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12.307 project 3: Fluids lab

# CONVECTION

## Abstract

This lab report will explore convection in the atmosphere and in the laboratory. In the laboratory, the fluid used is water which is incompressible whereas in the atmosphere, the fluid is compressible and obeys the Ideal Gas Law equation. For this reason, we go into some detail about convection in both compressible and almost-incompressible fluids. Atmospheric data will then be used to study convection and similarities from the experiment drawn.

## 1. Introduction

The radiation from the sun warms up the ground. Due to a layer of water vapor, terrestrial radiation from the ground is absorbed thus the only terrestrial radiation comes from the upper troposphere. The the earth has to emit energy to reach equilibrium. Warming at the surface triggers convection which transports heat vertically upward.

Convection will be simulated in the laboratory using a tank of water with a heating pad at the bottom. For the atmosphere, dry convection over the earth will be studied and compared to convection in the tank.

## 2. Convection in water (an almost-incompressible fluid)

Incompressible fluids do not obey the Ideal Gas Law equation and as a result, density is independent of pressure but density is temperature dependent. A good approximation for water is

$$\rho = \rho_{ref}(1 - \alpha T), \tag{1}$$

where $\rho_{ref}$ is a constant reference value of the density and $\alpha$ is the coefficient of thermal expansion.

Consider a horizontally uniform state with temperature T(z), and $\rho(z)$ defined by (1). Let's focus on a

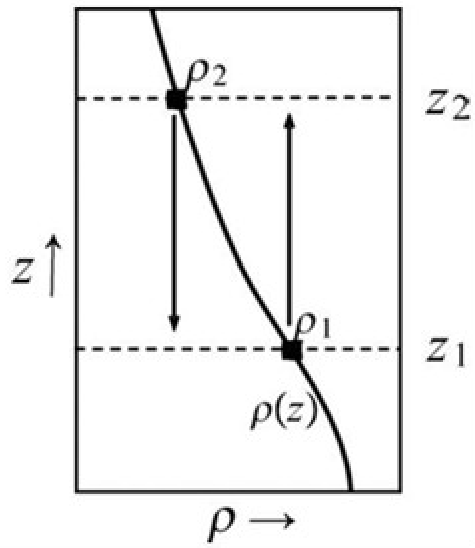



*Figure 1: Parcel Stability*

single parcel of fluid P, initially located at $z_1$ .It has temperature $T_1 = T(z_1)$ and density $\rho_1 = \rho(z_1)$ which is the same as its environment. It is in equilibrium as it is neutrally buoyant. See Fig.1 above.

Now let's consider a case whereby the parcel is displaced from $z_1$ to $z_2 = z_1 + z\delta$ where $z_2$ is vertically above $z_1$. T will be conserved during the displacement if the displacement is done quick enough since the internal energy of a liquid depends only on its temperature. The temperature and density of the parcel will remain the same as before but the environment will now have density

$$\rho(z_2) \sim \rho_1 + z\delta^*(d\rho/dz)_{e}, \qquad (2)$$

where $(d\rho/dz)_e$ is the environmental density gradient.

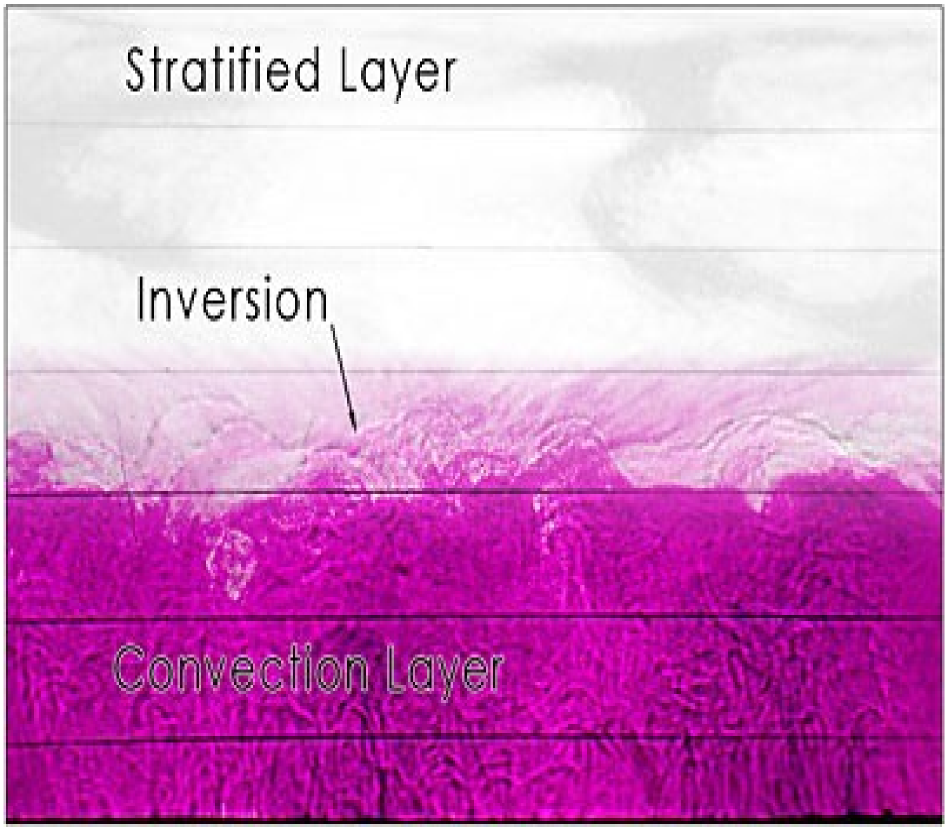
The buoyancy of the parcel depends on the difference between its density and that of its environment. It is positively buoyant when the environmental density gradient is positive, neutrally buoyant when environmental density gradient is zero and negatively buoyant if environmental density gradient is negative.

