Meteorology*. Fourth. Amsterdam: Esevier Inc., 2004.