If a parcel is positively buoyant, it accelerates upwards. Therefore, for stability to be achieved in an incompressible fluid, density should decrease with height. If the liquid is unstable, convection comes in to correct this. In summary, parcel is unstable when $d\rho/dz < 0$, stable when $d\rho/dz$ is $> 0$ and neutral when $d\rho/dz = 0$.

## 3. Laboratory Experiment

To study convection in the lab, a glass tank with a heating pad on its base is used. A piece of white paper is glued on to one face of the tank. The paper has horizontal lines 3 cm apart. A bright light is placed on the opposite side of the tank away from the side with the white paper. Porous sponges are first placed in on top of heating pad and water is then poured into the tank. The water temperature starts from 25 °C to 45 °C . The tank is linearly stratified by temperature since we can measure heat by thermometers. Salt can also be used to stratify using density but it's hard to measure the density in each layer even though it would provide a better stratification. For this experiment, ten layers were used. Ten thermometers were attached in the inner wall of the tank with 3 cm separating each one of them. So once we get to the maximum temperature and all the thermometers are under water, we carefully remove the porous sponges with minimal disturbance and switch on the heating pad. A sprinkle of permanganate is dropped in the water and started observing plumes forming as convection was occurring. A timer is then used to measure how long it takes for a bubble to move from the bottom of the tank to the top and the distance and time recorded. The times it took for various convective layers to be reached after complete mixing had occurred was also recorded. The experiment is left to run for approximately 30 minutes and the data from the thermometers is taken and manipulated on a computer. The heating from the base triggers the convection.

*Figure 1: Photograph showing the convection layer from lab experiment*



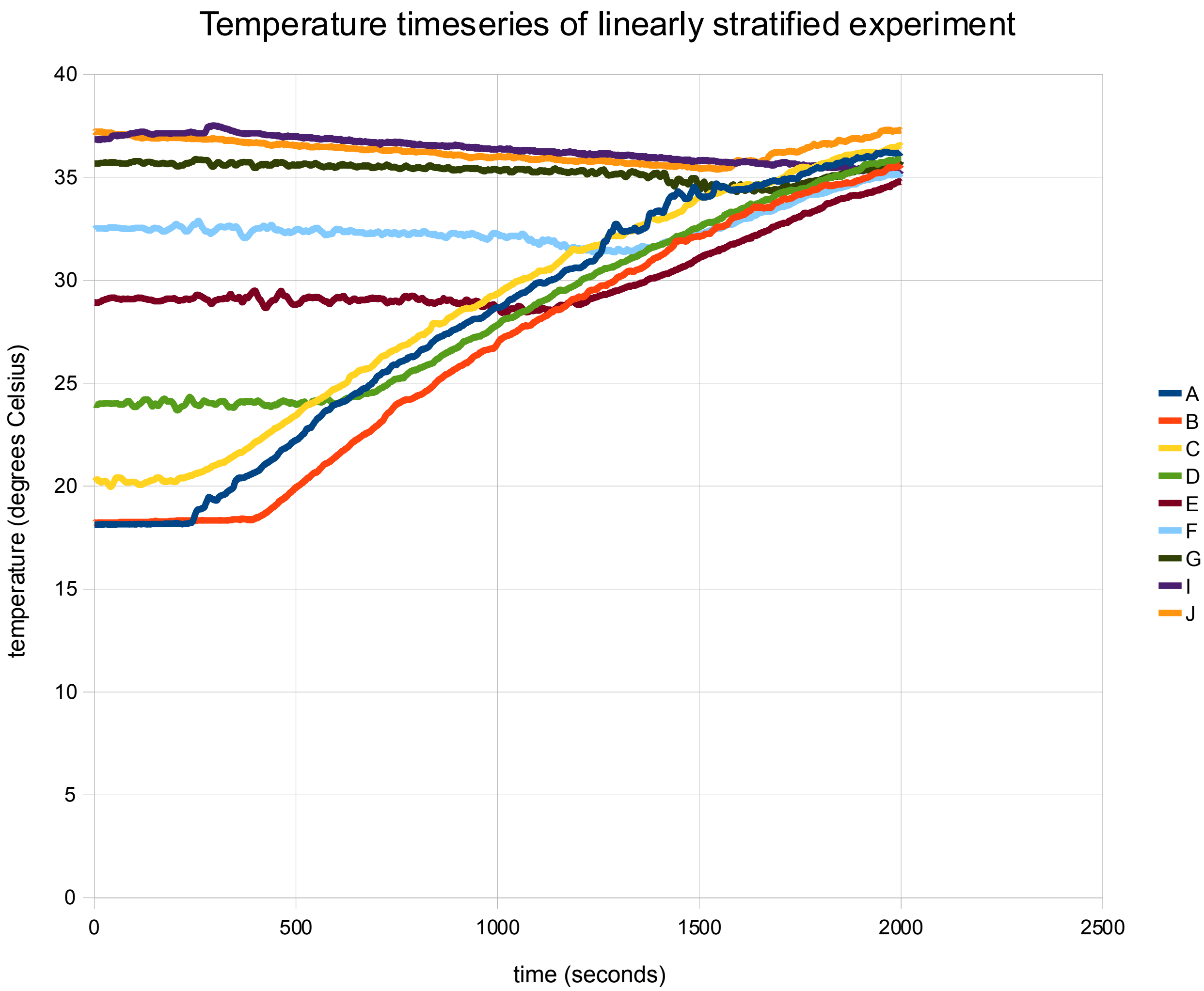



*Figure 2: Plot of how temperature varies with time in the stratified experiment with ten thermometers*

## 4. Dry Convection in a compressible fluid

The stability of a parcel of air in a compressible fluid such as the atmosphere is going to be considered. As the parcel of air rises, it moves into a zone of lower pressure and expands. As a result, the parcel of air cools adiabatically.

The rate at which the temperature decreases with height under adiabatic displacement is called the dry

adiabatic lapse rate $\Gamma_d = -g/c_p \sim 10$ Kelvin per kilometer.

The stability of the profile depends on how dT/dz of the environment varies relative to $\Gamma_d$ . According to the notes by Lodovica Illari and John Marshall on convection, when dT/dz is less than $-\Gamma_d$ , the profile is unstable. If dT/dz is greater than $-\Gamma_d$ , then the profile is stable. The profile is neutral in the case whereby $dT/dz = -\Gamma_d$ .

Since T is not conserved under adiabatic displacement, it is not an ideal measure of atmospheric thermodynamics. Therefore, potential temperature is used. The definition of potential temperature is the temperature an air parcel would have if it were compressed adiabatically from its existing pressure and temperature to a standard pressure. It is denoted by the symbol $\Theta$.

$$\Theta = T(p_0/p)^k \qquad (3)$$

with $k = R/c_p = 2/7$ and conventionally $p_0 = 1000$ mb.

Now the stability criterion in terms of potential temperature becomes stable if the change of potential temperature with height is positive, unstable if the change of potential temperature with height is negative and neutral if potential temperature does not change with height. ($d\Theta/dz > 0$ when stable,

$d\Theta/dz < 0$ when unstable and $d\Theta/dz = 0$ when neutral.

Since the behavior and some properties of compressible fluid (air) have been covered, some atmospheric examples that give similar results as what was observed in the laboratory experiment follow in the section below.

## 5. Atmospheric Examples of Convection

The stratified experiment done in the lab is comparable to the atmospheric observations of dry convection occurring in a region in Saudi Arabia and also in Yuma desert in Arizona. Vertical profiles of potential temperature instead of temperature are then plotted since air is a compressible fluid.

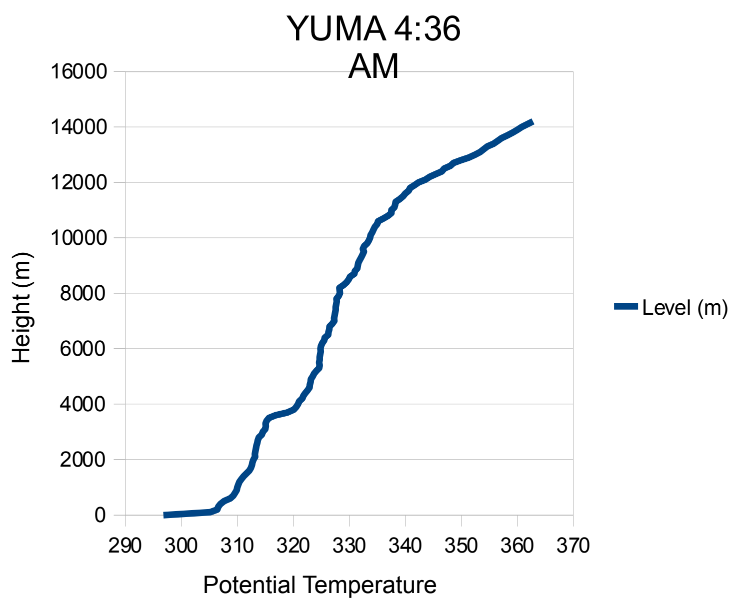



*Figure 3: Yuma desert temperature profile at 4:36 AM (June 18, 2007)*

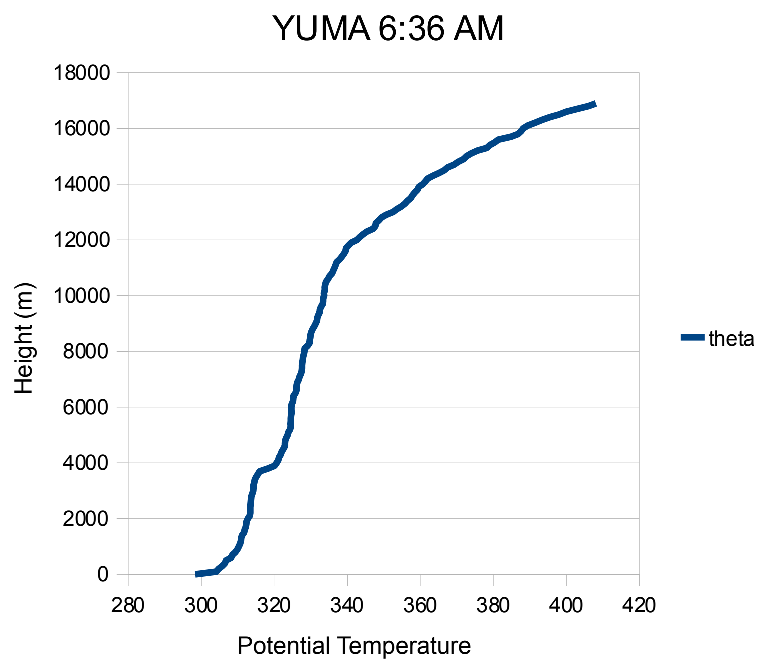



*Figure 4: Yuma desert temperature profile at 6:36 AM (June 18, 2007)*

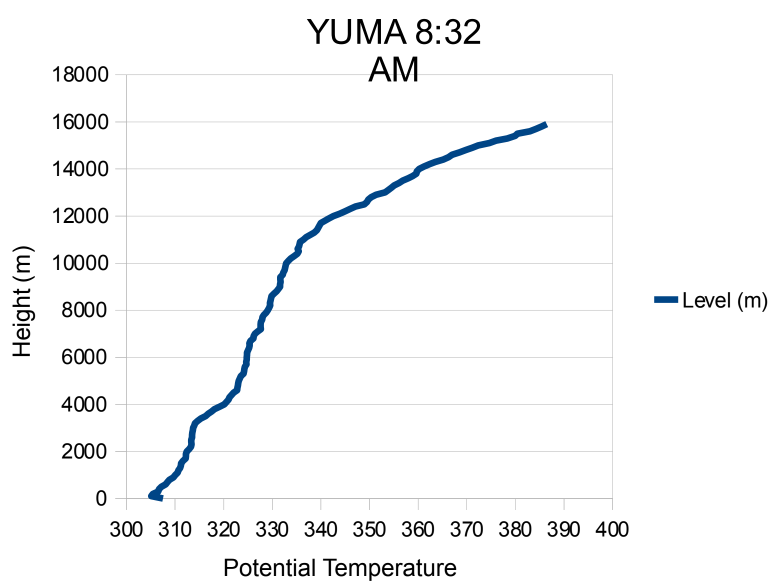



*Figure 5: Yuma desert temperature profile at 8:32 AM (June 18, 2007)*

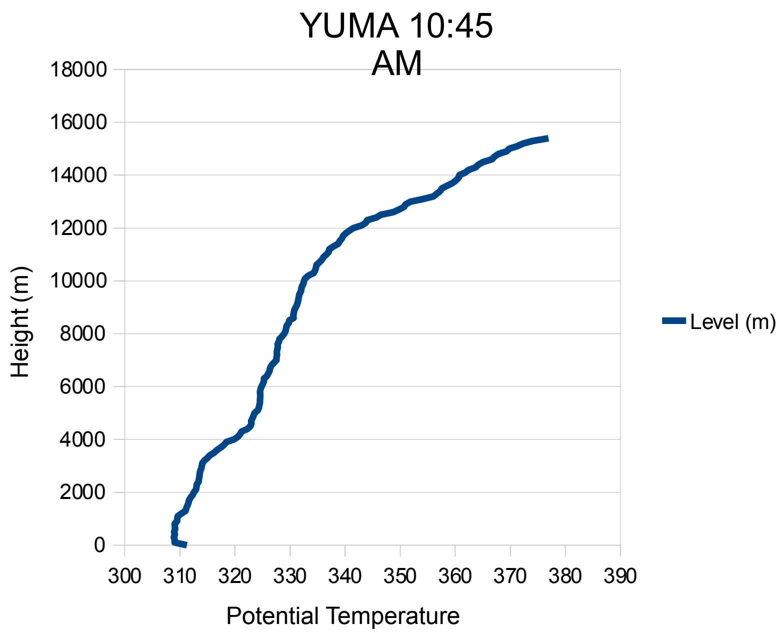



*Figure 6: Yuma desert temperature profile at 10:45 AM (June 18, 2007)*

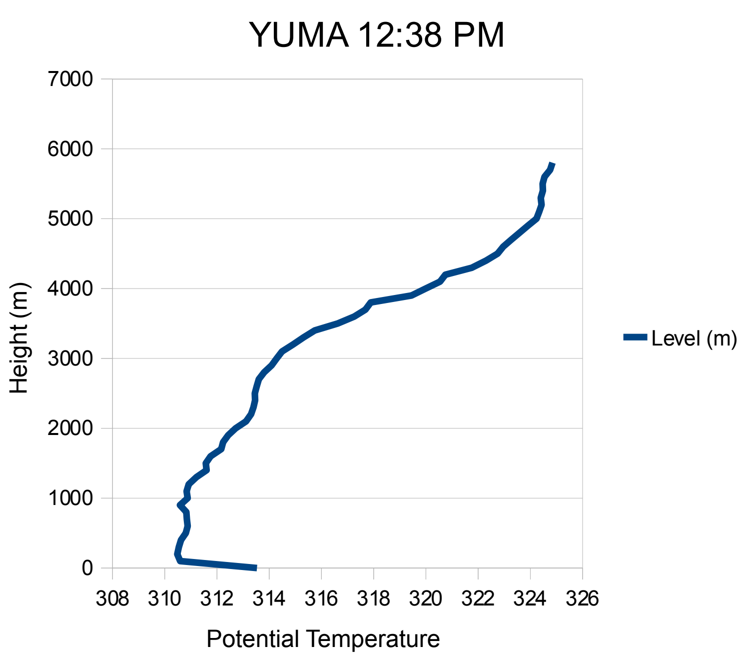



*Figure 7: Yuma desert temperature profile at 12:38 PM (June 18, 2007)*

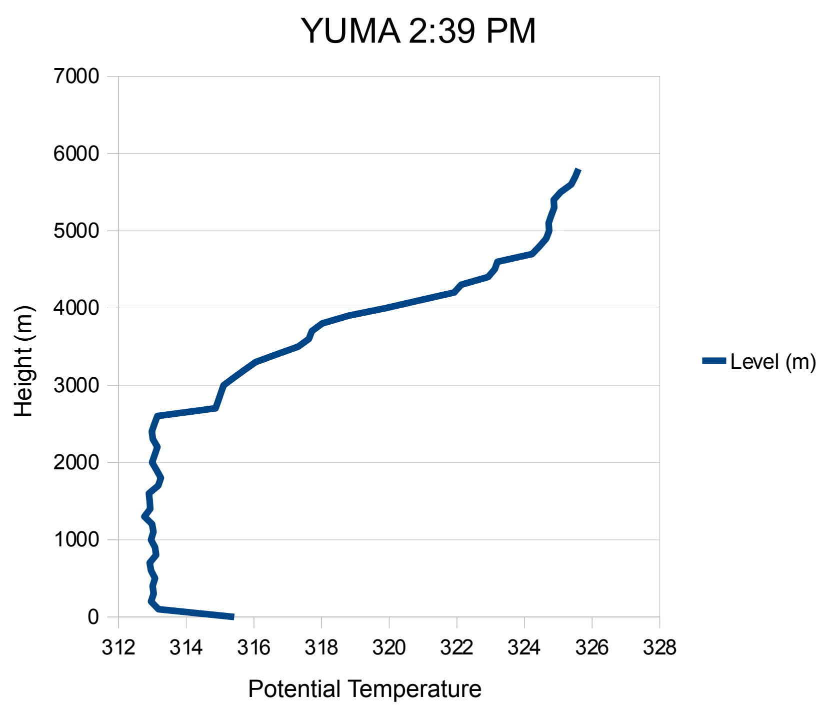



*Figure 8: Yuma desert temperature profile at 2:39 PM (June 18, 2007)*

From the Yuma desert temperature profiles above, it's evident how the the temperature profile in the morning is stable. This is because the sun is not yet out and the temperature is constant and no convection takes place (see Fig. 3). Later on just before 9 am, the sun comes out and heats the ground and triggers a little bit of convection and the convection layer starts to form as illustrated by Fig. 5 and 6. In the afternoon, the intensity of the sun is strongest and we see a well more pronounced convection layer where the potential temperature is fairly constant as you go higher in altitude for a about three kilometer (see Fig. 8). Afterwards, the solar intensity weakens and the convective layer gets less pronounced and assumes a stable profile.

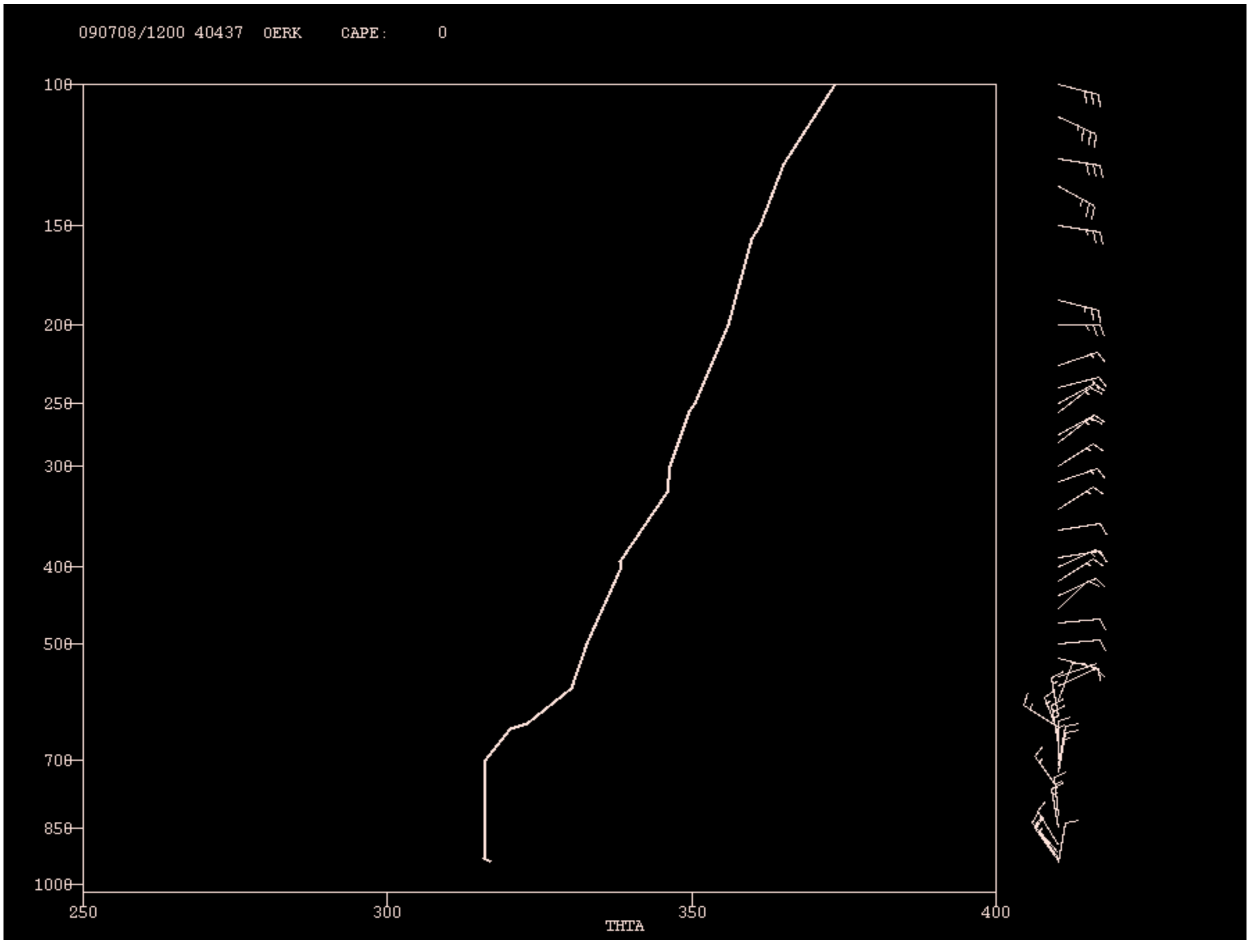


*Figure 9: Vertical potential temperature profile of a Saudi Arabia region at 3 pm.*

The above temperature profile is from Saudi Arabia during a dry hot summer at 3 pm. The convection layer is clearly evident until the 700 mb pressure level where the potential temperature is uniform. Afterwards, it gets stable as we see the change of potential temperature with height is positive. Whenever the profile is unstable, convection corrects that. The bottom tip of the above plot implies instability around the 900 mb pressure level. Convection happens and the convection layer is formed and finally we achieve stability in the system.

Fig. 10 below is another temperature profile from Saudi Arabia at 3 am. This happens early in the morning when the sun is not out and the profile is stable as no convection occurs. The bottom end of the curve actually shows how the profile is very stable near the ground and thus no convection can occur.

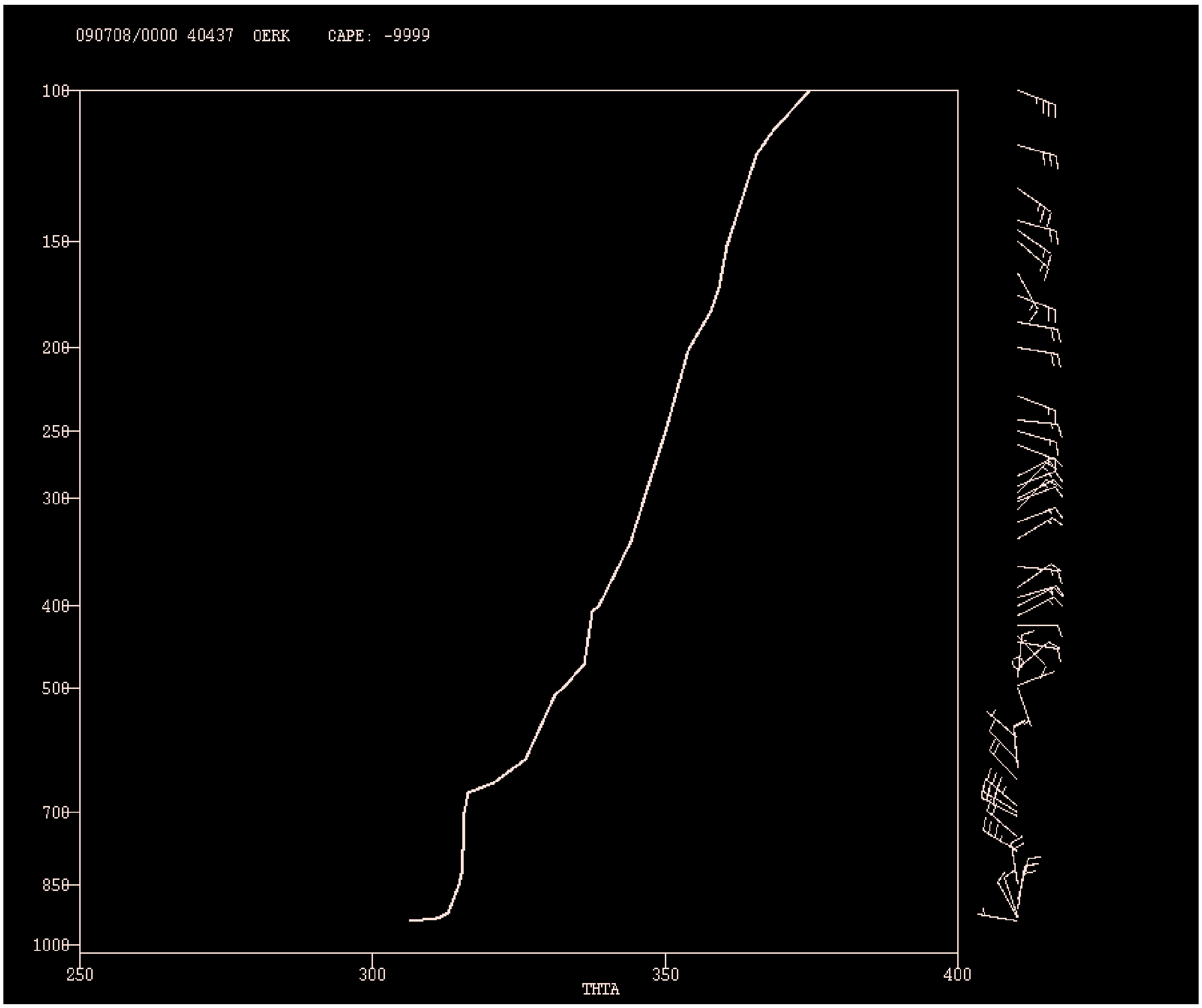


*Figure 10: Vertical potential temperature profile of a Saudi Arabia region at 3 am.*

## 6. Conclusion

Seeing how the temperature profiles evolve under the effect of dry convection, a conclusion can be made that the tank experiment was a very good analogy as the behavior in the tank was similar. Fig. 11 below shows how the tank experiment relates to the atmospheric examples.

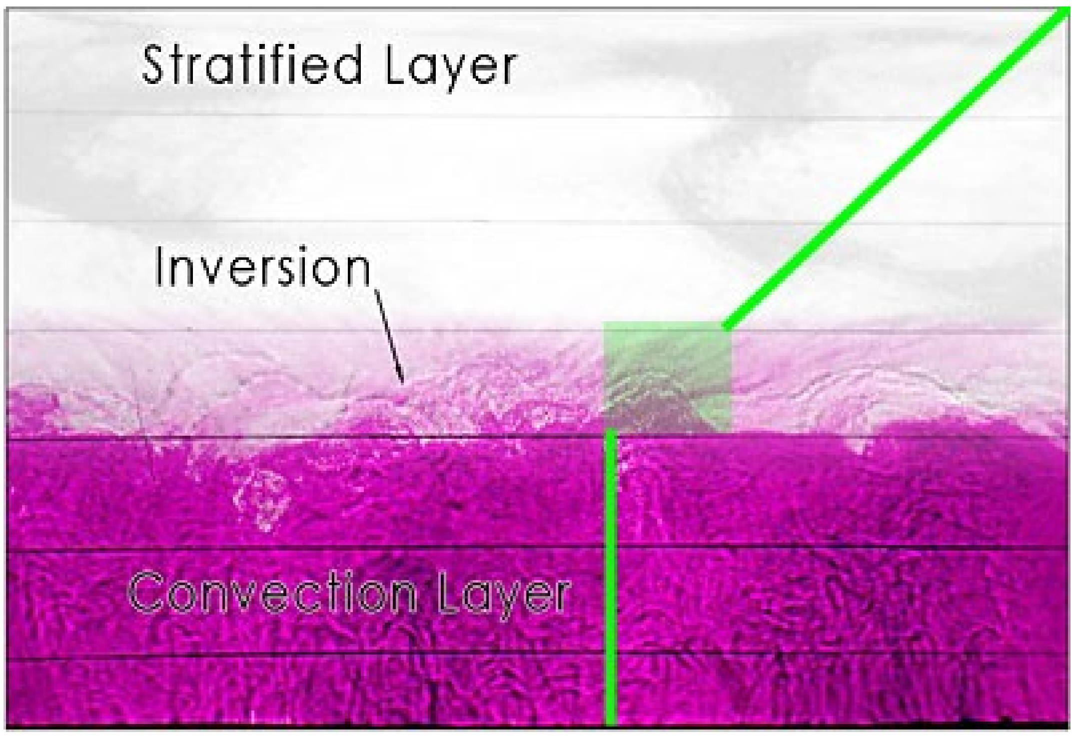



*Figure 11: Tank experiment result showing similarities as atmospheric examples*

### References


1. Lodovica Illari and John Marshall. 12.307 project 3: Convection, 2010.
2. John Marshall, Lodovica Illari and Alan Plumb. Notes on project 3: Convection in air, 2004
3. John Marshall, Lodovica Illari and Alan Plumb. Notes on project 3: Convection in water, 2004